

Energy levels, B(E2) and Potential energy surface for ¹³⁰⁻¹³⁶Nd Isotopes

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Abstract : In this study the spectra, B(E2) and effort power surface are studied in the Model of (IBM-1). It is found that ¹³⁰⁻¹³⁴Nd are in the region SU(3)-U(5). For ¹³⁶Nd in the region of SU(3)-O(6). The values of B(E2) decreased when the neutron number increases, the branching ratios proved the isotopes in them transition regions. (P.E.S) it demonstrates this isotopes have high deformation.

Keywords : IBM-1, B(E2), Potential Energy Surface, Nuclear Structure, Nd, Isotopes.

Introduction

To term nuclear properties like spins and powers of the little levels, decay probabilities for the emission of gamma quanta, probabilities (spectroscopic factors) of transfer reactions, multipole moments and so on, a pattern of the atomic nucleus has to be capable of characterize them [1]. The IBM (sometimes named IBA Model) who has demonstrated that aprimeagent to the light nuclei (up to 50 nucleons), it is fundamentally firm in shell model and decreases the figure of states heavily. It constitutes only (26) configurations for the (2⁺) state. The greater numeral of nucleons falls more shells have to be possessed into consideration and number of nuclear states soon after turns into so big this shell model ought be intractable[2].

2- The IBM Description

The Model of (IBM) has been rather successful at characterizing the collective properties of several medium and heavy nuclei. In the current work, we used the Model to study the low-lying combined state of even-even Nd isotopes, the Hamiltonian operator can be written down[3,4]

$$\hat{H} = \varepsilon \hat{n}_d + a_0 \hat{P}^\dagger \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (1)$$

ε is the energy of d_ bosons, where $\varepsilon = \varepsilon_d - \varepsilon_s$, $\varepsilon_s = 0$, therefore $\varepsilon = \varepsilon_d$, while the parameters (a_0, a_1, a_2, a_3, a_4) represents the interaction strength for paring, angular momentum, quadrupole momentum, octupole and hexadecapole between bosons respectively. Where \hat{n}_d operator produced the number of d bosons, \hat{P} stand for the paring operator, \hat{L} it represents the angular momentum operator, \hat{Q} , is the quadrupole operator, \hat{T}_3 and \hat{T}_4 stand for the octupole and hexadecapole operators. In O(6) the equation of Hamiltonian is [5]

$$\hat{H} = a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \quad (2)$$

The full strength of quadrupole (E2) transitions between low lying states of even-even nuclei is aremarkable observable to experiment with nuclear paradigms, which term collective phenomena. Fully

distorted even nuclei display great quadrupole transference force's $B(E; 2_1^+ \rightarrow 0_1^+)$ [6]. At a modest form that forcerises smoothly because increasing number of valence nucleons or holes along an isotopic or isotonic chain as collectivity increases. Considering symmetry of particles and punctures this model yields a maximum at mid-shell [7].The equation of electric quadrupole transition can be written by using the operators in the following form[8]

$$\hat{T}_m^{(E2)} = \alpha_2 [d^\dagger \tilde{s} + s^\dagger \tilde{d}]_m^{(2)} + \beta_2 [d^\dagger \tilde{d}]_m^{(2)} \tag{3}$$

Where α_2 and β_2 are two parameters used for fitting the experimental results.

The P.E.S ($V(N, \beta, \gamma)$) affords a final shape to the nucleus that agrees to the function of Hamiltonian, as display in Eq.(4) [9] :

$$E(N, \beta, \gamma) = \langle N, \beta, \gamma | H | N, \beta, \gamma \rangle / \langle N, \beta, \gamma | N, \beta, \gamma \rangle \tag{4}$$

The anticipation value of IBAM-1 Hamiltonian with the cohesive state ($|N, \beta, \gamma\rangle$) is utilized to make IBM energy surface[9].

The potential energy surface can be represented as a function of (β) and (γ) in following equation [9]:

$$V(N, \beta, \gamma) = \frac{N \epsilon_d \beta^2}{(1 + \beta^2)} + \frac{N(N + 1)}{(1 + \beta^2)^2} (a_1 \beta^4 + a_2 \beta^3 \cos 3\gamma + a_3 \beta^2 + a_4) \tag{5}$$

β Is a measure of the total deformation of the nucleus, γ Is the quantity of perversion from centralize symmetry and correlate with the nucleus.

3-Results and Discussion

3.1 energy levels

The Nd isotopes are neutron rich, have $Z=60$, the numbers of boson proton $N\pi = 5$ and boson neutron $N\nu$ different from 11 in ^{130}Nd isotope to 8 in ^{136}Nd isotope (*means hole boson). To study the levels in Nd isotopes, we want to evaluation the parameters used in (IBM-1), by applying the BOS. In package, the fitted values of those parameters are listed in Table (1), they were patronized as a free parameter and their values were estimated by appropriate to experimental level. Generally our observation that the (\hat{L}, \hat{Q}) parameters increasing with neutron numbers increases. Figure (1-4) shows a comparison between theoretical and available experimental energy levels for all studied Nd isotopes [10].

