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Analysis of PI controller by model based tuning method in Real Time Level Control of Conical Tank System using block box modeling

K.Indhumathi[1*], D.Angeline Vijula[2], S.Gurunagappan[3]

[1,3]Cheran College of Engineering, India
[2]PSG Institute of Technology, Coimbatore, India

Abstract : Conical tank Systems find wide applications in process industries because it prevents the accumulation of solid at the bottom of the tank. Control of liquid level in a conical tank is nonlinear due to the variation in the area of cross section of the tank system with its change in shape. In this paper the model of the process is identified using block box modeling and approximated to be a first order plus dead time (FOPTD) model. Also Non linear conical tank is linearized into four linearized zones based on the variation in area of CTS using piecewise linearization method. The Proportional plus Integral (PI) controller is commonly used to control the level in process industries because if its simplicity of implementation. Tuning of the PI controller is setting the proportional (K_p) and integral constant (K_I). In this paper PI controller is analyzed for Real Time conical tank system (CTS) and tuned by Ziegler Nichols method(Z-N method), Internal Model Control Tuning(IMC) and Model Reference Adaptive Control(MRAC) tuning method. The PI controller is simulated using MATLAB/SIMULINK environment and the tuning methods are implemented in real time for controlling the level in CTS.
Keywords : Conical tank system, Block box modeling, Piecewise Linearization, Ziegler Nichols tuned PI,Internal Model Control tuned PI, Model Reference Adaptive Control.

1 Introduction

Control of industrial processes is a challenging task for several reasons due to their nonlinear dynamic behavior, uncertain and time varying parameters, constraints on manipulated variable, interaction between manipulated and controlled variables, unmeasured and frequent disturbances, dead time on input and measurements. The control of liquid level in tanks is a basic problem in process industries. In many processes such as distillation columns, evaporators, reboliers and mixing tanks, the particular level of liquid in the vessel is of great importance in process operation. The conical tank system is widely used in many process industries like Petrochemical industries, Cement manufacturing industries, Food processing industries, Waste water treatment because it contributes better drainage of the liquids solid mixture, slurries. The level control of the conical tank is difficult because of its nonlinearity and constantly varying cross section with respect to height. Here the conical tank which has nonlinear characteristics is represented as piecewise linearized models. The PI and PID controllers are widely used in many industrial control systems because of its simple structure and robustness. Tuning of the PI controller is setting the proportional, integral constant. The most common classical controller tuning methods are the Ziegler Nichols (Z-N) and Cohen-Coon methods. Since it is easier to use than other methods. Internal model control (IMC) tuning offers an alternative tuning to increase the controller's performance. Model Reference Adaptive tuning differs from previous tuning methods. MRAC uses Non linear model. Based on reference model selected the controller adapts its parameter to obtain the desired performances.

2 Literature Review

Many research works have been carried out in the level control of the conical tank process. S.Anand et al., [2], explained about the Adaptive PI Controller which eliminates the oscillation at low level and provides a more consistent response. N.S. Bhuvaneswari et al., [5] carried out experiments in conical tank level control using Neural Network controllers. D.Marishiana et al.,[9] designed the Ziegler Nichols tuning controller and comparison is made between the values of P, PI, PID. Direct Synthesis Proportional Integral (DSPI) and Model Predictive Control (MPC) were designed by H.Kala et al.,[7]. P.Sowmyl et al., [15] has designed the fuzzy logic controller and the model of the process is identified using standard step response based system identification technique and it is approximated to be a first order plus dead time (FOPTD) model.D.AngelineVijula et al.,[16] has simulated the MRAC tuned PI in MATLAB. IMC tuned PI is implemented and proved as better tuning method for Real time conical tank.

In this work CTS is modeled by block box modeling and PI controller is tuned by Z-N tuning, IMC tuning, MRAC tuning. Controller is simulated in MATLAB and implemented to control Level of Conical tank. Real time Performances of tuning methods are analyzed and presented in this paper.

The section two deals with the hardware description of the conical tank system. The section three explains the modeling of the system. The section four provides the PI controller design. Z-N tuning and IMC tuning is explained. The section five presents the results and discussion for both simulation and real time implementation.

