



Charge density distributions and charge form factors of some even-A p-shell nuclei

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Abstract : The ground state charge density distributions, elastic electron scattering form factors and the corresponding rms radii for some 1p-shell nuclei with $Z = N$ (such as ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei) have been calculated using the single particle radial wave functions of harmonic oscillator (HO) and Woods-Saxon (WS) potentials. The calculated results are discussed and compared with the experimental data. It is found that the contributions of the quadrupole form factors in ${}^{10}\text{B}$ and ${}^{14}\text{N}$ nuclei, which are described by the undeformed p-shell model, are essential for obtaining a remarkable agreement between the theoretical and experimental form factors.

Keywords : Charge density distributions, quadrupole form factors, Woods-Saxon and harmonic oscillator potentials.

Introduction

The charge density distributions (*CDD*) and form factors are the most important quantities in the nuclear structure which were well studied experimentally over a wide range of nuclei. This interest in the *CDD* is related to the basic bulk nuclear characteristics such as the shape and size of nuclei, their binding energies, and other quantities which are connected with the *CDD*. Besides, the density distribution is an important object for experimental and theoretical investigations since it plays the role of a fundamental variable in nuclear theory [1]. Electron–nucleus scattering is known to be one of the powerful tools for investigating nuclear charge density distributions. Charge density distributions for stable nuclei have been well studied with this method [2-4]. The comparison between calculated and measured electron scattering form factors has long been used as a successful test of nuclear models which have been adopted through the last fifty years. One of these models is the shell model [5] which is the most modern microscopic nuclear structure calculations for finite nuclei and has been very successful in describing the nuclear structure [6]. The Woods–Saxon potential, as an important mean field nuclear potential describes the interaction of single nucleon with whole nuclei and widely used in nuclear structure, nuclear reactions, nuclear scattering and particle physics, has been attracted a great deal of interest for some decades. In order to study the structure of nuclei contain, particle-hole theory, many nucleon configuration, electromagnetic transitions and nuclear decay the Woods–Saxon basis has been used as better choice than harmonic oscillator basis in both relativistic and non-relativistic theories of nuclear mean-field shell model [7]. Mirea [8] has been computed the single-particle energies and wave functions of an axially two-center Woods-Saxon potential. The spin-orbit interaction was included in the Hamiltonian. Lojewski and Dudek [9] evaluated the proton and neutron separation energies and mean square charge radii within the Woods–Saxon plus BCS model for even-even nuclei with $40 \leq A \leq 256$.

In this work, we study the ground state charge density distributions, charge form factors and the corresponding rms radii for some 1p-shell nuclei with $Z = N$ such as ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei using the radial wave function of harmonic oscillator and Woods-Saxon potentials and compared the calculated results with experimental results

Theory

The single nucleon wave function, as fundamental parameter for studying nuclear structure and constructing the wave function of nuclei, is derived from single nucleon Schrodinger equation. The single-nucleon radial Schrodinger equation for a central potential can be written as [7]:

$$\left[\frac{-\hbar^2}{2m} \left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) + V(r) \right] \psi(r) = E\psi(r) \tag{1}$$

By introducing the reduced radial wave function, $R(r) = r \psi(r)$, which is normalized by $\int \psi(r) d^3r = 1$, Eq. (1) is rewritten as

$$\frac{d^2 R(r)}{dr^2} + \frac{2m}{\hbar^2} \left[E - V(r) - \frac{\hbar^2}{2m} \frac{\ell(\ell+1)}{r^2} \right] R(r) = 0 \tag{2}$$

For the local potential $V(r)$, the Woods-Saxon shape is used in the compact form [10,11]:

$$V(r) = V_0(r) + V_{so}(r) \vec{\ell} \cdot \vec{s} + V_c(r), \tag{3}$$

where $V_0(r)$ is the spin-independent central potential:

$$V_0(r) = \frac{-V_0}{1 + [e^{(r-R_0)/a_0}]} \tag{4}$$

$V_{so}(r)$ is the spin-orbit potential:

$$V_{so}(r) = V_{so} \frac{1}{r} \left[\frac{d}{dr} \frac{1}{1 + [e^{(r-R_{so})/a_{so}}]} \right] \tag{5}$$

and $V_c(r)$ is the Coulomb potential generated by a homogeneous charged sphere of radius R_c [12]:

$$V_c(r) = \frac{Ze^2}{r} \quad \text{for } r \geq R_c \tag{6}$$

$$V_c(r) = \frac{Ze^2}{R_c} \left[\frac{3}{2} - \frac{r^2}{2R_c^2} \right] \quad \text{for } r \leq R_c \tag{7}$$