Table (1) IBM-1 Parameter for even-even Nd isotopes

A	N	EPS(MeV)	$\hat{P} \cdot \hat{P}$ (MeV)	$\hat{L} \cdot \hat{L}$ (MeV)	$\hat{Q} \cdot \hat{Q}$ (MeV)	$\hat{T}_3 \cdot \hat{T}_3$ (MeV)	$\hat{T}_4 \cdot \hat{T}_4$ (MeV)	CHI	SO6
^{130}Nd	11	0.1920	0.0000	0.0065	-0.0450	0.0000	0.0000	-1.3228	1.0000
^{132}Nd	10	0.0661	0.0000	0.0203	-0.0350	0.0000	0.0000	-1.3228	1.0000
^{134}Nd	9	0.0948	0.0000	0.0210	-0.0463	0.0000	0.0000	-1.3228	1.0000
^{136}Nd	8	0.0000	0.0812	0.0244	-0.1024	0.0000	0.0000	-1.3228	1.0000

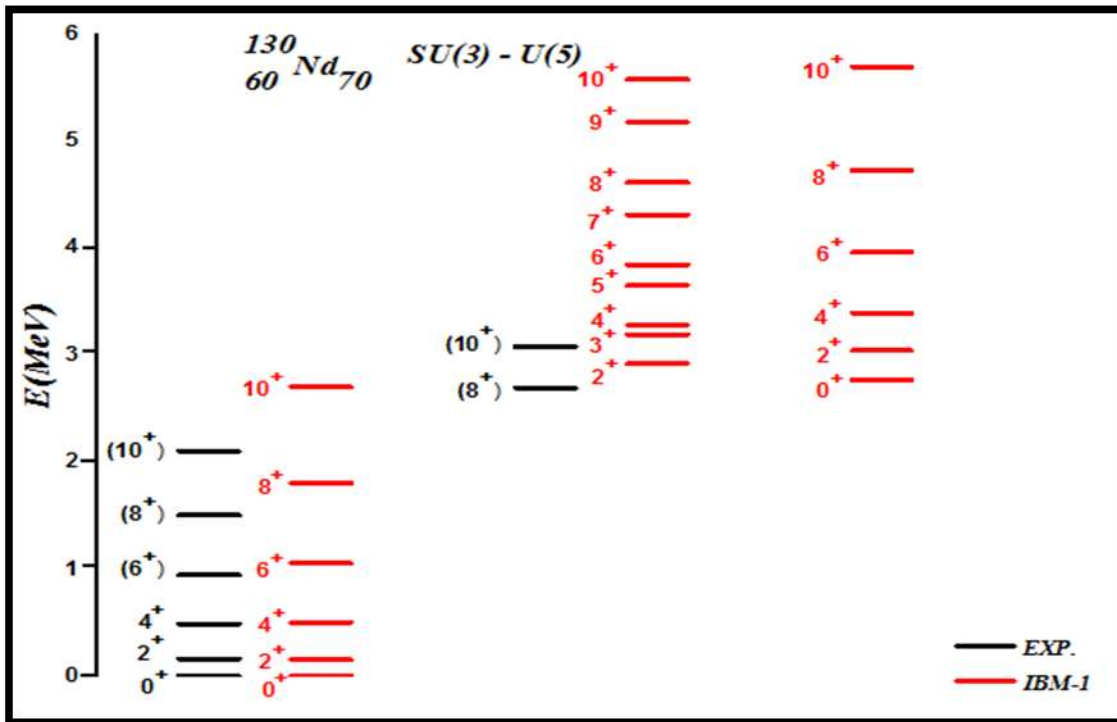


Figure (1) Comparison IBM-1 calculations with the experimental data for ^{130}Nd isotope