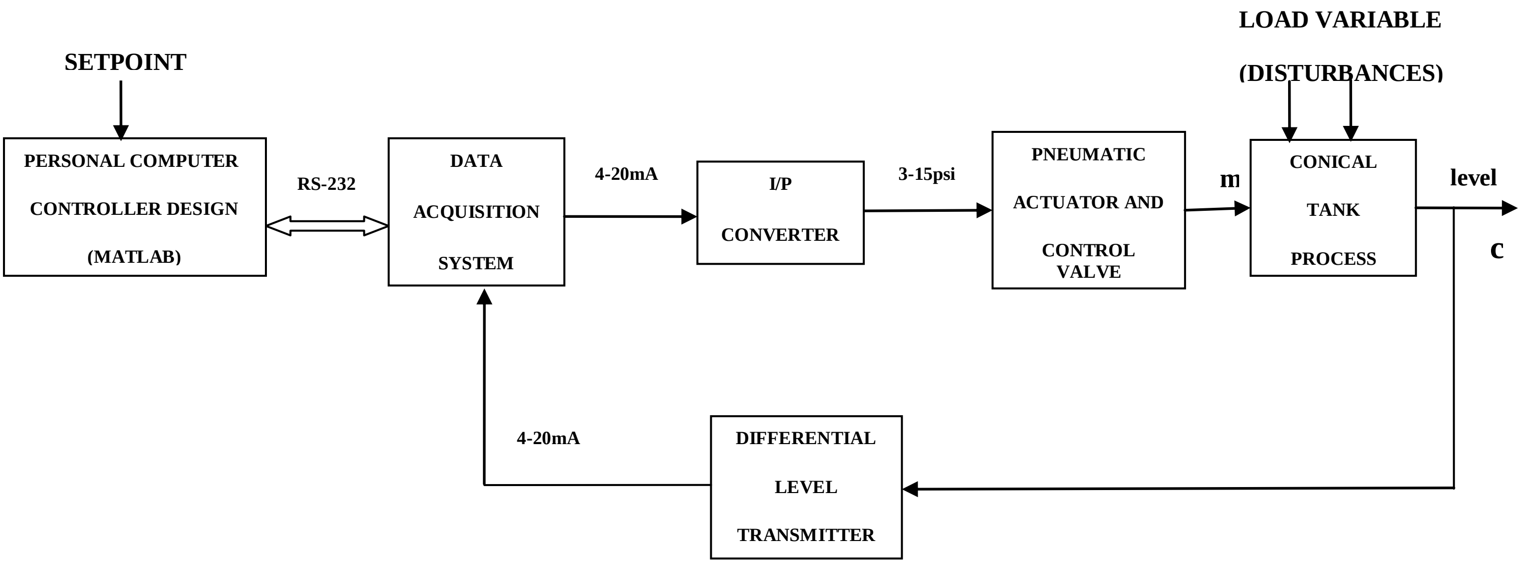

Figure 1: Block diagram of CTS

2 Hardware Description

The conical tank is made up of stainless steel and is mounted vertically on the stand. The water enters into the tank from the top and leaves to the reservoir, which is placed at the bottom of the tank. The Block diagram of CTS is shown in Fig.1. The level of the water in the conical tank is quantified by means of the DPT. The quantified level of water in the form of current in the range of (4-20) mA is sent to the DAQ in which ADC converts the analog data to digital data and feed it to the PC. The PC acts as the controller and data logger. The controller considers the process variable as feedback signal and finds the manipulated variable as the output based on the predefined set point. The DAC module of the DAQ converts this manipulated variable to analog form into 4-20 mA current signals.The I/P converter converts the current signal to pressure in the range of (3-15) psi, which regulates the flow of water into the conical tank based on the outflow rate of the tank. Block diagram of the CTS is shown in figure 1.

The conical tank system which exhibits the property of non-linearity is considered here. The photograph of the conical tank system is shown in the Figure 2 with its components.

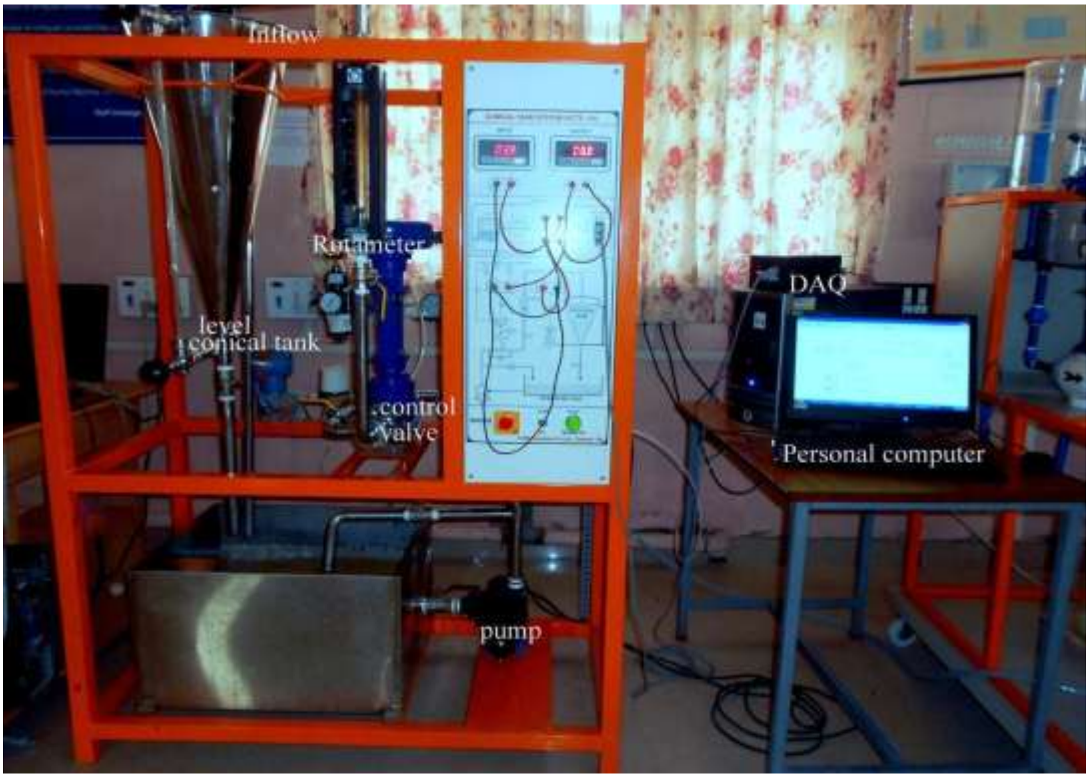

Figure 2: Setup of conical tank system

3 Modeling of Conical Tank Process

Modeling is the development of mathematical equations, constraints and logical rules of real world process. In this paper three types ofmodeling approaches are carried on. They are as follows,

1. Analytical Modeling – where the entire system is represented as mathematical equations.
2. Black Box Modeling - CTS is modeled by experimentation.
3. Model using Piecewise Linearization - Four linearized models are obtained using piecewise linearization.

3.1 Analytical Model

The cross section ofthe CTS is shown in Figure 3.

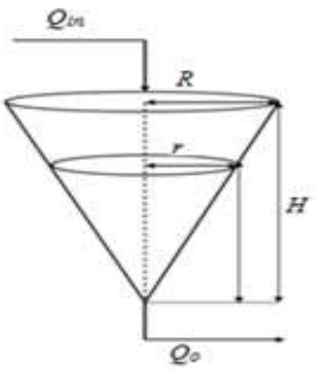

Figure 3: Conical tank

Where,

Q_{in} - inflow rate in lph
Q_o- outflow rate in lph
R- Maximum radius of the tank in cm
H- Maximum height of the tank in cm
r- Radius of the tank tank at steady state in cm
h- Height of the tank tank at steady state in cm
According to mass balance equation
Accumulation =Input-Output

$$\frac{dv}{dt} = Q_{in} - Q_0 \qquad (1)$$

The volume of the conical tank can be expressed as equation (2)

$$A\frac{dh}{dt} = Q_{in} - Q_0 \qquad (2)$$

Where A is the Area of the Tank

$$A = \pi r^2 \qquad (3)$$

So the height of the tank is,

$$\frac{dh}{dt} = \alpha Q_{in} h^{-2} - \beta h^{\frac{-3}{2}} \qquad (4)$$

Where $\alpha = \frac{1}{\pi}\left(\frac{H}{R}\right)^2$ (5)

$$\beta = k_v \alpha \qquad (6)$$

Above equation (4) is Non linear form,
Linearisation is done using Taylor series method
Linearisation of $Q_{in}h^{-2}$ is,

$$f(h,Q) = f(h_s - Q_s) + \frac{\partial f(h-h_s)}{\partial h} + \frac{\partial f(Q-Q_s)}{\partial Q} \qquad (7)$$