The radii R_0 , R_{so} and R_c are usually expressed as:

$$R_i = r_i A^{1/3} \tag{8}$$

The point density distributions of neutrons, protons, and matter can be written respectively as [1]:

$$\rho_{n,porm}(r) = \frac{1}{4\pi} \sum_{nlj} X_{n,porm}^{nlj} |R_{nlj}(r)|^2 \tag{9}$$

where $X_{n,p,orm}^{nlj}$ represents the number of neutrons, protons, or nucleons in the nlj -subshell.

From the proton density $\rho_p(r)$ and the intrinsic charge distribution f_p of one proton, one can obtain the charge distribution of the nucleus with the following folding relation [13]:

$$\rho_{ch}(r) = \int \rho_p(r) f_p(r' - r) dr' \quad (10)$$

The root mean square (rms) radii of the neutron, proton and charge distributions can be obtained from these densities as follows [13]:

$$r_g = \langle r_g^2 \rangle^{1/2} = \left[\frac{\int r^2 \rho_g(r) dr}{\int \rho_g(r) dr} \right]^{1/2} \quad g = n, p, ch \quad (11)$$

The elastic electron scattering form factors from considered nuclei are studied by the plane wave Born approximation (PWBA). In the PWBA, the incident and scattered electron waves are represented by plane waves. The elastic electron scattering form factor is simply given by the Fourier-Bessel transform of the ground state charge density distribution $\rho_{ch}(r)$ [14], *i.e.*

$$F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) j_0(qr) r^2 dr, \quad (12)$$

where

$$j_0(qr) = \sin(qr)/qr \quad (13)$$

is the zeroth order spherical Bessel function and q is the momentum transfer from the incident electron to the target nucleus.

The quadrupole form factors $F_2(q)$ (which are important in ^{10}B and ^{14}N nuclei) are described by the undeformed p-shell model as [15]:

$$F_2(q) = \frac{\langle r^2 \rangle}{Q} \left(\frac{4}{5P_J} \right)^{1/2} \int \rho_{2ch}(r) j_2(qr) r^2 dr \quad (14)$$

Where $j_2(qr)$ is the second order of the spherical Bessel functions, Q is the quadrupole moment and $\rho_{2ch}(r)$ is the quadrupole charge density distribution assumed, according to the undeformed p-shell model, to be the same as that of ground state charge density distribution $\rho_{ch}(r)$.

Here, P_J is a quadrupole projection factor given as:

$$P_J = J(2J - 1)/(J + 1)(2J + 3) \quad (15)$$

Where J is the angular momentum of the ground state.

Results and discussion

The charge density distributions, charge form factors and charge and matter root mean square (rms) radii of ^6Li , ^{10}B , ^{12}C and ^{14}N nuclei have been calculated using single-particle radial wave functions of Woods-Saxon (WS) and harmonic oscillator (HO) potentials. The calculated CDD of ^6Li nucleus has been compared with the experimental data of two parameter Fermi model CDD [16], while the calculated CDD's of ^{10}B , ^{12}C and ^{14}N nuclei have been compared with the experimental data of the harmonic-oscillator CDD [17,18]. Table-1

displays the parameters of Woods-Saxon V_0 , V_{so} , r_0 , r_{so} , a_0 , a_{so} and r_c employed in the present calculations for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei. Table-2 shows the calculated results of the charge and matter rms radii for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei compared with the experimental results [11,17,19]. It is clear from this table that the obtained results are in a good agreement with experimental results within quoted error. In Table-3 the calculated single-particle energies (ϵ) of the considered nuclei are compared with those obtained by the shell model OXBASH code [20]. This comparison demonstrates that the calculated single-particle energies are in very good coincidence with the results of the shell model OXBASH code.

