Steady State Optimal Power Flow

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Abstract : An optimal load-flow solution gives the optimal active and reactive power dispatch for a static power system loading condition. Computationally, it is a very demanding nonlinear programming problem, due to the large number of variables and in particular to the much larger number of variables and in particular to the much larger number and types of limit constraints which define the boundaries of technical feasibility This paper solves the standard 26 bus power system network.

Keywords : optimal power flow; OPF; Lamda iteration

I. Introduction

One of most important and frequently used analysis and computational procedures for the planning, design, and operation of an electric power system is the ac power-flow program. This program constitutes a simulation of the steady-state ac power flows and voltages in the network under study. It is used to simulate the flows and voltages corresponding to several future load conditions for various design alternatives. If the system-design alternative under consideration is not capable of supplying the assumed loads then the set of values for some of the control variables such as voltage levels or power productions; capacitors are adjusted. Much of this time-consuming trial-and error process is reduced by the optimum power flow, and provide a set of feasible values of the control variables. Optimum power-flow solutions may be used not only for system planning but also for system operation, which is a real-time function. In operation, it provides the most economical operating point that meets all the flow and voltage constraints related to power-system security and quality of service. The standard 26 bus test system which has 6 generators, 7 transformers and 9 shunt capacitor is considered in this work.

II. Problem formulation

The steady state Optimal Power Flow problem is a minimization problem which is stated as follows. The objective is to minimize the generation cost.

Min $C_t = \sum_{i=1}^{ng} \alpha_i + \beta_i P_i + \gamma_i P_i^2$ (1)

Where, C_t - Total cost function, α, β, γ are cost coefficients, P_i is the ith bus real power generation

Subject To

$$\sum_{i=1}^{ng} P_i = P_D + P_L \qquad (2) \qquad \sum_{i=1}^{ng} Q_i = Q_D + Q_L \qquad (3)$$

$$V_{i(min)} \le V_i \le V_{i(max)} \quad \text{for i=1 to Nbus} \quad (4) \qquad P_{i(min)} \le P_i \le P_{i(max)} \qquad \text{for i=1 to ng} \qquad (5)$$

$$Q_{i(min)} \le Q_i \le Q_{i(max)} \text{ for i=1 to ng} \qquad (6) \quad t_{i(min)} \le t_i \le t_{i(max)} \qquad \text{for i=1 to Ntrans} \qquad (7)$$

Where, P_i, Q_i – Real and Reactive Power generation of ith bus, P_D, Q_D – Real and Reactive Power demand in i^{th} bus

P_L, Q_L – Real and Reactive Power loss ,V_i – Voltage magnitude of the ith bus ,t_i – Transformer tap position of the ith transformer, Nbus – Number of bus, ng – number of generator Ntrans – number of transformer

Lagrange Function $\quad L = C_t + \lambda(P_D + P_L - \sum_{i=1}^{ng} P_i) + \sum_{i=1}^{ng} \mu_{i(max)}(P_i - P_{i(max)}) + \sum_{i=1}^{ng} \mu_{i(min)}(P_i - P_{i(min)})$ (8)

Where, λ – Lagrange multiplier, μ_i – Khun-tucker function multiplier

The minimum of this unconstrained function is found at the point where partials of the function to its variables are zero

$$\frac{\partial L}{\partial P_i} = 0 \qquad (9) \qquad\qquad \frac{\partial L}{\partial \lambda} = 0 \qquad (10)$$

$$\frac{\partial L}{\partial \mu_{i(max)}} = 0 \qquad (11) \qquad\qquad \frac{\partial L}{\partial \mu_{i(min)}} = 0 \qquad (12)$$

$\mu_{i(max)}$ and $\mu_{i(min)}$ are zero when Pi is within its limit

From Equation – (9),

$\dfrac{\partial C_t}{\partial P_i} + \lambda(\dfrac{\partial C_t}{\partial P_i} - 1) = 0$ We know that $\dfrac{\partial C_t}{\partial P_i} = \dfrac{\partial C_i}{\partial P_i}$ since Ct = C1+   C2+ ….. Cng

Therefore $\dfrac{\partial C_i}{\partial P_i} + \lambda \dfrac{\partial P_L}{\partial P_i} = \lambda$ for i=1 to ng   (13)

$\dfrac{\partial C_i}{\partial P_i} = \lambda - \lambda \dfrac{\partial P_L}{\partial P_i}$ $\dfrac{\partial C_i}{\partial P_i} = \lambda(1 - \dfrac{\partial P_L}{\partial P_i})$ $\dfrac{1}{(1 - \dfrac{\partial P_L}{\partial P_i})}.(\dfrac{\partial C_i}{\partial P_i}) = \lambda$ $\lambda = \dfrac{\partial C_i}{\partial P_i}.L_i$ (14)

Where, Penalty Factor, $L_i = \dfrac{1}{(1 - \dfrac{\partial P_L}{\partial P_i})}$

$$\frac{\partial L}{\partial P_i} = 2\sum_{j=1}^{ng} B_{ij}P_j + B_{0i} \tag{15}$$

From equation 13,

$$\beta_i + 2\gamma_i P_i + 2\lambda \sum_{j=1}^{ng} B_{ij}P_j + B_{0i}\lambda = \lambda$$

$$(\frac{\gamma_i}{\lambda} + B_{ii})P_i + \sum_{\substack{j=1 \\ j \neq i}}^{ng} B_{ij}P_j = \frac{1}{2}(1 - B_{0i} - \frac{\beta_i}{\lambda}) \tag{16}$$