Linearisation of $\beta h^{\frac{-3}{2}}$ is,

$$h^{\frac{-3}{2}} = h_s^{\frac{-3}{2}} - \frac{3}{2} h_s^{\frac{-5}{2}}(h - h_s) \qquad (8)$$

Now applying steady state values *y*= *(h-h$_s$)* and *u*= *(Q-Q$_s$)*
The approximate linear model obtained as,

$$\tau\frac{dy}{dt} + y = kU \qquad (9)$$

$$\tau = \frac{2h_s^{\frac{5}{2}}}{3\beta} \qquad (10)$$

$$k = \frac{2\alpha h_s^{\frac{5}{2}}}{\beta} \qquad (11)$$

Above equation implies that the conical tank system is First Order System.

The transfer function obtained for 20cm is expressed in equation (12)

$$G(s) = \frac{0.578}{62.5s + 1} \qquad (12)$$

Using the above Transfer function set point will not be reached exactly. Offset error will be present in the system. So, instead of analytical modeling Block box modeling is done.

3.2 Block Box Modelling

Block box modelling is used to obtain the parameters of the transfer function of the FOPDT model by letting the response of the actual system and that of the model to meet at two points, which describe the two parameters τ and t_d.

The loop is made open and a step increment (230 lph) is given in inflow rate and the readings are noted till the system reaches the steady state value. The experimental data are approximated to a First Order plus Dead Time (FOPDT) model to obtain the open loop parameters of the conical tank process. Theopenloop response is shown in Figure 4.

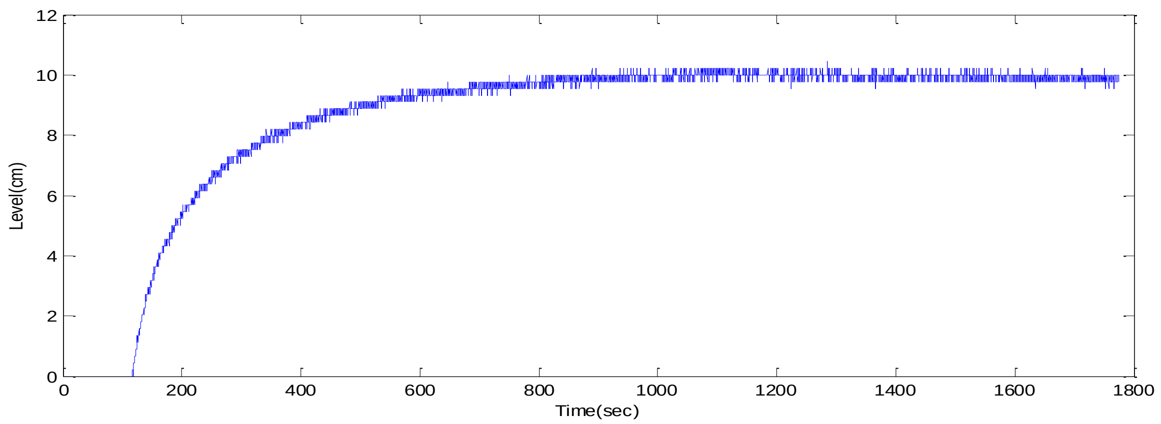

Figure 4: Open loop response of conical tank system

The transfer function is approximated as FOPDT using open loop response and expressed by equation (13).

$$G(s) = \frac{0.04e^{-98s}}{157s+1} \quad (13)$$

3.3 Model using Piecewise Linearisation

Linearization is a term used in general for the process by which a nonlinear system is approximated as a linear process model. In practice, all physical processes exhibit some non-linear behavior. The conical tank exhibits nonlinearity by constantly varying its cross section with respect to height. For fixed input water flow rate and output water flow rate of the conical tank, the tank is allowed to fill with water from (0-45) cm. Using the open loop method, for a given change in the input variable the corresponding steady states of the system is recorded and the transfer function is obtained for different zones. Thus the nonlinear model of CTS is linearized into four linear models. The flow rate and the corresponding zones are shown in the Table 1.

Table 1: Piecewise Linearization

Zone	Flow rate(lph)	Time taken(sec)	Steady state level(cm)
1	230	1750	10.01
2	280	5050	19.79
3	320	10170	30.48
4	350	16000	38.21

The Response for piecewise linearization shown in the Figure 5.