Table -1: The parameters of Woods-Saxon employed in the present study for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$, and ${}^{14}\text{N}$.

Nuclei	V_0 (MeV)	V_{so} (MeV)	a_0 (fm)	a_{so} (fm)	r_0 (fm)	r_{so} (fm)	r_c (fm)
${}^6\text{Li}$	47.417	6.0	0.526	0.526	1.381	1.381	1.552
${}^{10}\text{B}$	49.587	6.0	0.542	0.542	1.236	1.236	1.306
${}^{12}\text{C}$	58.587	6.0	0.539	0.539	1.267	1.267	1.351
${}^{14}\text{N}$	62.962	6.0	0.527	0.527	1.319	1.319	1.431

Table -2: The calculated matter and charge rms radii for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei along with the experimental data.

Nuclei	$\langle r_{ch}^2 \rangle_{cal}^{1/2}$ (fm)		$\langle r_{ch}^2 \rangle_{exp}^{1/2}$ (fm)	$\langle r_m^2 \rangle_{cal}^{1/2}$ (fm)		$\langle r_m^2 \rangle_{exp}^{1/2}$ (fm)
	HO	WS		HO	WS	
${}^6\text{Li}$	2.43	2.46	2.43±0.06	2.51	2.48	2.46±0.21
${}^{10}\text{B}$	2.45	2.60	2.45±0.12	2.39	2.52	2.56±0.23
${}^{12}\text{C}$	2.48	2.47	2.47±0.01	2.41	2.38	2.35±0.02
${}^{14}\text{N}$	2.53	2.49	2.49±0.03	2.455	2.398	2.47±0.03

Table-3: The calculated single-particle energies for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei along with results of the Ref. [20].

Nuclei	nlj	proton		neutron	
		ϵ_{cal} (MeV)	ϵ (MeV) [20]	ϵ_{cal} (MeV)	ϵ (MeV) [20]
${}^6\text{Li}$	1s _{1/2}	18.880	18.880	20.167	20.167
	1p _{3/2}	3.182	3.182	4.238	4.238
${}^{10}\text{B}$	1s _{1/2}	21.972	21.972	24.555	24.555
	1p _{3/2}	6.703	6.703	8.915	8.915
${}^{12}\text{C}$	1s _{1/2}	31.770	31.770	34.819	34.819
	1p _{3/2}	15.691	15.691	18.448	18.448
${}^{14}\text{N}$	1s _{1/2}	38.635	38.635	41.988	41.988
	1p _{3/2}	23.260	23.260	26.376	26.376
	1p _{1/2}	19.667	19.667	22.795	22.795

The dependence of the ground state CDD's (in fm⁻³) on r (in fm) for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei are displayed in figures 1(a) to 1(d), respectively. In these figures, the filled circle symbols are the experimental data [16,17,18] whereas the blue and red curves are the calculated CDD's by Woods-Saxon (WS) and harmonic oscillator (HO) potentials, respectively. Fig.1 demonstrates that both red and blue curves are in excellent agreement with the experimental data throughout the whole range of r .