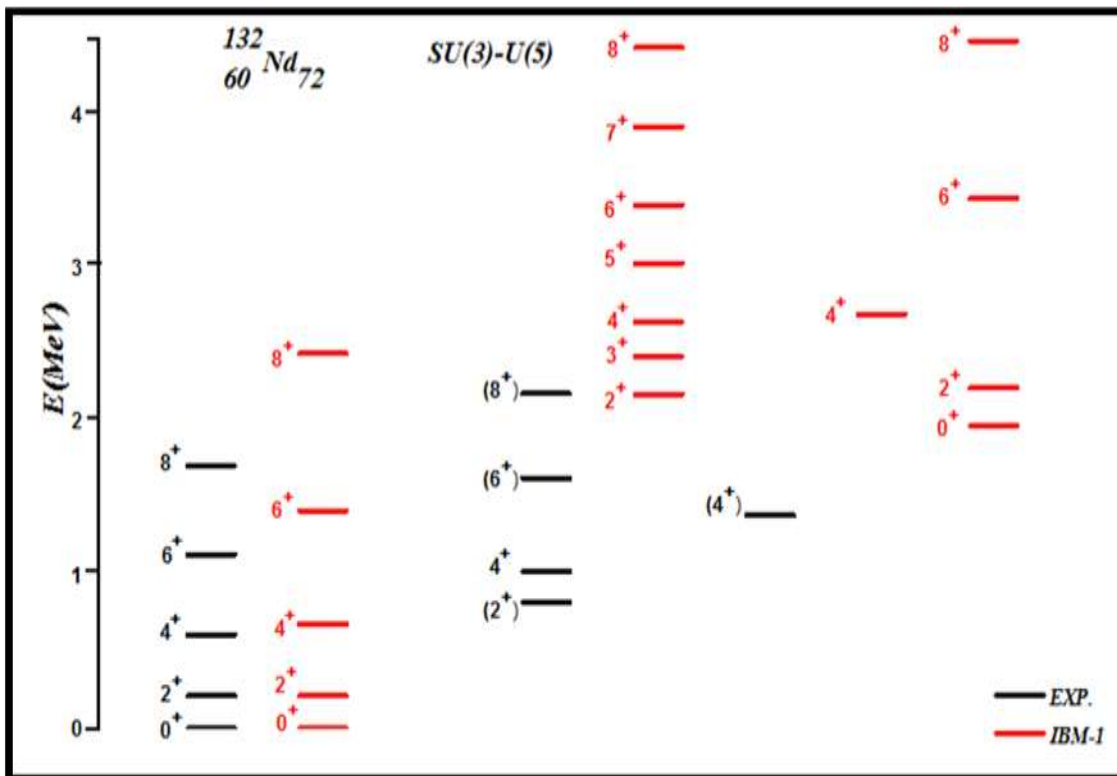


Figure (2) Comparison IBM-1 calculations with the experimental data for ^{132}Nd isotope

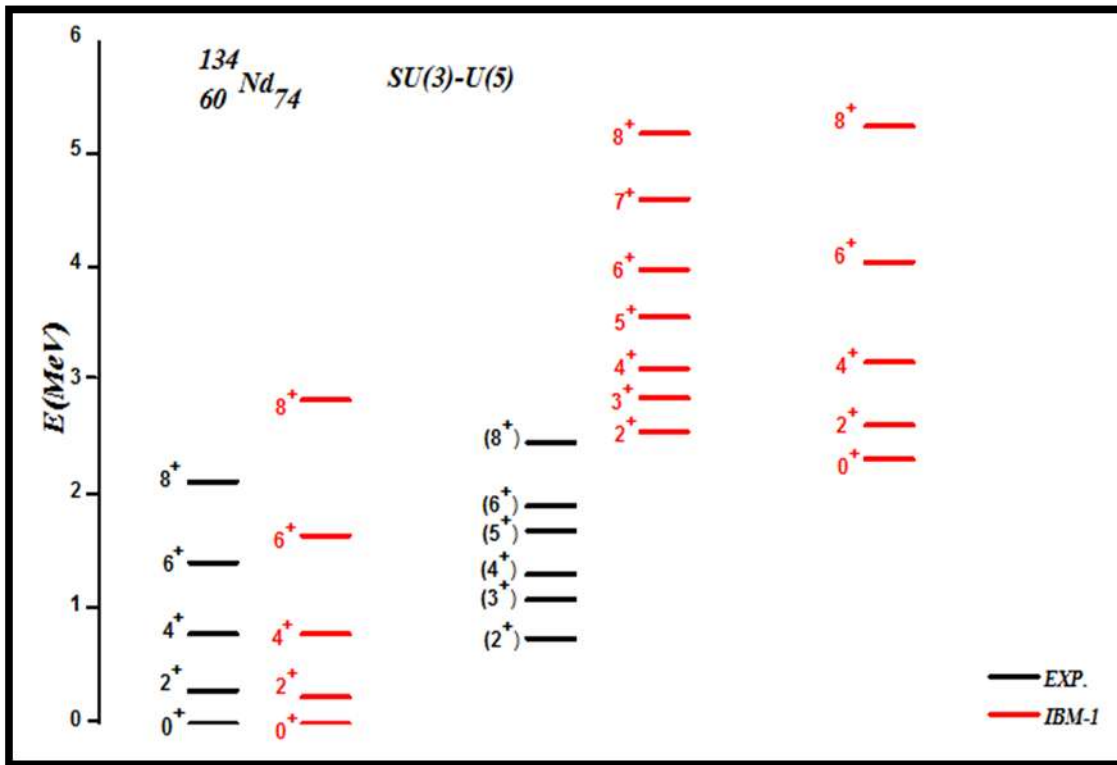


Figure (3) Comparison IBM-1 calculations with the experimental data for ^{134}Nd isotope