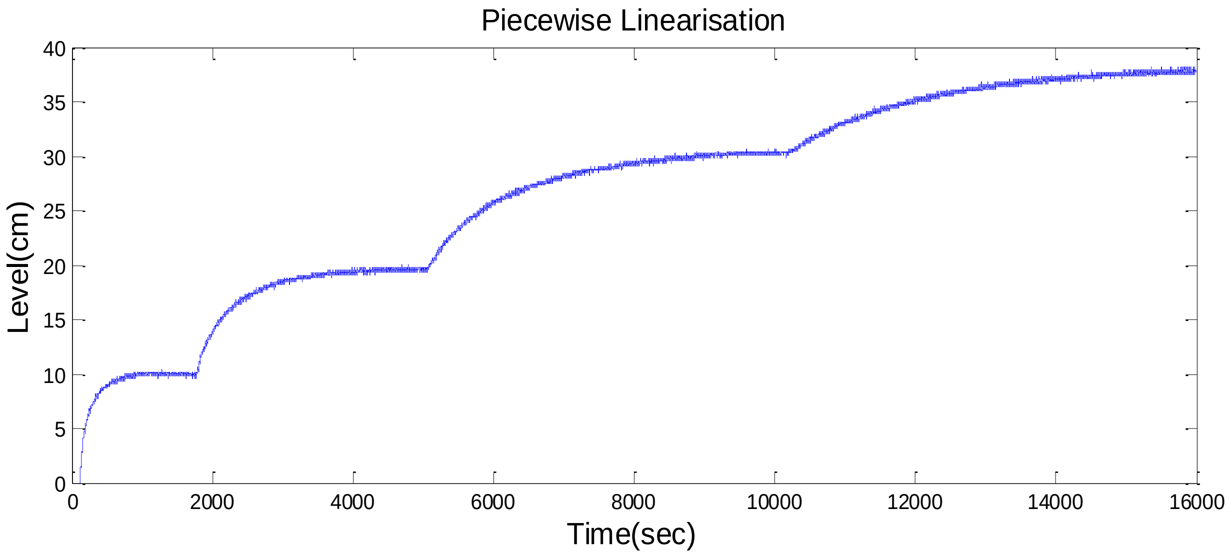

Figure 5: Open loop response of Piecewise linearization

A step increment (230lph) is given as an input to the CTS.It reaches the steady state value at 10cm.So the height (0-10cm) is considered as zone I. The inflow is incremented to 280lph it reaches the steady state at 19.79cm.The height (10.01-19.79cm) is considered as zone II. Similarly it is repeated for 320,350lph and zone III, IV is considered. Once it reaches the time after settling then it goes to the second region.

The transfer functions obtained for the four zones are shown in the Table 2

Table.2 Linearized model for different levels

Level(cm)	Linearized first order Model	
0-10	Zone I	$G_1(s)=\dfrac{0.04e^{-98S}}{157s+1}$
10-20	Zone II	$G_2(s)=\dfrac{0.0706e^{-83S}}{544s+1}$
20-30	Zone III	$G_3(s)=\dfrac{0.0952e^{-75S}}{1195s+1}$
30-40	Zone IV	$G_4(s)=\dfrac{0.109e^{-70S}}{2100s+1}$

4 Controller Design

The PI controller consists of proportional and integral term. The proportional term changes the controller output proportional to the current error value. Large values of proportional term make the system unstable. The Integral term changes the controller output based on the past values of error. So, the controller attempts to minimize the error by adjusting the controller output. The most common classical controller tuning methods are the Z-N and Cohen-Coon methods. The Z-N method can be used for both closed and open loop systems, while Cohen-Coon is typically used for open loop systems.

4.1 Ziegler Nichols Tuning

Z-N open loop tuning formula for PI controller is given in the equations

$$K_p = \frac{0.09\tau}{kt_d} \quad (18)$$

$$K_I = \frac{Kp}{\tau_I} \quad (19)$$

The calculated PI gain parameters can be given as,
K_p= 36.04
K_I= 0.11

4.2 Internal Model Control Tuning

Internal model control tuning also referred as Lambda tuning method offers a robust alternative tuning aiming for speed. Lambda tuning is a form of internal model control (IMC) that endows a PI controller with the ability to generate smooth, non-oscillatory control efforts when responding to changes in the set point.The IMC based tuning parameters for PI controller can be obtained by determining the controller equation. Otherwise directly the parameters can be

calculated by using the formulae

$$K_p = \frac{\tau}{(\lambda + t_d)k} \quad (20)$$

$$K_I = \frac{K_p}{\tau_I} \quad (21)$$

Assuming λ =5 sec,
The calculated PI gain parameters can be given as
K_p= 38.106
K_I= 0.242

3.3 Model Reference Adaptive Control Tuning

A tuning system of an adaptive control will sense these parametric variations and tune the controller parameters in order to compensate forit. The parametric variation may be due to the inherent non-linearity of the system such as conical tank. In a conical tank the cross section area varies as a function of level which in turn leads to parametric variations. The time constant and gain of the chosen process vary as a function of level.

In MRAC tuning a reference model describes the system’s performance. The adaptive controller is then designed to force the system (or plant) to behave like the reference model. Model output is compared to the actual output and the difference is used to adjust feedback controller parameters.

MRAC has two loops: an inner loop (or regulator loop) that is an ordinary control loop consisting of the plant and the regulator, and an outer (or adaptation) loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero. Block diagram of MRAC is shown in Figure 6.

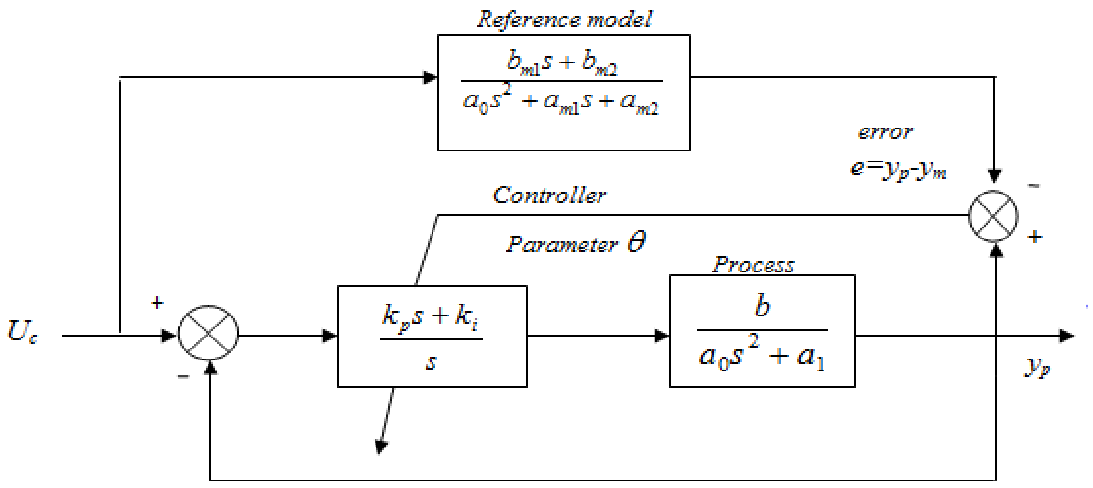

Figure 6. Block diagram of MRAC tuned PI controller

Tracking error is,

$$e = y_p - y_m \quad (16)$$

The cost function is,

$$J(\theta) = \frac{1}{2}\left(e^2\theta\right) \quad (17)$$

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta} \quad (18)$$

where 'e' denotes the model error and 'θ' is the controller parameter vector.'γ' denotes the adaptation gain. Instead of 'θ' the PI controller parameters K_P,K_I are considered.

