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Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology

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Abstract : The purpose of this article is to extend some more topics of topology in neutrosophic topology. Eerily introduce neutrosophic topological space and open sets, closed sets, semi-open and semi closed sets, now in this manuscript we extend up to neutrosophic pre-open and pre-closed (NPO and NPC).
Keywords : Neutrosophic set, Neutrosophic Topology, Neutrosophic pre-open set, Neutrosophic pre-closed set.

1.1Introduction:

Neutrosophictechnique is a special technique is based on Neutrosophy. This theory developed by "FlorentinSmarandache"in 1995. In Neutrosophy consider every notion or idea is together with $\langle \text{A} \rangle$, $\langle \text{Anti-A} \rangle$ and $\langle \text{Neut-A} \rangle$ here $\langle \text{Anti-A} \rangle$ is the opposite or negation of $\langle \text{A} \rangle$, $\langle \text{Neut-A} \rangle$ is the field of "neutralities".Neutrosophic method is derived fromFuzzy logic or in intuitionistic Fuzzy logic. The first book was published on Neutrosophy as a title of book is Neutrosophy probability, set and logic in 1998. $[1]$, Rehoboth. The word Neutrosophy introduced from Latin "neuter" – neutral, Greek "Sophia"- skill/wisdom. Neutrosophy means sill on neutrals. The main task of this study is to apply neutrosophic method to the general theory of relativity, aiming to discover new hidden effects. Here it's why we decided to employ neutrosophic method in this field. Neutrosophic method means to find common features to uncommon entities. $\text{T}, \text{I}, \text{F}$ are called neutrosophic components will represent the truth value, indeterminacy value and false hood value respectively referring to neutrosophic methods.

1.2 Notations:

$\langle \text{A} \rangle \to$ Proposition

$\langle \text{NON-A} \rangle \to$ Not $\langle \text{A} \rangle$

$\langle \text{Anti-A} \rangle \to$ Opposite of $\langle \text{A} \rangle$

$\langle \text{Neut-A} \rangle \to$ Neither $\langle \text{A} \rangle$ nor $\langle \text{Anti-A} \rangle$

$\langle \text{A}' \rangle \to$ Aversion of $\langle \text{A} \rangle$

1.3 Remark:

$\langle \text{NON-A} \rangle$ is different to $\langle \text{Anti-A} \rangle$.

Example 1.1:

$\langle A \rangle = pink$, $\langle Anti\text{-}A \rangle =$ yellow But $\langle \text{NON-A} \rangle$ =green, red, blue…etc $(any\,colour\,\text{exp}ect\ pink)$ $\langle \text{Neut-A} \rangle$ =green, red, blue…etc $(any\,colour\,\text{exp}ect\ pink\ and\ yellow)$. ⟨ A' ⟩ = any shade of pink.

1.4Remark: Between an idea ⟨ A ⟩ and its opposite ⟨ Anti- A ⟩ there is a continuum power spectrum of neutralities ⟨ Neut- A ⟩ . Neutrosophic theory was studied in neutrosophic metric space, smooth topological spaces, arithmetic operations, rough sets, neutrosophic geometry, and neutrosophic probability….etc. Now, in natural way make longer neutrosophic theory in neutrosophic pre-open and pre-closed set and neutrosophic topological space.

1.5 Neutrosophic logic:

A logic in which each proposition is estimated to have the percentage of truth in subset T the percentage of indeterminacy in subset I and percentage of falsity in a subset F where T, I, F are defined above is called neutrosophic logic. (T,I,F) Truth values where T, I, F are standard or non-standard subsets of non-standard interval $]^{-}0,1^{+}[$, where

$\text{n}_{\text{inf}} = \inf \text{T} + \inf \text{I} + \inf \text{F} \geq 0$ And $\text{n}_{\text{sup}} = \sup \text{T} + \sup \text{I} + \sup \text{F} \leq 3$ .they are many neutrosophic rules of inference[2].

The degree of membership to $\text{-}\,\text{A}(\text{M})$, degree of non-membership-to $\text{-}\,\text{A}(\text{N})$, such that $\text{M+N} \leq 1$ when M+N=1 one obtain the fuzzy set, and if M+N<1 there is an indeterminacy I=1-M-N.Zadeh et. al. $[3]$ fuzzy set theory as a mathematical tool in 1965 for dealing with uncertainties where each element had each element had degree of membership acquire form [4-7].