This equation is extended to all generating plants results in following linear equation in matrix form

$$\begin{bmatrix} \frac{\gamma_1}{\lambda} + B_{11} & B_{12} & . & . & B_{1ng} \\ B_{11} & \frac{\gamma_2}{\lambda} + B_{22} & B_{23} & . & B_{2ng} \\ . & & . & & . \\ . & & & . & . \\ B_{ng1} & B_{ng2} & . & . & \frac{\gamma_{ng}}{\lambda} + B_{ngng} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ . \\ . \\ P_{ng} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ . \\ . \\ 1 - B_{0ng} - \frac{\beta_{ng}}{\lambda} \end{bmatrix} \tag{17}$$

To find the optimal dispatch for an estimated λ the simultaneous linear equation 17 is solved to find P matrix. Then the iteration process is continued using the gradient method. To do this, from equation 16, Pi at the kth iteration is

$$P_i^k = \frac{\lambda^k(1 - B_{0i}) - \beta_i - 2\lambda^k \sum_{j \neq i} B_{ij}P_j^k}{2(\gamma_i + \lambda^k B_{ii})} \tag{18}$$

Substitute equation 19 in equation 10,

$$\sum_{i=1}^{ng} \frac{\lambda^k(1 - B_{0i}) - \beta_i - 2\lambda^k \sum_{j \neq i} B_{ij}P_j^k}{2(\gamma_i + \lambda^k B_{ii})} = P_D + P_L \tag{19}$$

This can be written as $\quad f(\lambda) = P_D + P_{Lk}$ \hfill (20)

Expand the equation 21 L.H.S. using Taylor's series about an operating point λk and neglecting the higher order terms

$$f(\lambda^k) + \frac{\partial f(\lambda^k)}{\partial \lambda} \Delta\lambda^k = P_D + P_L^k \tag{21}$$

$$\Delta\lambda^k = \frac{\Delta P^k}{\frac{\partial f(\lambda^k)}{\partial \lambda}} \qquad \Delta\lambda^k = \frac{\Delta P^k}{\sum (\frac{\partial P_i}{\partial \lambda})^k} \tag{22}$$

Where $\quad \sum_{i=1}^{ng} (\frac{\partial P_i}{\partial \lambda})^k = \sum_{i=1}^{ng} \dfrac{\gamma_i(1 - B_{0i}) - B_{ii}\beta_i - 2\gamma_i \sum_{j \neq i} B_{ij}P_j^k}{2(\gamma_i + \lambda^k B_{ii})^2}$ \hfill (23)

$$\lambda^{k+1} = \lambda^k + \Delta\lambda^k \qquad (24) \qquad \Delta P^k = P_D + P_L^k - \sum_{i=1}^{ng} P_i^k \qquad (25)$$

The process is continues until ΔP_k is less than a specified accuracy

III. Test System

26 bus power system network is considered. Bus 1 is taken as slack bus its voltage adjusted to 1.025 angle 0° and $P_{1Max} = 500Mw$, $P_{1Min}=100$ Mw, the data for the other generator buses are

Bus No	Voltage Mag. (pu)	Q_{Min} (Mvar)	Q_{Max} (Mvar)	P_{Min} (Mw)	P_{Max} (Mw)
2	1.02	40	250	50	200
3	1.02	40	150	80	300
4	1.05	40	80	50	150
5	1.04	40	160	50	200
26	1.01	15	50	50	120

Transformer Data

Between Buses	Tap setting (pu)
2 -3	0.96
2-13	0.96
3-13	1.017
4-8	1.05
4-12	1.05
6-19	0.95
7-19	0.95

Shunt Capacitive Data

Buses No	Mvar
1	4.0
4	2.0
5	5.0
6	2.0
9	3.0
11	1.5
12	2.0
15	0.5
19	5.0

Generators operating costs in $/h

$C_1 = 240 + 7.0 P_1 + 0.007 \ P_1^2$
$C_2 = 200 + 10 \ P_2 + 0.0095 P_2^2$
$C_3 = 220 + 8.5 P_3 + 0.009 \ P_3^2$
$C_4 = 200 + 11 \ P_4 + 0.009 \ P_4^2$
$C_5 = 220 + 10.5 P_5 + 0.008 P_5^2$
$C_{26} = 190 + 12 P_{26} + 0.0075 P_{26}^2$

IV. Test Result

Test result for the 26 bus power system is summarized in the following table

Description	Base Case Power Flow	Optimal Power Flow	Savings
Total Generation Cost ($/h)	16760.73	15447.72	1313.01
Total system loss (Mw)	15.53	12.807	2.723
System Power Generation by the generators (Mw)	474.1196 173.7886 190.9515 150.0000 196.7196 103.5772	447.6919 173.1938 263.4859 138.8142 165.5884 87.0260	
Incremental cost of delivered power (λ) ($/MWh)	13.911780	13.538113	

V. Conclusion

The optimum power flow has been defined and its advantages over the ordinary power flow have been shown to be greatly reduced trial and error. It has been shown that Newton's method of power flow solution can be extended to yield an optimal power flow solution that is feasible with respect to all relevant inequality constraints. The main features of the method are a gradient procedure for finding the optimum and the use of penalty functions to handle functional inequality constraints.

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