So the $K_p$is,

$$k_p = \frac{1}{s}\left(-\gamma_p e\left(\frac{bs}{a_0s^2+(a_1+bk_p)s+bk_i}\right)(U_c-Y_p)\right) \quad (19)$$

Similarly K_I Parameter is,

$$k_i = \frac{1}{s}\left(-\gamma_i e\left(\frac{b}{a_0s^2+(a_1+bk_p)s+bk_i}\right)(U_c-Y_p)\right) \quad (20)$$

5 Results and Discussions

5.1 Simulation Results of Block box model (without linearization)

The PI Controller is tuned using ZN, IMC, MRAC tuning and are simulated with block box model of CTS in MATLAB/SIMULINK environment and the responses are obtained & presented in this section. Figure 7 presents the response of ZN tuned PI for nonlinear model of CTS. It is observed that controller takes more time to reach the desired level. Also disturbance is not rejected by controller which is given at 2500 sec.

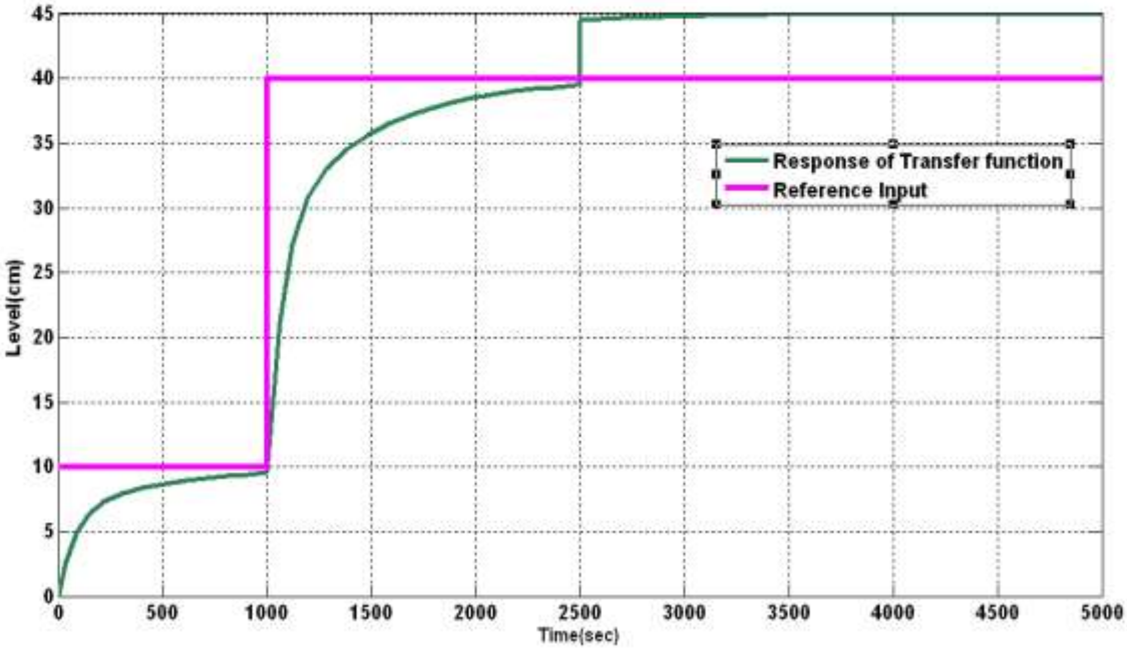

Figure 7: Response of Z-N tuned PI controller without linearization

The lambda tuning rules also called as IMCtuning, offers an alternative tuning aiming for speed. The tuning is very robust meaning that the closed loop will remain stable even if the process characteristics change dramatically.As shown in Figure 8, set point is reached with minimum time in other operating zone. Even if the disturbance given controller rejects it and tank level is maintained at setpoint.

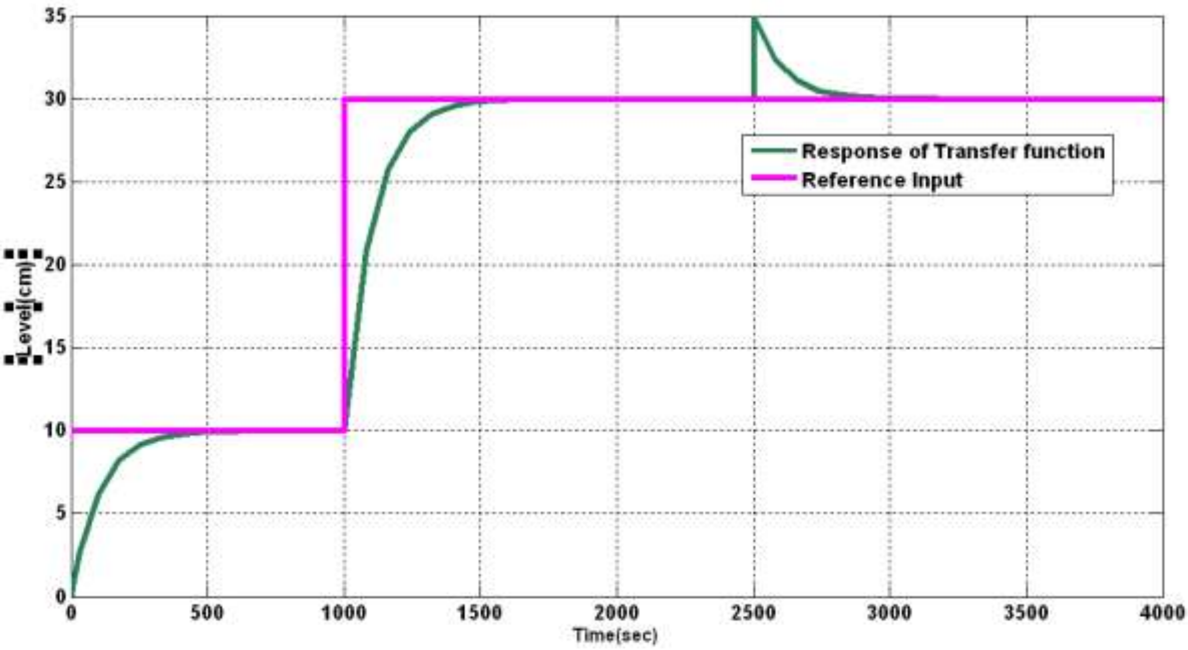

Figure 8: Response of IMC tuned PI controller without linearization

Figure 9 shows Response of MRAC Tuned PI. It is shown that desired level is reached in minimum time. But peak overshoot presented in MRAC tuning method.

