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Most Economical Design of Axially Loaded Columns

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Abstract : The main objective of this paper is to achieve an optimal design of the reinforced concrete columns. Optimization of columns results in saving in cost. The objective function is to minimize the total cost of the column. The objective function is taken as the cost per unit length of column consisting the cost of concrete, reinforcement, formwork and ties. In the formulation of the optimum design problem, the breadth, depth of the column, number and the diameter of reinforcement bars and diameter of lateral ties and their spacing were treated as design variables. The optimal design was carried out using MATLAB (The Mathworks, Inc.) software. Optimization problem was formulated as a Nonlinear constrained minimization problem. This was solved using fmincon SQP Algorithm. Many problems were formulated and the optimal solutions were obtained. It was found that the solutions give the most economical design.
Keywords : Columns, Constrained Nonlinear Minimization, Matlab, Nonlinear Programming Problem, Optimization, SQP Algorithm

1.0 Introduction

Since reinforced concrete structures are used extensively in the world, they are very important for civil engineers. In the traditional design procedure, the designer must verify problem requirements and find the solutions through mathematical analysis. If such requirements are not satisfied, then dimensions and/or reinforcement of RC element are changed and a new solution is performed based on engineering perception which will satisfy the design requirements. This repeated process consumes considerable time, until a suitable section is found. Optimal design procedure which consists of selecting the design variables so as to minimize the objective function under some constraints is an alternative to the traditional design method.

Optimization is the art of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the efforts required or to maximize the desired benefits. Since the efforts required or the benefits required in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of function.

A large number of articles have been published on optimization of structures and following are some of the articles related to optimal design of columns.

Fouad A Mohammad and Dana A Seyan carried out the optimum design of reinforced concrete rectangular columns subjected to axial compression and biaxial bending moments. For the optimization process, the Generalised Reduced Gradient (GRG) technique was implemented. The formulated models for the design of reinforced concrete columns and the imposed constraints were based on the provisions of the Eurocode 2 (EC2). The design variables were the cross sectional dimensions (width and depth) and the reinforcing steel area[1].

Kirpka and Bordignon optimized the design of reinforced concrete rectangular columns, subjected to uniaxial flexural compression using Simulated Annealing. The amount and diameters of the reinforcement bars and the dimensions of the columns cross sections were considered as discrete variables. The results obtained were compared with those obtained from the conventional design procedure and other optimization methods, in an attempt to verify the influence of resistance class, variations in the magnitudes of bending moment and axial force, and material costs on the optimal design of reinforced concrete columns subjected to uniaxial flexural compression [2]. Salim T Yousif and Rabi' M Najem determined the optimum cost of reinforced concrete columns with all loading conditions (axially, uniaxially and biaxially loaded) using Genetic Algorithm. Many design constraints were used to cover all the reliable design results, such as limiting the cross sectional dimensions, limiting the reinforcement ratio and even the behavior of the optimally designed sections[2,3]. Jagbir Singh and Sonia Chutani carried out the optimal design of reinforced concrete column. The optimization was performed using one of the direct search methods - Complex Iterative Method. Complex Iterative Method, being a gradient free method was adopted to test its efficiency[4]. Preethi and Prince Arulraj carried out optimization of axially loaded columns. The optimal design was carried out using MATLAB's (The Math works, Inc.) software[5,6]. They reported that the developed program is efficient and versatile.

While going through the literature, it was found that no one has considered the cost of lateral ties along with other costs. An attempt has been made to include the cost of lateral ties along with the cost of concrete, steel and formwork in the objective function for the design the columns subjected to axial loads.