The intuitionistic fuzzy set was introduced y Atanassov in $1983[4]$ as a generalization of fuzzy set where besides the degree of membership and the degree non-membership of each element. The neutrosophic set was introduced by Smarandache and explained neutrosophic set is a generalization of intuitionistic fuzzy set. In 2012 Salama,Alblow $[5]$ introduced the concept of topological space. In 2016 introduced the concept of neutrosophic semi-open sets in neutrosophic topological space by p.Ishwarya and dr.k.Bageerathi.

Currently, in this manuscript we recall the neutrosophic pre-open sets and neutrosophic per-closed in neutrosophic topological spaces.

2.1 Terminologies:

We recollect some important basic preliminaries, and in particular, the work of Smarandache in [1], Atanassov in [4, 5] and Salama [8, 9]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0,1^{+}[$ is nonstandard unit interval.

Definition 2.2: Let X be a non-empty fixed set. A Neutrosophic set (NSfor short) A is an object having the form $\text{A} = \{\langle \text{x}, \mu_{\text{A}}(x), \sigma_{\text{A}}(x), \gamma_{\text{A}}(x) \rangle\}, x \in X$ where $\mu_A(x), \sigma_A(x), \gamma_A(x)$ which represent the degree ofmember ship function $\mu_A(x)$ the degree of indeterminacy $\sigma_A(x)$ and the degree ofnon-member ship $\gamma_A(x)$ respectively of each element $x \in X$ to the set A [7].

Definition 2.3: The Neutrosophic subsets 0_N and 1_N in X as follows:

0_N may be defined as:

$(0_1)\ 0_N = \{\langle x,0,0,1\rangle : x \in X\}$
$(0_2)\ 0_N = \{\langle x,0,1,1\rangle : x \in X\}$
$(0_3)\ 0_N = \{\langle x,0,1,0\rangle : x \in X\}$
$(0_4)\ 0_N = \{\langle x,0,0,0\rangle : x \in X\}$

1_N may be defined as

$(1_1)\ 1_N = \{\langle x,1,0,0\rangle : x \in X\}$
$(1_2)\ 1_N = \{\langle x,1,0,1\rangle : x \in X\}$
$(1_3)\ 1_N = \{\langle x,1,1,0\rangle : x \in X\}$
$(1_4)\ 1_N = \{\langle x,1,1,1\rangle : x \in X\}$

Definition 2.4: Let $\mathrm{A}=\langle \mu_A, \sigma_A, \gamma_A \rangle$ be a NS on X , then the complement set of the $\mathrm{A}[\mathrm{C}(\mathrm{A})]$ may be defined as three kinds of complements.

$C_1\ C(A)=\{<x,1\text{-}\mu_A,1\text{-}\sigma_A(x),1\text{-}\gamma_A(x)>:x \in X\}$
$C_2\ C(A)=\{<x,\gamma_A,\sigma_A,\mu_A>:x \in X\}$
$C_3\ C(A)=\{<x,\gamma_A,1\text{-}\sigma_A,\mu_A,>:x \in X\}$

Example 2.1: The complement of O_N is I_N and the complement of I_N is O_N.

$O_N=\{<x,0,0,1>:x \in X\}$
$C_1=C(O_N)=\{<x,1,1,0>:x \in X\}$
$C_2=C(O_N)=\{<x,1,0,0>:x \in X\}$
$C_3=C(O_N)=\{<x,1,1,0>:x \in X\}$
O_N and I_N are complement each other

Definition 2.5: Let x be a non empty set and neutrosophic sets A and B in the form

$A=\{<x,\mu_A(x),\sigma_A(x),\gamma_A(x)>:x \in X\}$
$B=\{<x,\mu_B(x),\sigma_B(x),\gamma_B(x)>:x \in X\}$ then we consider two possible definitions for subsets.
$1.A \subseteq B \Leftrightarrow \mu_A(x) < \mu_B(x), \sigma_A(x) \le \sigma_B(x), and\ \gamma_A(x) \ge \gamma_B(x) \forall x \in X.$
$2.A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x), and\ \gamma_A(x) \ge \gamma_B(x) \forall x \in X.$

Proposition 2.1: For any neutrosophicset A, then the following conditions are holds.