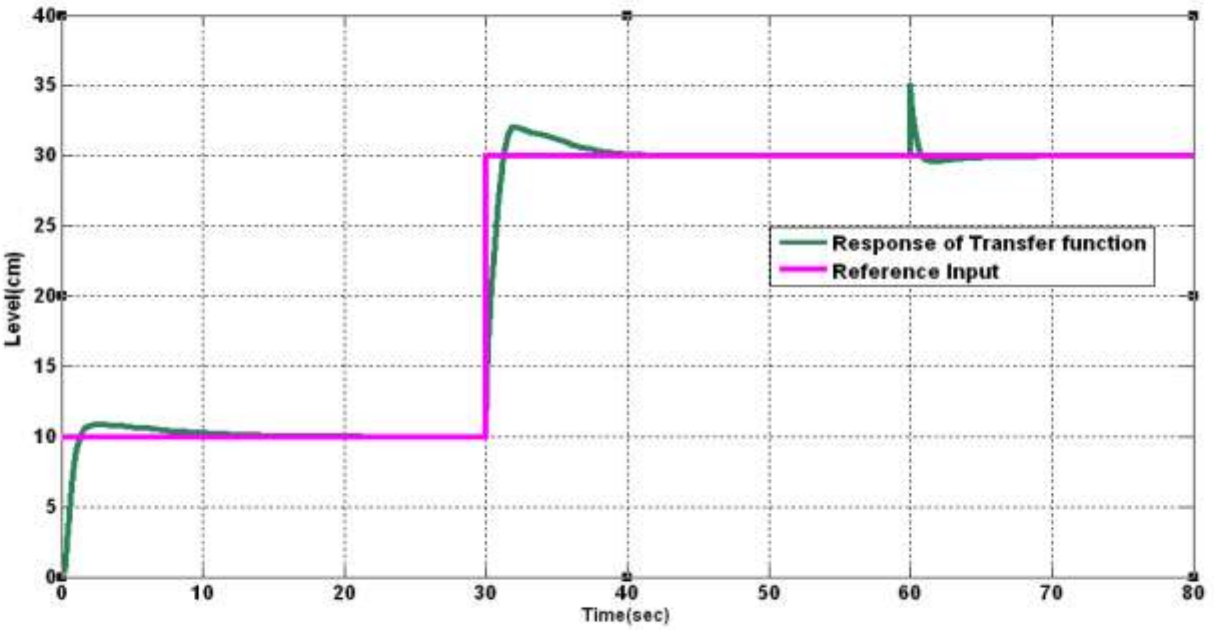

Figure 9.Response of MRAC tuned PI

5.2 Simulation Results of Piecewise models (Linearised Model)

The PI parameters obtained for the four linearised models using Z-N tuning and IMC tuning. The simulated responses are shown below.