2.0 Formulation of Optimization Problem

2.1 Objective function

The cost of a reinforced concrete column consists of the cost of concrete, steel, formwork and lateral ties. The cost of the column per unit length can be obtained using the following relationship:

$C = C_c*A_c + C_s*A_s+ C_f* P_f +C_s * A_t* P_t$

C = total cost of column

C_c = cost coefficient of concrete /m length

A_c = area of concrete in the column

C_s = cost coefficient of steel/ m length

A_s = area of steel in the column

C_f = cost coefficient of formwork /m^2

P_f = perimeter of column

A_t = area of lateral ties

P_t= Perimeter of ties

2.2 Design constraints

The constraints reflect design requirements in the optimization problem. In other words they limit the range of acceptable designs in the problem. To achieve the optimum solution, design constraints for the problem should be defined. For the axially loaded column, the used design constraints were:

1. Axial load on the member should be less than or equal to the load carrying capacity of the column.
2. Maximum reinforcement percentage should be less than or equal to value specified in codes.
3. Minimum reinforcement percentage should be greater than or equal to the value specified in codes.
4. Diameter of lateral tie shall be 6mm or one fourth of the main diameter whichever is larger.
5. Spacing of ties shall not be more than 16times the diameter of the main bars or least lateral dimension or 300mm.

The columns were designed as per IS 456:2000.

3.0 Optimal Design of Columns

The objective function is the total cost of the column. The cost of column includes the cost of concrete, steel, formwork and ties. The optimization was carried out for different grades of concrete, different grades of steel and different diameters of reinforcement in order to understand the effect of grade of concrete, grade of steel and diameter of reinforcement on the optimal design of axially loaded columns. The optimal design was carried out using MATLAB's (The Mathworks, Inc.) software. Optimization problem formulated as a Non-linear constrained minimization problem. This is solved using fminconSQP Algorithm.

3.1 Objective function:

$0.0057*(x_1*x_2-\frac{\pi}{4}*\phi^2*x_3) + (0.43175*\frac{\pi}{4}*\phi^2*x_3) + 2*(x_1+x_2)*0.430 + (0.43175*\frac{\pi}{4}*x_4^2*2*[x_1+x_2-160+4*x_4]*1000/x_5)$

Where,

Breadth of column= x_1 =b
Depth of column = x_2 = d
Number of rods = x_3
Diameter of ties = x_4
Spacing for lateral ties = x_5
Cost factor for concrete = C_c = 0.0057
Cost factor for steel = C_s =0.43175
Cost factor for formwork= C_f= 0.430

3.2 Constraints:

1. The load carrying capacity of the column must be greater than the factored load. This is ensured by the constraint

   $P_u \leq 0.4 * f_{ck}*((x_1*x_2)-(\pi/4*\phi^2*x_3))+0.67*f_y*(\pi/4*\phi^2*x_3)$

2. $\frac{\frac{\pi}{4}*\phi*\phi*x3*100}{x1*x2} \geq 0.8$

   The second constraint will ensure that the percentage of compression reinforcement is more than 0.8

3. $\frac{\frac{\pi}{4}*\phi*\phi*x3*100}{x1*x2} \leq 6$

   The third constraint will ensure that the percentage of compression reinforcement is less than 6.

4. $x_1 \geq 230$
5. $x_2 \geq 230$

   The constraints 4 and 5 will ensure that the minimum dimensions of the column will be 230 x 230

6. $x_3 \geq 4$

   Constraint 6 will ensure that the minimum number of rods will be four in a square column.

7. $x_4 \geq 6$
8. $x_4 \geq \phi/4$

   Constraints 7 and 8 will ensure that the diameter of the lateral ties will be ¼ of the diameter of the main rod or 6 mm whichever is larger.

9. $x_5 \leq 300$
10. $x_5 \leq 16*\phi$

11. $x_5 \leq x_1$

12. $x_5 \leq x_2$

Constraint 9, 10, 11 and 12 will take care of the spacing of the lateral ties.

The optimal design of column has been done for various loads by sequential quadratic programming algorithm using Matlab software.

4.0 Effect of Grade of Concrete

In order to determine the effect of grade of concrete on the cost of columns, the optimal design of columns were found for various grades of concrete. The diameter of the main reinforcement was taken as 12mm and Fe415 steel was adopted. The details of the optimal design of columns carrying factored loads of 500kN, 1000kN, 1500kN and 2000kN are given in Table 1, Table 2, Table 3 and Table 4 respectively.