(1) $O_N \subset A$, $O_N \subseteq O_N$
(2) $A \subseteq I_N$, $I_N \subseteq I_N$

Definition 2.6: Let X be a non-empty set and A,B are neutrosophic sets then

$(i) A \cap B$ May be defined as
$(I_1) A \cap B = < x, \mu_A \wedge \mu_B, \sigma_A \wedge \sigma_B, \gamma_A \vee \gamma_B >$
$(I_2) A \cup B = < x, \mu_A \vee \mu_B, \sigma_A \wedge \sigma_B, \gamma_A \wedge \gamma_B >$

We can easily generalize the operations of intersection and union in def 5 to arbitrary family of neutrosophic sets.

Proposition 2.2: For all A and B are two neutrosophic sets then the following conditions are true.

(1) $C(A \cap B) = C(A) \cup C(B)$
(2) $C(A \cup B) = C(A) \cap C(B)$

Definition 2.7: A neutrosophic topology (NT) is a non-empty set X is a family τ of a neutrosophic sets in X satisfying the following condition.

(NT_1) $O_N, I_N \in \tau$
(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(NT_3) $\cup G_i \in \tau$ *for every* $\{G_i : i \in j\} \subseteq \tau$

In this case (X,τ) is called a neutrosophic topological space.

Example 2.2:

Let $X = \{x\}$ and
$A = \{< x,0.5,0.5,0.4 > : x \in X\}$
$B = \{< x,0.4,0.5,0.8 > : x \in X\}$
$C = \{< x,0.5,0.6,0.4 > : x \in X\}$
$D = \{< x,0.4,0.6,0.8 > : x \in X\}$

Then the family $\tau = \{O_N, I_N, A, B, C, D\}$ of neutrosophic set in X is neutrosophic topology X.

Definition 2.8: The elements of τ is neutrosophic open sets the complement of neutrosophic open set is called neutrosophic closed set.

Definition 2.9: Let (X,τ) be neutrosophic topological space and $A = < x, \mu_A, \sigma_A, \gamma_A >$ be a neutrosophic set in X then neutrosophic closure and neutrosophic interior of A are defined by

$Ncl(A) = \cap\{K : k \text{ is a Ncs in } X \text{ and } A \subseteq K\}$
$Nint(A) = \cup\{G : g \text{ is a Nos in } X \text{ and } G \subseteq A\}$

It can also show that $\mathrm{Ncl}(\mathrm{A})$ is neutrosophic closed set and $\mathrm{Nint}(\mathrm{A})$ is neutrosophic open set in X

(i) A is neutrosophic open set $\Leftrightarrow \mathrm{A}=\mathrm{Nint}(\mathrm{A})$

(ii) A is neutrosophic closed set $\Leftrightarrow \mathrm{A}=\mathrm{Ncl}(\mathrm{A})$

Proposition 2.3: Let (X,τ) be neutrosophic topological space and $A\in(X,\tau)$ we have

(a) $\mathrm{Ncl}(C(A))=C(\mathrm{Nint}(A))$

(a) $\mathrm{Nint}(C(A))=C(Ncl(A))$

Proposition 2.4: Let (X,τ) be neutrosophic topological space and A,B are two neutrosophic set in X then the following properties are hold.