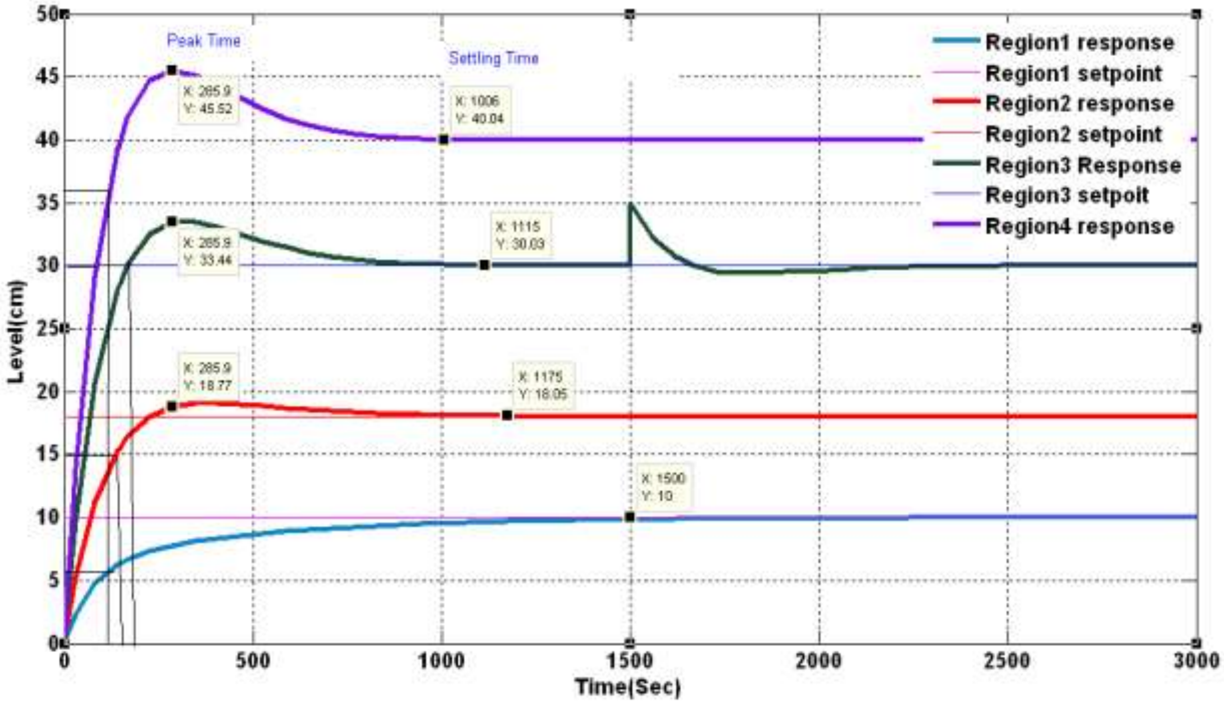

Figure 10:Response of Z-N tuned PI controller with piecewise linearised models

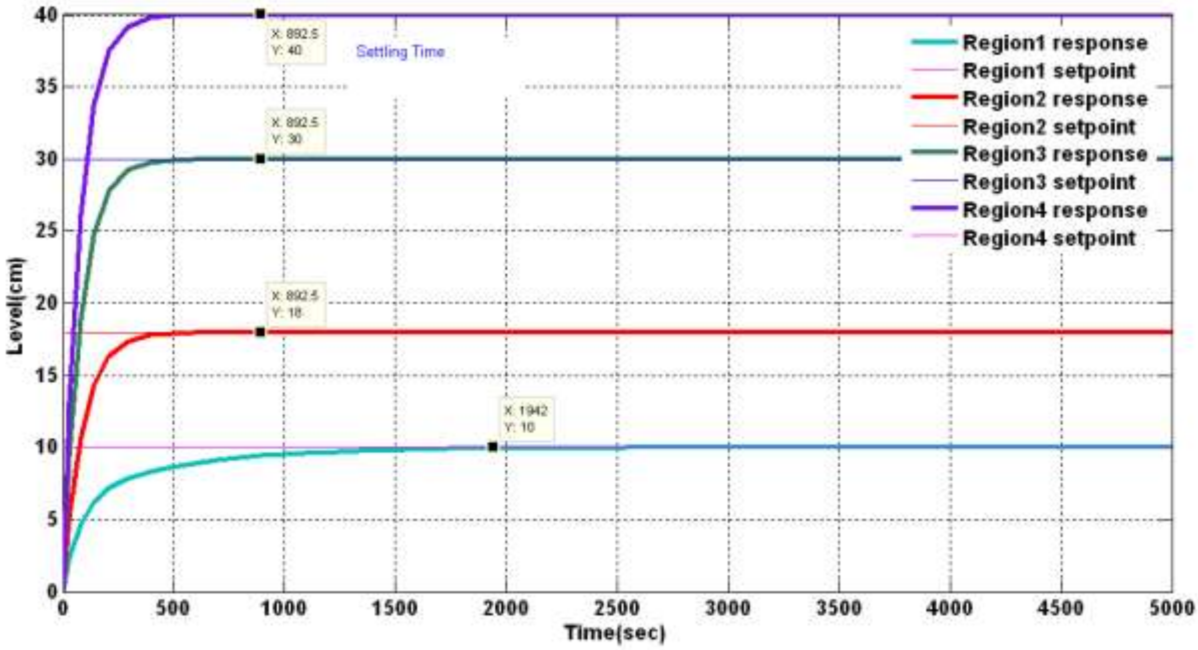

Figure 11: Response of IMC tuned PI controller with piecewise linearised models

Comparing both the simulation results it is clear that IMC tuned PI produces less overshoot and minimum settling time for linearised models.MRAC tuned produces less overshoot and minimum settling time for Non linear model.

Table 3:Comparison of performance of Z-N tuned PI, IMC tuned PI and MRAC tuned PI controllers through simulation

Performance Indices	Block Box model(Nonlinear Model)						Piecewise Model(Linear model)							
	Zone I			Zone III			Zone I		Zone II		Zone III		Zone IV	
	ZN PI	IMC PI	MRAC PI	ZN PI	IMC PI	MRAC PI	ZN PI	IMC PI	ZN PI	IMC PI	ZN PI	IMC PI	ZN PI	IMC PI
Settling Time (sec)	1000	600	15	2000	1700	15	1500	1942	1000	892	1000	892	1000	892
Peal Time (sec)	0	0	5	0	0	2	0	0	300	0	250	0	250	0
Peak overshoot (%)	0	0	1	0	0	2	0	0	1	0	3	0	5	0
IAE	2223	1033	1570	6818	3099	41570	2269	2273	1807	1587	3675	2400	4124	3007
ISE	7867	5149	7853	708000	463400	870700	7868	7867	13870	14200	36540	35900	60230	59000

5.3 Real Time level control using PI Controller

The simulated PI Parameters for the four linearized models are implemented in real time CTS. The responses are shown in figure 12.

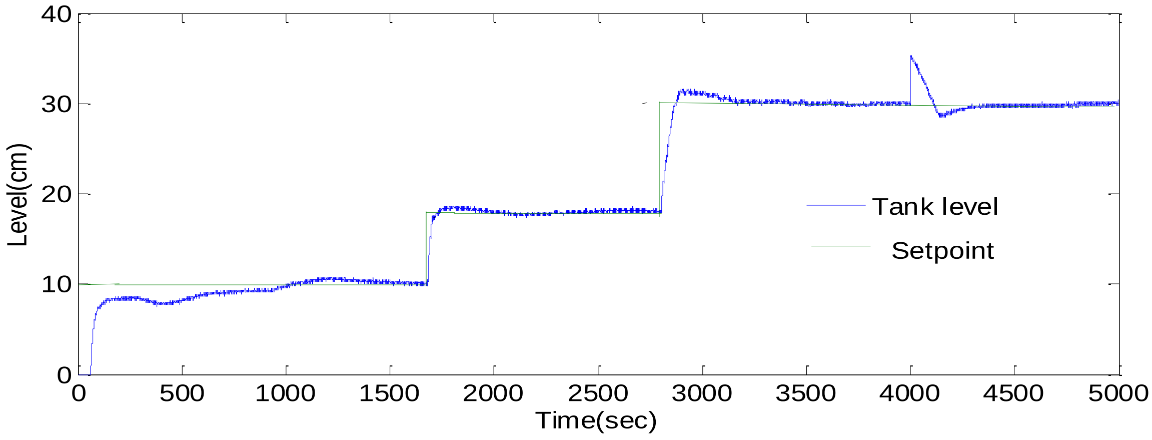

Figure 12: closed loop response of conical tank system using Z-N tuned PI Controller with linearization

As shown in above figure controller takes long time to reach the set point. In order to speed up the response IMC tuning parameters are implemented in real time for linearized models. The response is shown in Figure 13.