Table 1: Details of the optimal design for the columns with a factored load of 500 kN

Grade of concrete	b (mm)	d (mm)	No of bars	Diameter of ties (mm)	Spacing of ties (mm)	Cost/m (Rs)
M20	230	230	4	6	200	950
M25	230	230	4	6	200	992
M30	230	230	4	6	200	1029
M35	230	230	4	6	200	1064
M40	230	230	4	6	200	1094
M45	230	230	4	6	200	1116
M50	230	230	4	6	200	1179

It can be seen from the Table 1that, M20 grade concrete gives the least cost when the load on the column is 500kN. The percentage difference in the cost between the maximum and the least value is 24.10%.

Table 2: Details of the optimal design for the columns with a factored load of 1000kN

Grade of concrete	b (mm)	d (mm)	No of bars	Diameter of ties (mm)	Spacing of ties (mm)	Cost/m (Rs)
M20	325	325	8	6	200	1607
M25	300	300	6	6	200	1450
M30	275	275	6	6	200	1366
M35	250	250	6	6	200	1269
M40	250	250	4	6	200	1210
M45	230	230	4	6	200	1116
M50	230	230	4	6	200	1179

It can be seen from the Table 2, that M45grade concrete gives the least cost when the load on the column is 1000kN. The percentage difference in the cost between the maximum and the least value is 43.99%.

Table 3: Details of the optimal design for the columns with a factored load of 1500kN

Grade of concrete	b (mm)	d (mm)	No of bars	Diameter of ties (mm)	Spacing of ties (mm)	Cost/m (Rs)
M20	400	400	12	6	200	2239
M25	375	375	10	6	200	2101
M30	350	350	8	6	200	1929
M35	325	325	8	6	200	1833
M40	300	300	6	6	200	1623
M45	275	275	6	6	200	1490
M50	275	275	6	6	200	1580

From Table 3, it can be seen that M45 grade concrete gives the least cost when the load on the column is 1500kN. The percentage difference in the cost between the maximum and the least value is 50.25%.

Table 4: Details of the optimal design for the columns with a factored load of 2000kN

Grade of concrete	b(mm)	d (mm)	No of bars	Diameter of ties (mm)	Spacing of ties (mm)	Cost/m (Rs)
M20	450	450	14	6	200	2663
M25	425	425	12	6	200	2543
M30	400	400	10	6	200	2381
M35	375	375	10	6	200	2290
M40	350	350	8	6	200	2079
M45	325	325	8	6	200	1937
M50	325	325	8	6	200	2064

From Table 4, it can be seen that M45 grade concrete gives the least cost when the load on the column is 1500kN. The percentage difference in the cost between the maximum and the least value is 37.50 %.

5.0 Effect of Diameter of Reinforcement on The Cost of Columns

Designers normally find area of steel and determine the number of rods assuming the diameter of rods. In case of columns, the designers can select the diameter from a set of {12, 16, 20, 25,32,40 mm} which are available in the market. Only one of them will give the least cost. In order to determine the optimal diameter of reinforcement, the optimal design of columns were carried out for various diameters of reinforcement.

The details of the optimal design for the column with a factored load of 500kN are given in Table 5. The grade of concrete assumed was M20 and Fe415 steel was adopted.

Table 5: Effect of diameter of reinforcement on optimal cost of column with a factored load of 500kN

Diameter (mm)	12	16	20	25
No of rods	4	4	4	4
Area provided (mm^2)	452	804	1256	1963
Cost/m (Rs)	950	1092	1285	1628

From Table 5, it can be seen that the cost will be least when 12mm diameter rods are adopted. The reduction in the cost of column with respect to the cost of column with 25mm diameter rods is 71.21%.

The details of the optimal design for the column with a factored load of 1000 kN are given in Table 6.