a) $\mathrm{Nint}(A)\subseteq A$

b) $A\subseteq Ncl(A)$

c) $A\subseteq B\Rightarrow \mathrm{Nint}(A)\subseteq \mathrm{Nint}(B)$

d) $A\subseteq B\Rightarrow \mathrm{Ncl}(A)\subseteq \mathrm{Ncl}(B)$

e) $\mathrm{Nint}(\mathrm{Nint}(A))=\mathrm{Nint}(A)$

f) $\mathrm{Ncl}(\mathrm{Ncl}(A))=\mathrm{Ncl}(A)$

g) $\mathrm{Nint}(A\cap B)=\mathrm{Nint}(A)\cap \mathrm{Nint}(B)$

h) $\mathrm{Ncl}(A\cup B)=\mathrm{Ncl}(A)\cup \mathrm{Ncl}(B)$

i) $\mathrm{Nint}(O_N)=\mathrm{Ncl}(O_N)=O_N$

j) $\mathrm{Nint}(I_N)=\mathrm{Ncl}(I_N)=I_N$

k) $A\subseteq B\Rightarrow C(B)\subseteq C(A)$

l) $\mathrm{Ncl}(A\cap B)\subseteq \mathrm{Ncl}(A)\cap \mathrm{Ncl}(B)$

m) $N\mathrm{int}(A\cup B)\supseteq \mathrm{Nint}(A)\cup \mathrm{Nint}(B)$

Definition 2.10: Let (X,τ) be neutrosophic topological space and $A=<x,\mu_A,\sigma_A,\gamma_A>$ be a neutrosophic set in X then A is called a neutrosophic semi-open if $A\subseteq \mathrm{Ncl}(\mathrm{Nint}(A))$ and also neutrosophic semi-closed if $\mathrm{Nint}(\mathrm{Ncl}(A))\subseteq A$ .The complement of neutrosophic semi open set is a neutrosophic semi closed set.

3. Neutrosophic Pre -open:

In this section the concept of neutrosophic pre-open set of X is introduced and also characterizations of neutrosophic pre-open sets.

Definition 3.1: Let A be neutrosophic set of a neutrosophic topology. Then A is said to be Neutrosophic pre-open[NPO]set of X if there exists a neutrosophic open set NO such that $NO\subseteq A\subseteq NO(Ncl(A))$.

Theorem3.1:A subset A in neutrosophic topological spaces X is neutrosophicpre open set if and only if $A\subseteq N\mathrm{int}(Ncl(A))$.

Proof: Suppose $A \subseteq N\operatorname{int}(Ncl(A))$

Now we have to prove that A in neutrosophic topological space
i.e., A is neutrosophic pre-open set.
We know that $NO = N\operatorname{int}(A)$
$NO \subseteq A \subseteq NO(Ncl(A))$.

Therefore A is neutrosophic topological set.

Conversely suppose that

Let A be neutrosophic topological set in X .i.e. $NO \subseteq A \subseteq NO(Ncl(A))$ for some NO
But $NO \subseteq N\operatorname{int}(A)$, thus $NO(Ncl(A)) \subseteq N\operatorname{int}(Ncl(A))$
Hence $A \subseteq NO(Ncl(A)) \subseteq N\operatorname{int}(Ncl(A))$
Therefore $A \subseteq N\operatorname{int}(Ncl(A))$

Hence proved the theorem

Theorem3.2: In neutrosophic topology the union of two neutrosophic pre-open sets again a neutrosophic pre-open set.

Proof: Let $A\, and\, B\, are$ neutrosophic pre-open sets in X

$A \subseteq N\operatorname{int}(Ncl(A))$
$B \subseteq N\operatorname{int}(Ncl(B))$
$Therefore\ A \cup B \subseteq N\operatorname{int}(Ncl(A)) \cup Ncl(B)$
$A \cup B \subseteq N\operatorname{int}(Ncl(A) \cup Ncl(B))$
$A \cup B \subseteq N\operatorname{int}(Ncl(A \cup B))$

By the definition $A \cup B$ is a neutrosophic open set in X .

Theorem3.3: let (X, τ) be an NTS.If $\{A_\alpha\}_{\alpha \in \Delta}$ is a collection of NPO sets in a NTS X then $\cup_{\alpha \in \Delta} A_\alpha$ is NPO set in X.

Proof: for each $\alpha \in \Delta$,we have a neutrosophic open set NO_α such that

$NO_\alpha \subseteq A_\alpha \subseteq NO_\alpha(NCl(A))$, then $\cup_{\alpha \in \Delta} NO_\alpha \subseteq \cup_{\alpha \in \Delta} A_\alpha \subseteq \cup_{\alpha \in \Delta} NO_\alpha(Ncl(A))$
$\cup_{\alpha \in \Delta} A_\alpha \subseteq \cup_{\alpha \in \Delta} N\operatorname{int}_\alpha(Ncl(A))$

Hence theorem proved.