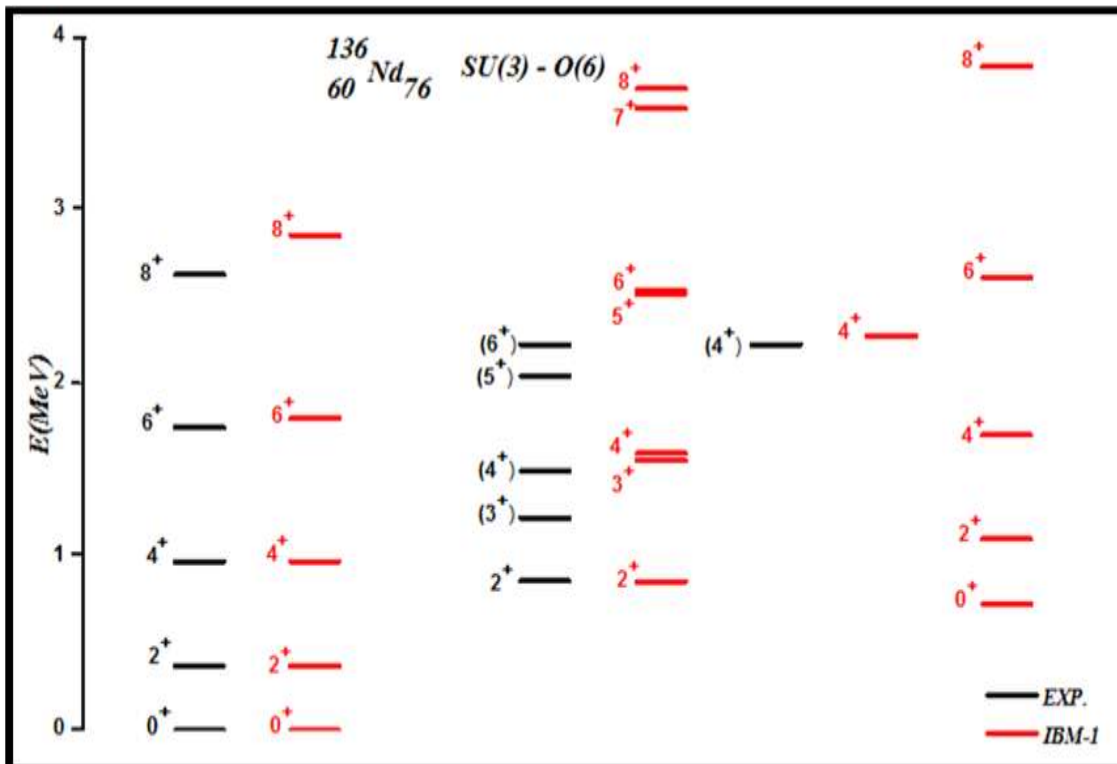


Figure (4) Comparison IBM-1 calculations with the experimental data for ^{136}Nd isotope

The energy ratios E_{4^+}/E_{2^+} , E_{6^+}/E_{2^+} , E_{8^+}/E_{2^+} has been calculated theoretically for the even-even $^{130-136}\text{Nd}$ isotopes and compared with their corresponding experimental values taken from ref. [10] and with the typical values for each limit [8,9] and shown in figures (5-7).

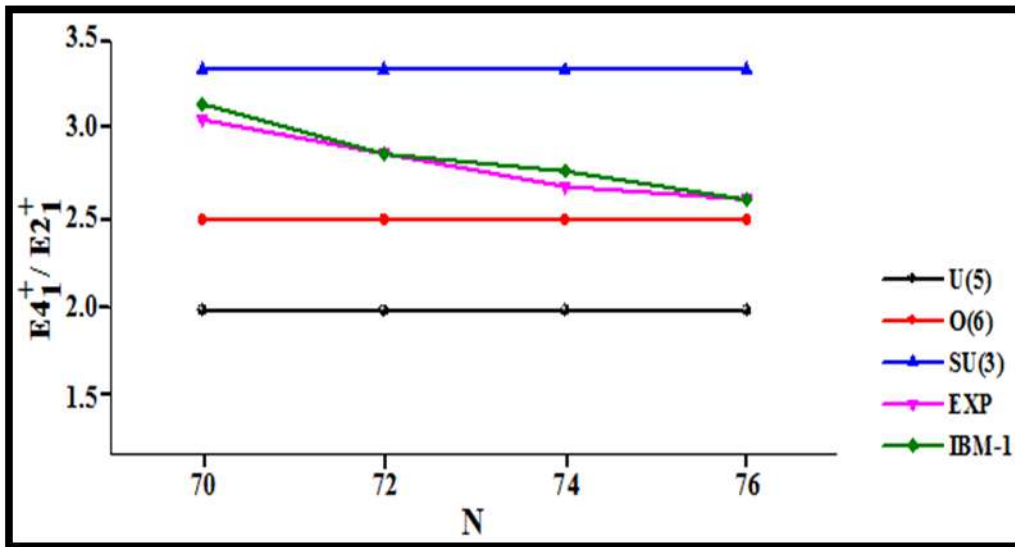


Figure (5) The Comparison of $E_{4_1^+}/E_{2_1^+}$ Theoretically, Experimentally and with Typical Values at Every Limit

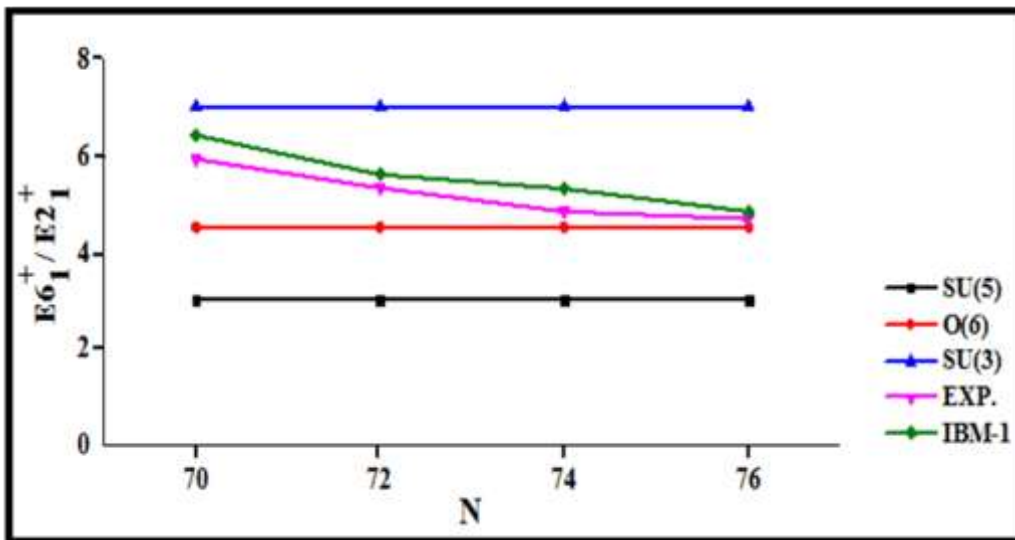


Figure (6) The Comparison of $E_{6_1^+}/E_{2_1^+}$ Theoretically, Experimentally and with Typical Values at Every Limit