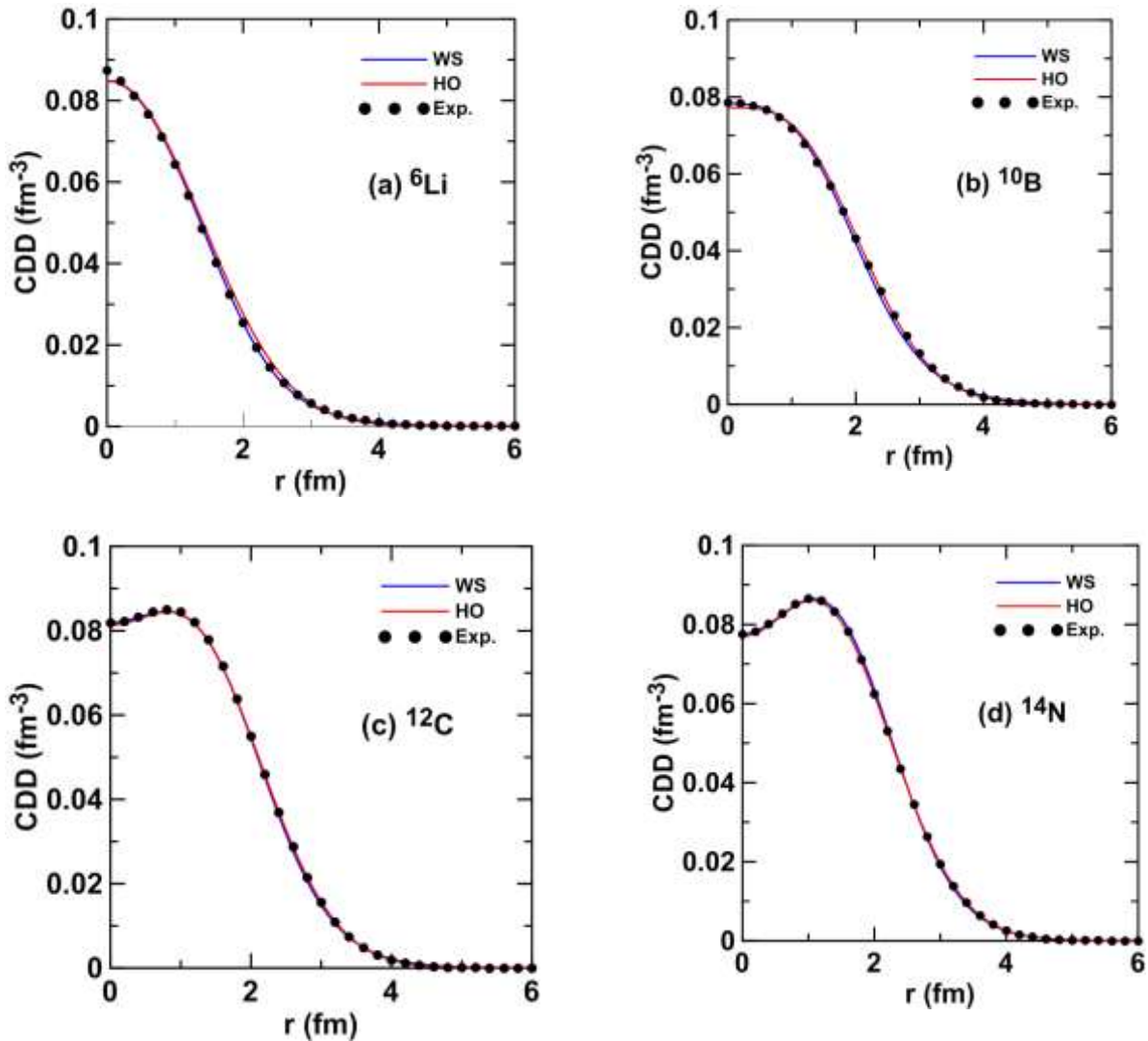


Fig. 1: The calculated charge density distributions for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei. The filled circle symbols are the experimental data.

In Fig. (2) the calculated charge form factors for ${}^{10}\text{B}$ (top panel) and ${}^{14}\text{N}$ (bottom panel) nuclei are compared with those of experimental data (filled circle symbols) [21,22]. Parts *a* and *b* of this figure are the calculated results obtained by HO and WS potentials, respectively. The individual contributions to the form factor are shown as dashed and dash-dotted curves for the monopole $|F_0(q)|^2$ and quadrupole $|F_2(q)|^2$ form factors, respectively, while the solid curves represent the total contribution $|F(q)|^2$, which is obtained as the sum of $|F_0(q)|^2$ and $|F_2(q)|^2$. The quadrupole form factors are calculated by the undeformed p-shell model. This figure shows that the results of monopole form factors for ${}^{10}\text{B}$ and ${}^{14}\text{N}$ nuclei don't describe the experimental data very well in the range of momentum transfers $q > 1.4 \text{ fm}^{-1}$ and 1.5 fm^{-1} for ${}^{10}\text{B}$ and ${}^{14}\text{N}$ nuclei, respectively. It is very clear that the contribution of the quadrupole form factors gives a strong modification to the monopole form factors and brings the calculated values very close to the experimental data.