Table 6: Effect of diameter of reinforcement on optimal cost of column with a factored load of 1000kN

Diameter(mm)	12	16	20	25
No of rods	8	4	4	4
Area provided (mm^2)	904	804	1256	1963
Cost/m (Rs)	1607	1544	1605	1709

From Table 6, it can be seen that 16mm diameter rods gives the least cost. The reduction in the cost of column with respect to the cost of column with 25mm diameter rods is 10.69%.

The details of the optimal design for the column with a factored load of 1500 kN are given in Table 7.

Table 7: Effect of diameter of reinforcement on optimal cost of column with a factored load of 1500kN

Diameter (mm)	12	16	20	25
No of rods	12	6	4	6
Area provided (mm^2)	1357	1206	1256	2945
Cost/m (Rs)	2239	2154	2176	2627

From Table 7, it can be seen that 16mm diameter rods gives the least cost. The reduction in the cost of column with respect to the cost of column with 25mm diameter rods is 21.94 %.

The details of the optimal design for the column with a factored load of 2000 kN are given in Table 8.

Table 8: Effect of diameter of reinforcement on optimal cost of column with a factored load of 2000kN

Diameter (mm)	12	16	20	25
No of rods	14	8	6	6
Area provided (mm^2)	1583	1608	1884	2945
Cost/m (Rs)	2663	2654	2772	2927

From Table 8, it can be seen that 16mm diameter rods gives the least cost. The reduction in the cost of column with respect to the cost of column with 25mm diameter rods is 10.28 %.

6.0 Effect of Grade of Steel on the Cost of Column

Inorder to study the effect of grade of steel on the cost of column, optimal designs were found outfor various grades of steel. The grade of concrete assumed was M20.The optimization was done with Fe415, Fe500 and Fe550D grades of steel. Eventhough IS 456-2000 permits the use of only three grades namely Fe 250,Fe415 and Fe500, Fe 550D which has a higher ductility isalso considered during the present research.

The details of columns designed with different grades of steel reinforcement are given in Table 9.

Table 9: Design details of columns

P_u (kN)	Area of concrete required (mm^2)			Area of steel required (mm^2)			cost/m (Rs)		
	Fe415	Fe500	Fe550D	Fe415	Fe500	Fe550D	Fe415	Fe500	Fe550D
500	52900	52900	52900	452.38	452.38	452.38	950	972	998
750	75625	75625	75625	678.58	678.58	678.58	1254	1284	1319
1000	105625	105625	105625	904.77	804.24	804.24	1607	1599	1640
1250	140625	122500	122500	1130.97	904.77	1130.97	1989	1785	1945
1500	160000	160000	160000	1357.16	1206.36	1206.36	2239	2224	2283
1750	180625	180625	180625	1357.16	1357.16	1357.16	2399	2455	2519
2000	202500	202500	202500	1583.35	1608.48	1608.48	2663	2739	2814

From the Table 9, it can be seen that grades of steel has a definite impact on the cost of columns. Fe415 is found to give optimal cost for loads of 500kN, 750 kN, 1750kN and 2000kN. Fe500 is found to give optimal cost for loads of 1000kN, 1250kN and 1500kN. The percentage reduction in cost is found to range between 2% and 11%.

7.0 Conclusions

The main conclusions drawn from the current research can be summarized as follows:

- Optimization of reinforced concrete columns subjected to axial loads indicates that minimum percentage of steel must be used as reinforcement. Higher percentage of steel results in higher cost.
- The effect of grade of concrete on the cost of columns was studied and it was found that grade of concrete has an impact on the cost of R.C columns. M20 grade of concrete is found to give least cost when the load on the column was 500kN. For other loads, M45 grade of concrete is found to give the least cost.
- The effect of grade of steel on the cost of column was studied and it was found that grade of steel has an impact on the cost of R.C columns.
- The effect of diameter of reinforcement on the cost of column was also studied and it was found that diameter of reinforcement has an impact on the cost of reinforced concrete columns. Instead of selecting the diameter of the reinforcement arbitrarily, diameter has to be found through optimization.

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9.0 References

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