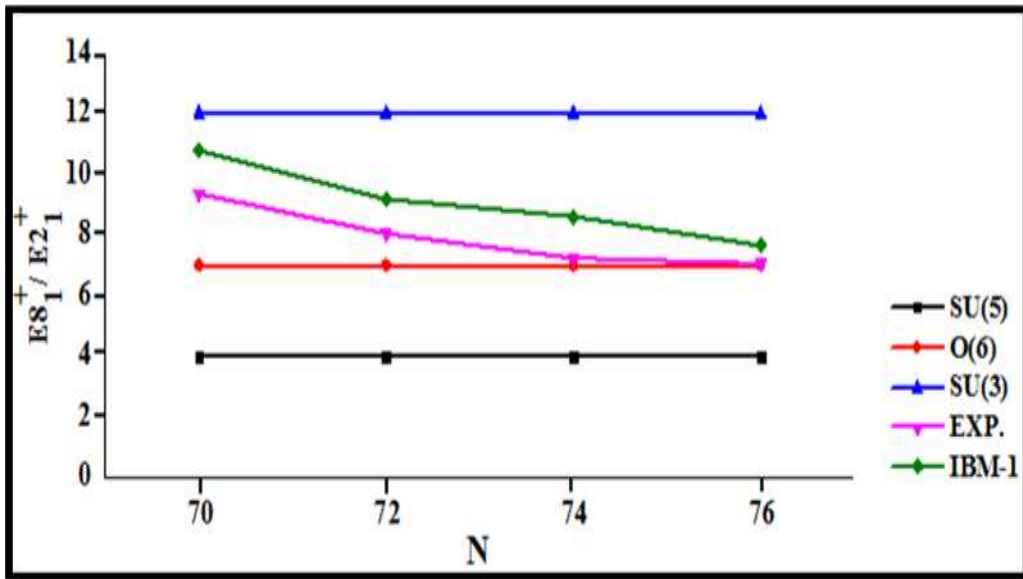


Figure (7) The Comparison of $E_{8_1^+}/E_{2_1^+}$ Theoretically, Experimentally and with Typical Values at Every Limit

3.2B(E2) and Quadruple moment $Q_{2_1^+}$

Many datum can be gained by researching reduced transition probabilities B(E2). The (IBMT-code) were employed (α_2, β_2). The parameters (E2SD) and (E2DD) applied in the existent calculations are fixed by the calculated values to the experimentally recognized once and displayed in Table (2).

Table (2) The experimental values of B(E2) and the coefficients(E2SD, E2DD) for $^{130-136}\text{Nd}$ used in the present work

A	$B(E2: 2_1^+ \rightarrow 0_1^+)$ Exp. e^2b^2	$B(E2: 2_1^+ \rightarrow 0_1^+)$ Theo. e^2b^2	E2SD (e b)	E2DD (e b)
^{130}Nd	0.9392 [10]	0.9374	0.130766	-0.172987
^{132}Nd	0.8346 [10]	0.8340	0.134705	-0.178197
^{134}Nd	0.3789 [10]	0.3785	0.100118	-0.132443
^{136}Nd	0.0324	0.0324	0.032646	-0.043186

A comparison between the experimental [10] and calculated $B(E2; 2_1^+ \rightarrow 0_1^+)$ are shown in figure (8) and prove that results are quite well for all isotopes under study.

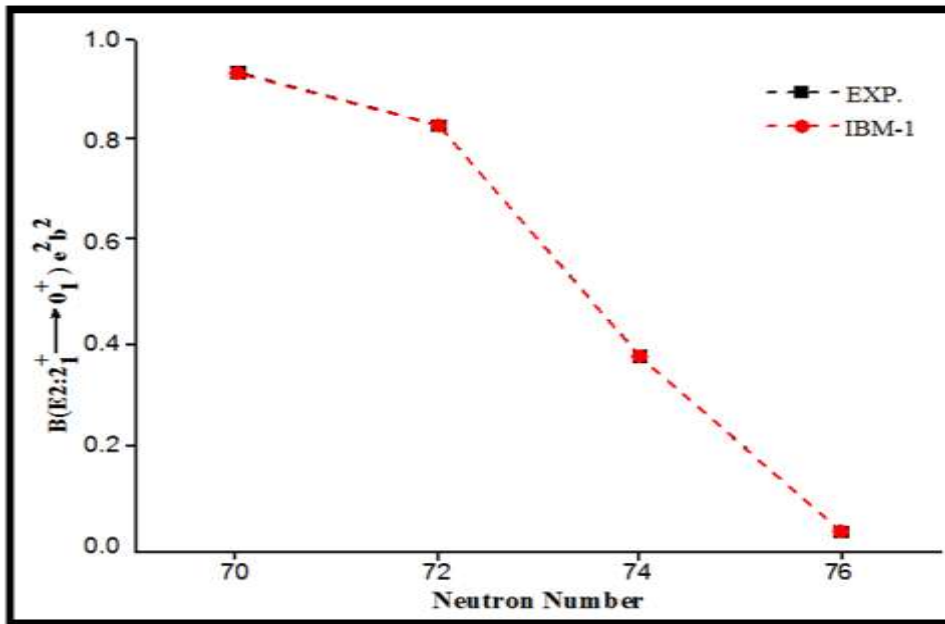


Figure (8) Comparison of the Experimental and Calculated $B(E2; 2_1^+ \rightarrow 0_1^+)$ for $^{130-136}\text{Nd}$ Isotopes