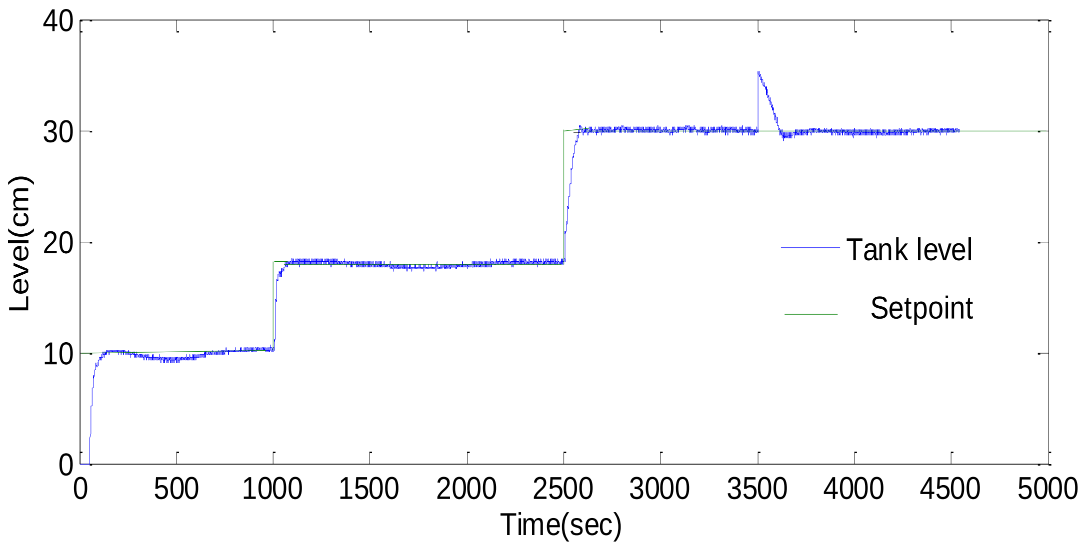

Figure 13: closed loop response of conical tank system using IMC based PI Controller with linearization

MRAC Tuned PI is implemented and shown in figure 14. It shows that conical tank System oscillatesand desired level is not reached for given set point.

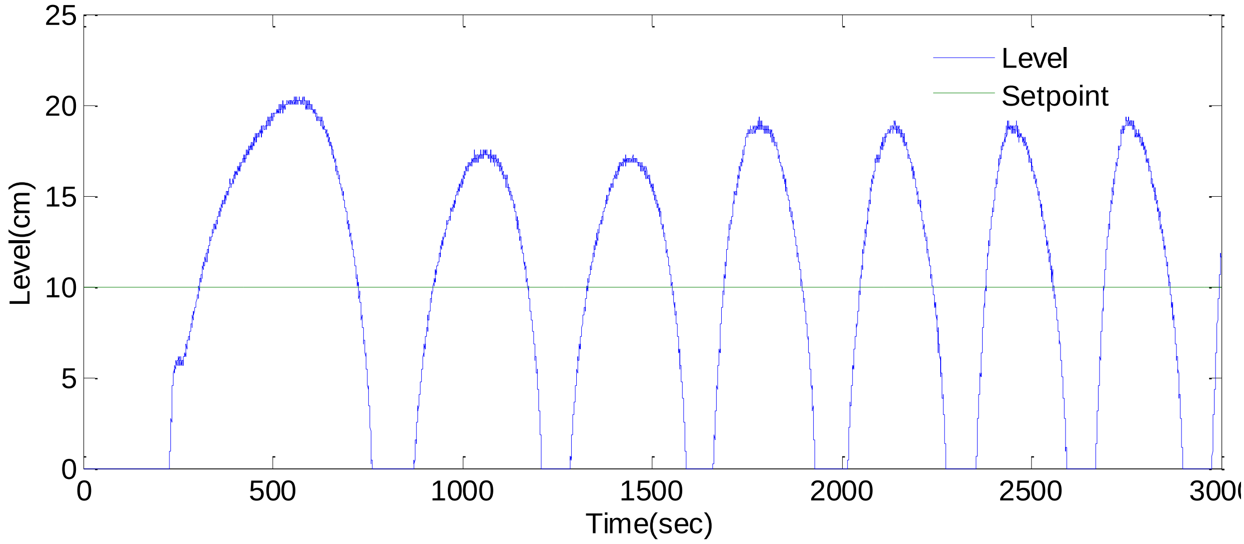

Figure 14: closed loop response of conical tank system using MRAC tuned PI Controller with Non linear Model

Comparison of Z-N, IMC tuned PI controller using real time results is shown in table 4.To compare the performances of the proposed controller various parameters such as rise time, peak overshoot and settling time, IAE are considered.

Table 4:Comparison of Z-N tuned PI, IMC tuned PI Controller through real time implementation

Performance Indices	**Piecewise Model(Linear model)**					
	Zone I		**Zone II**		**Zone III**	
	ZN PI	IMC PI	ZN PI	IMC PI	ZN PI	IMC PI
Settling Time	2000	600	2100	600	2200	600
Rise Time	600	250	900	250	900	300
Peak Time	1200	0	200	0	400	0
Peak overshoot(%)	1	0	0	0	0	0
IAE	2273	1033	4091	1820	6818	3099

From the table it is clear that four linearized models produces the better performance compared to single model.MRAC Tuned PI gives better performance in simulation. For linearized models IMC tuned PI produces better performance compared to Z-N tuned PI in real time.

6 Analysis and Conclusion

Conical tank system is a highly nonlinear system because of its variable cross section. In this paper three different modeling approaches are done. The Z-N tuned PI, IMC tuned PI, MRAC tuned PI controllers are implemented in real time CTS. From the results it has been observed that oscillations are more in the response of Non linear model with ZN tuned PI the controller. Moreover the system is not reaching the set point in all operating zones. MRAC tuned PI controller gives better performance using Non linear model. While implementing these tuning methods in Real time level control of CTS it is observed that MRAC tuned PI for Non linear model produces oscillations and desired level is not reached. So it is necessary to linearize the nonlinear system and Controller has to be designed for linearized model. Piecewise linearization based control system gives better performance with both the controllers and from the real time experimentation it is proved that IMC based PI gives even better performance than Z-N tuned PI.

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