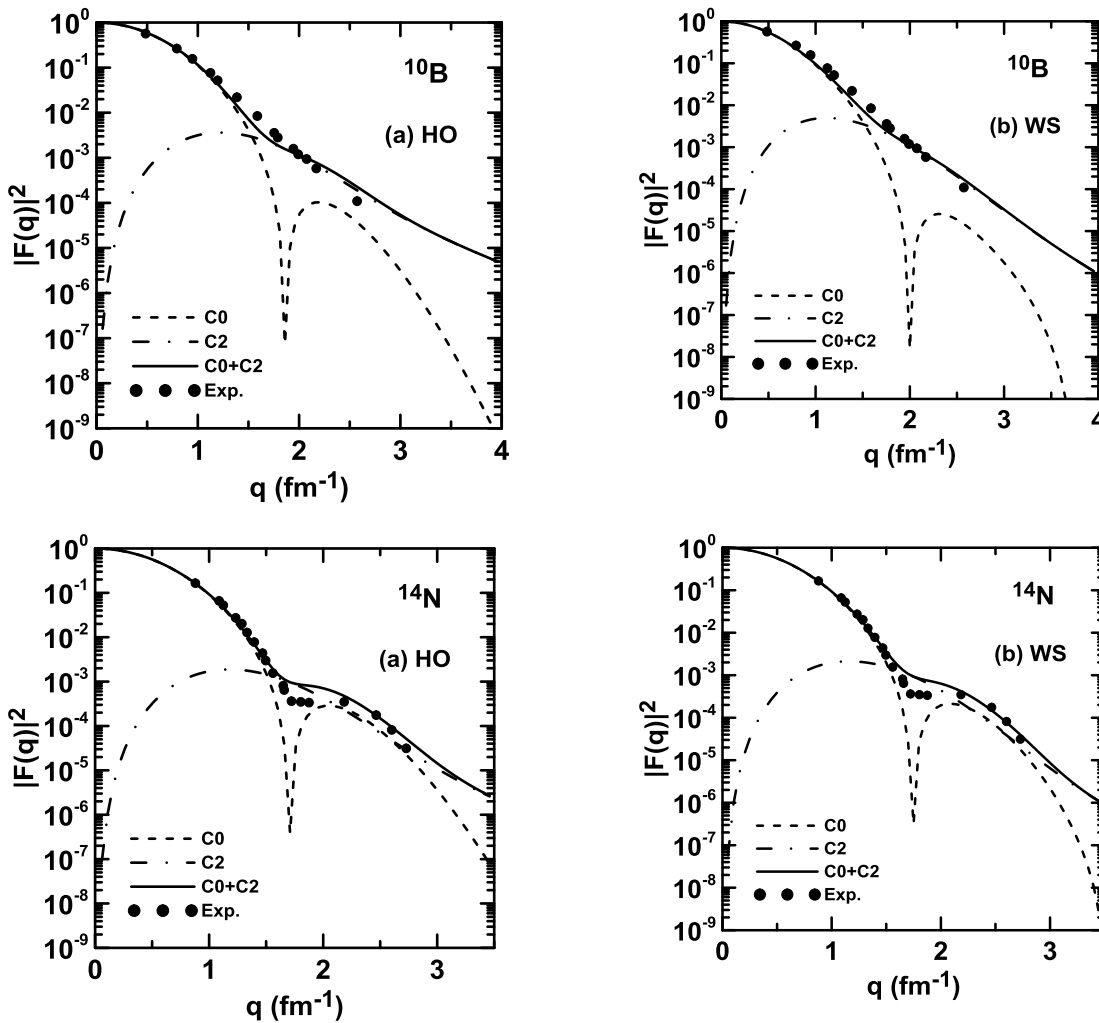


Fig. 2: The calculated form factors obtained by the HO and WS potentials for ^{10}B and ^{14}N nuclei. The experimental data are taken from Ref. [21] for ^{10}B and taken from Ref. [22] for ^{14}N .