The quadrupole moment (Q) is a remarkable property for nuclei and is defined as follows the variation from the spherical charge distribution inside the nucleus and from the quadrupole moment we can determine if the nucleus is spherical, deformed oblate or prolate shapes [11-16].

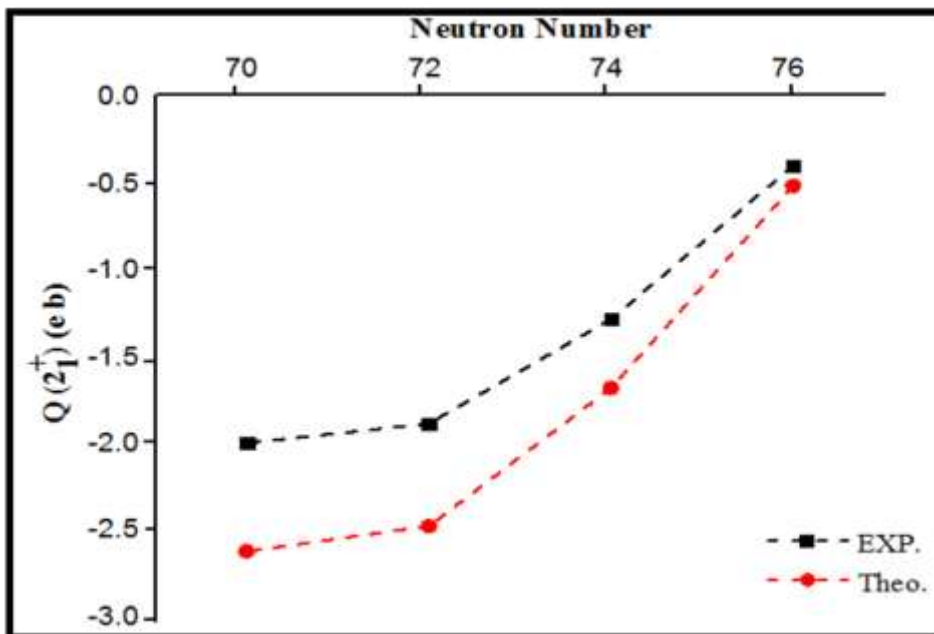


Figure (9) The Comparison Between the Experimental Quadrupole Moment $Q(\text{eb})$ Taken from Refs.[10] and the Calculation from Present Work for $^{130-136}\text{Nd}$ Isotopes

3.3 The B(E2) branching ratios

The significance to study branching ratios is to research the shape of the nucleus and its dynamic symmetries and to set which dynamic symmetries. The R , R' and R'' defined as follows [17, 18]:

$$R = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (6)$$

$$R' = \frac{B(E2; 2_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (7)$$

$$R'' = \frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (8)$$

The estimated them and their equivalent experimental values are presented in Table (3). The comparison experimental and calculated branching ratios, and the typical values at the limits are displayed in the figure (10).

Table(3) Comparison between experimental and theoretical B(E2)branching ratios for ¹³⁰⁻¹³⁶Nd isotopes

B(E2) Ratios	¹³⁰ Nd		¹³² Nd		¹³⁴ Nd		¹³⁶ Nd	
	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
$R = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}$	0.583	1.402	0.88	1.39	1.45	1.39	-----	1.38
$R' = \frac{B(E2; 2_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}$	-----	0.0003	-----	0.0001	-----	0.000	-----	0.000
$R'' = \frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}$	-----	0.0011	-----	0.0006	-----	0.000	-----	0.000