In Figs. 3(a) and 3(b) we explore the calculated results for the elastic charge form factors of ^6Li and ^{12}C nuclei, respectively. The dashed and solid curves are the elastic charge form factors obtained by Woods-Saxon (WS) and harmonic oscillator (HO) potentials, respectively whereas the filled circle symbols are the experimental results. In ^6Li nucleus, the calculations of both the dashed and solid curves are in very good agreement with those of experimental data of ref. [15] up to $q = 2.3 \text{ fm}^{-1}$ and 1.5 fm^{-1} for the dashed and solid curves, respectively and they underestimate slightly these data beyond this region of q . In ^{12}C nucleus, the calculated results of the dashed curve agree with the experimental data [23] up to $q = 1.8 \text{ fm}^{-1}$ and it is discredited these data at higher momentum transfer. On the other hand, the experimental data are described very well by the calculated results of solid curve throughout all values of momentum transfer q . In addition, the diffraction minimum which is known from the experimental data is very well reproduced by the calculations of the dashed and solid curves.

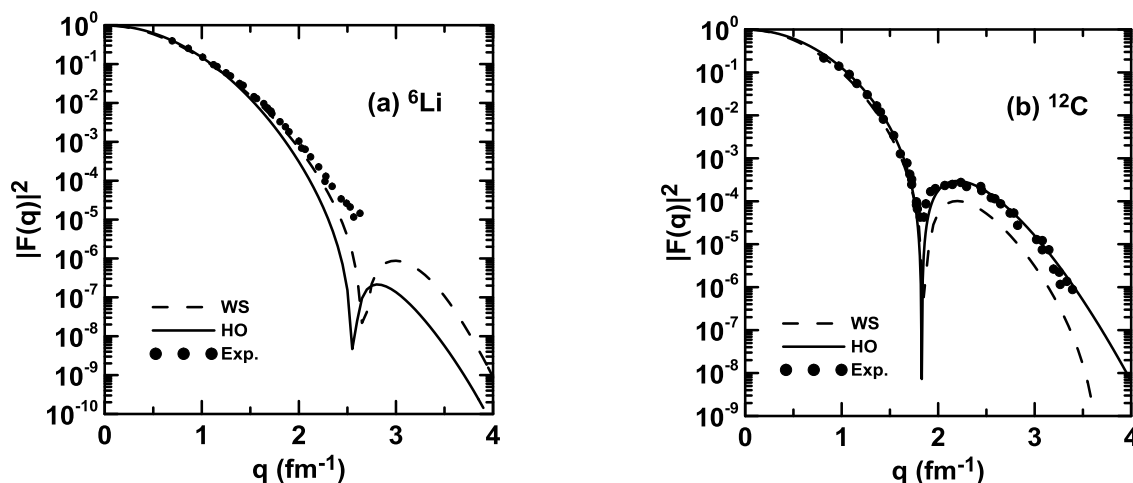


Fig. 3: The elastic charge form factors for ${}^6\text{Li}$ and ${}^{12}\text{C}$ nuclei calculated by the WS (dashed curve) and HO (solid curve) potentials. The experimental data (filled circle symbols) for ${}^6\text{Li}$ are taken from Ref. [15] while the experimental data for ${}^{12}\text{C}$ are taken from ref. [23].

Conclusions

The CDD and electron scattering form factors for ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{12}\text{C}$ and ${}^{14}\text{N}$ nuclei are calculated by the single particle radial wave functions of Wood-Saxson (WS) and harmonic oscillator (HO) potentials. From this work, it is possible to draw the following conclusions:

- 1- The calculated results of CDD for consider nuclei obtained by both WS and HO potentials are in very good accordance with experimental data.
- 2- The calculated charge and matter rms radii for the investigated nuclei are in a good agreement with the experimental data.
- 3- The results of elastic charge form factors for ${}^6\text{Li}$ nucleus calculated by WS potential are more close to the experimental data than the calculated results with HO potential.
- 4- The calculated elastic form factors of ${}^{12}\text{C}$ nucleus using HO potential are in the best agreement with the experimental data.
- 5- The contribution of the quadrupole form factors in ${}^{10}\text{B}$ and ${}^{14}\text{N}$ nuclei, which are described by the undeformed p-shell model, are essential in obtaining a remarkable agreement between the theoretical and experimental form factors.

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