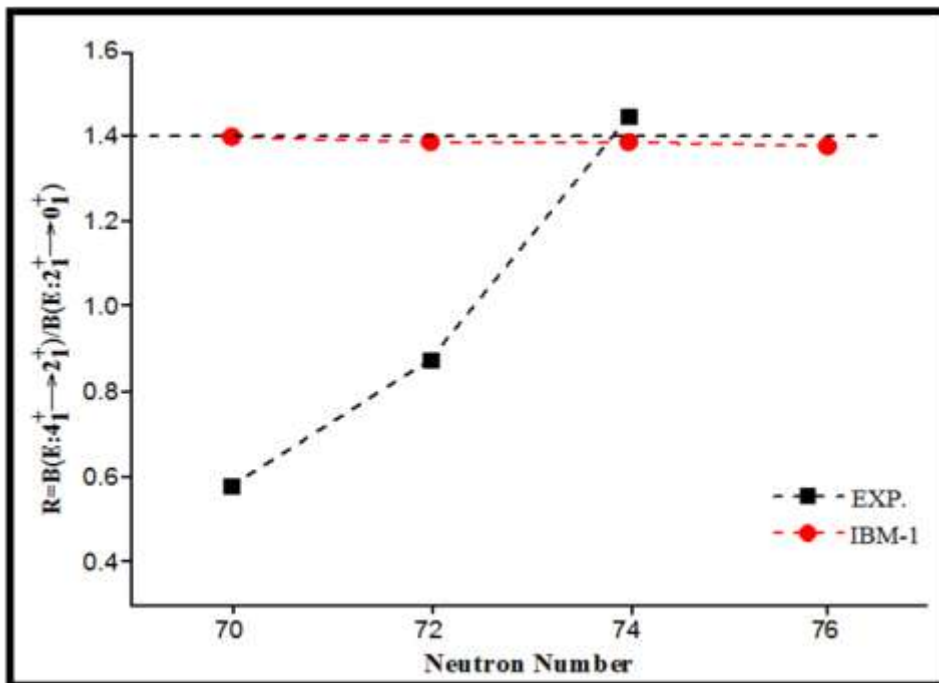


Figure (10) Comparison between the Experimental and Calculated B(E2) Branching Ratios for Even-Even ¹³⁰⁻¹³⁶NdIsotopes with the Typical Values of SU(3), U(5) and O(6) Limits

3.4 Potential Energy Surface (P.E.S)

The PES.FOR program is applied to calculate P.E.S $V(N, \beta, \gamma)$. In this work, it has been calculated from Eq.(5). In the figures (11-14), the contour plots in the $(\gamma-\beta)$ planes resulting from $E(N, \beta, \gamma)$ are shown for $^{130-136}\text{Nd}$ isotopes. The triaxial disfigured assists to know the prolate to oblate shape transition that occurs in the considered Nd isotopes.

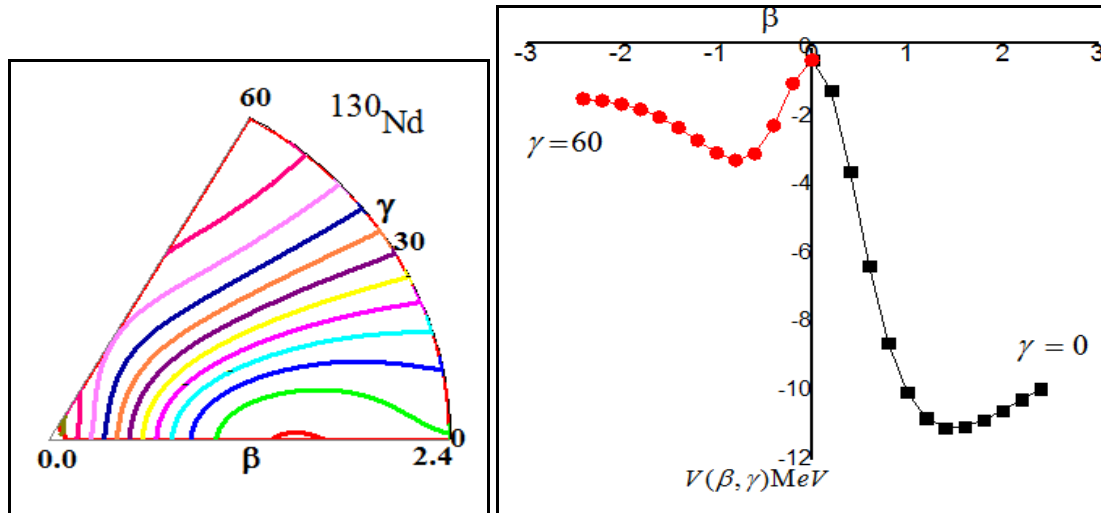


Figure (11) The potential energy surface in γ - β planes for ^{130}Nd isotope

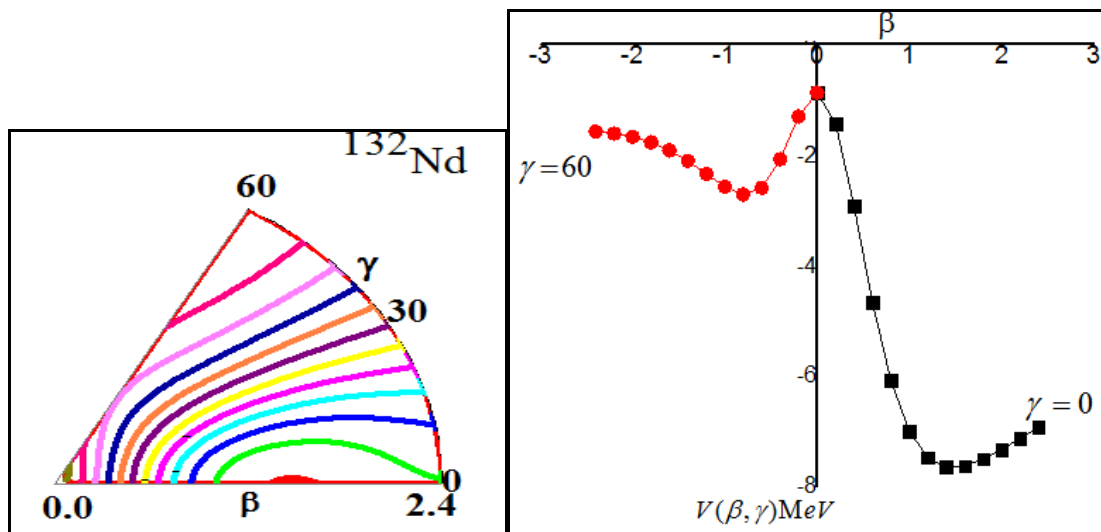


Figure (12) The potential energy surface in γ - β planes for ^{132}Nd isotope

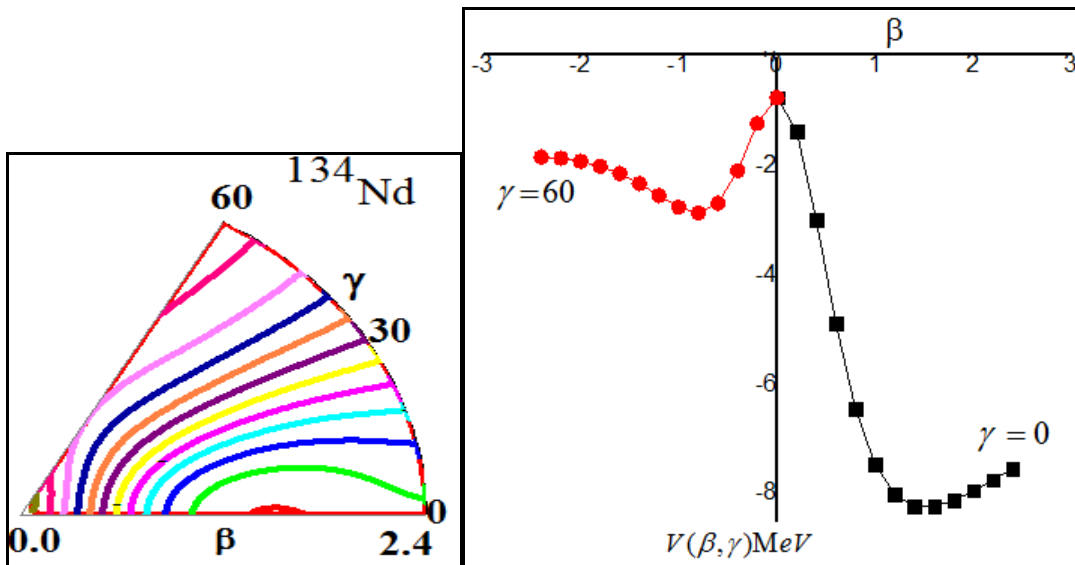


Figure (13) The potential energy surface in γ - β planes for ^{134}Nd isotope

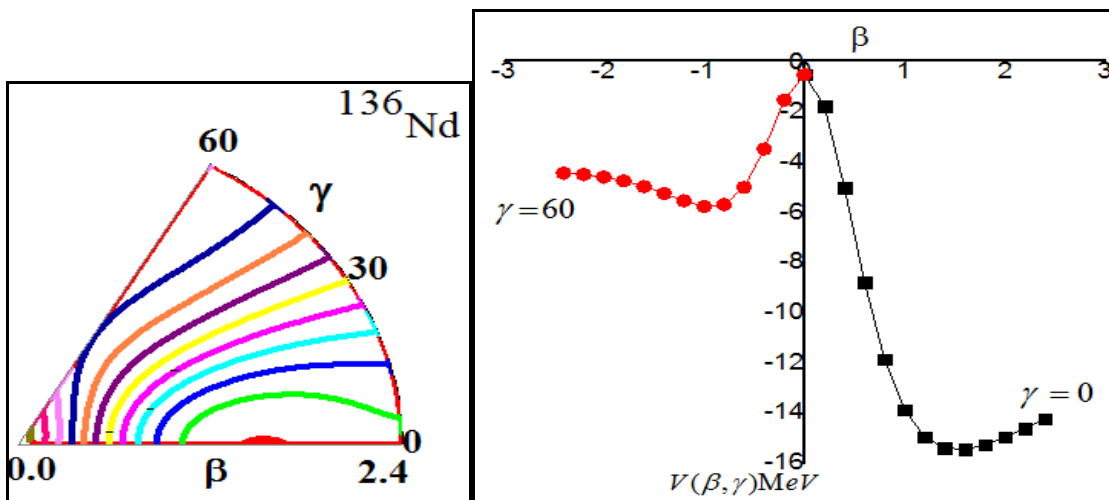


Figure (14) The potential energy surface in γ - β planes for ^{136}Nd isotope

4-Conclusions

1. The calculated spectrum of $^{130-134}\text{Nd}$ is compared with experimental spectrum the ground band is well reproduced up to $L=6$. In this isotope the parameter has a large descent. This produces ^{130}Nd more near to the rotational limit.
2. The study of the reduced transition probability that it decreases as the neutron number increase, and this is a key signature that the nuclei deformation less when near to closed shell and become stable. The calculations of $B(E2)$ values display a good agreement with the available experimental data. However, in some cases, they can be higher or lower than the experimental values, due to the deformation effects of these nuclei.
3. The study of the quadrupole moment for ^{130}Nd is more oblate distorted shape than the other isotopes under study and the oblate perversion from spherical symmetry reduces with the increase of mass number.
4. The ratios of the reduced transition probabilities' R , R' and R'' have been proved in agreement both experimentally and theoretically in their transition regions that pass through it.
5. From the axially symmetric for the isotopes $^{130-136}\text{Nd}$ have high deformation.

5- References

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