Remark3.1: The intersection of any two NPO sets need not be a NPO set in X as shown by the following example.

Example: let $X = \{a, b\}$

$A = \{\langle 0.4, 0.8, 0.9\rangle, \langle 0.7, 0.5, 0.3\rangle\}$

$B=\{\langle 0.5,0.8,0.6\rangle,\langle 0.8,0.4,0.3\rangle\}$
$C=\{\langle 0.4,0.7,0.9\rangle,\langle 0.6,0.4,0.4\rangle\}$
$D=\{\langle 0.5,0.7,0.5\rangle,\langle 0.8,0.4,0.6\rangle\}$
Then $\tau=\{0_N,A,B,C,D,1_N\}$ is NTS

Consider

$A_1=\{\langle 0.6,0.8,0.6\rangle,\langle 1.0,0.5,0.5\rangle\}$
$A_2=\{\langle 1.0,1.0,0.3\rangle,\langle 0.7,0.3,0.6\rangle\}$

From this example $A_1\cap A_2$ is not NPO set.

Theorem 3.4: Every neutrosophic open set in the NTS in X is NPO set in X.

Proof: let A be NO set in NTS.

Then $A=N\operatorname{int}(A)$
Clearly $A\subseteq Ncl(A)$
$N\operatorname{int}(A)\subseteq N\operatorname{int}(Ncl(A))$
$A\subseteq N\operatorname{int}(Ncl(A))$
A is a NPO set in X .

Theorem 3.5: Let A be neutrosophic pre-open set in the neutrosophic topological space X and suppose $A\subseteq B\subseteq Ncl(A)$ then B is neutrosophic pre-open set in X .

Proof: Let A be NO set in neutrosophic topological space X .

Then $A=N\operatorname{int}(A)$ also
$A\subseteq Ncl(A)$
$N\operatorname{int}(A)\subseteq N\operatorname{int}(Ncl(A))$
$A\subseteq N\operatorname{int}(Ncl(A))$

Hence the theorem proved.

Lemma3.1: Let A be an NO set in X and B a neutrosophic pre-open set in X then there exists an NO set G in X such that

$B\subseteq G\subseteq Ncl(B)$ it follows that
$A\cap B\subseteq A\cap G\subseteq A\cap Ncl(B)\subseteq Ncl(A\cap B)$

Now, since $A\cap G$ is open, from the above (theorem 5) lemma, $A\cap B$ is neutrosophic pre-open set in X .

Proposition 3.1: Let X and Y are neutrosophic topological space such that X is neutrosophic product related to Y then the neutrosophic product $A\times B$ of a neutrosophicpre open set A of X and a neutrosophic pre open set B of Y is a neutrosophic pre open set of the neutrosophic product topological space $X\times Y$.

Proof: let $0_1\subseteq A\subseteq Ncl(0_1)$ and $0_2\subseteq B\subseteq Ncl(0_2)$

Then $0_1 \times 0_2 \subseteq A \times B \subseteq Ncl(0_1) \times Ncl(0_2)$
$0_1 \times 0_2 \subseteq A \times B \subseteq Ncl(0_1 \times 0_2)$
$NInt(0_1 \times 0_2) \subseteq NInt(A \times B) \subseteq NInt(Ncl(0_1 \times 0_2))$
$0_1 \times 0_2 \subseteq A \times B \subseteq N\operatorname{int}(Ncl(0_1 \times 0_2))$
Hence $A \times B$ is NPO set in $X \times Y$

4. Neutrosophic Pre-closed sets:

Definition 4.1: Let A be neutrosophic set of a neutrosophic topology spaces X . Then A is said to be neutrosophic pre-closed sets of X if there exists a NC set such that $Ncl(NC) \subseteq A \subseteq NC$.

Theorem4.1:A subset A in a NTS X is NCS set if and only if $Ncl(N\operatorname{int}(A)) \subseteq A$

Proof: consider $Ncl(N\operatorname{int}(A)) \subseteq A$

Then $NC = Ncl(A)$ clearly $Ncl(N\operatorname{int}(A)) \subseteq A \subseteq NC$
Therefore A is NPC set
Conversely, suppose that Let A be NPC set in X
Then $NC(N\operatorname{int}(A)) \subseteq A \subseteq NC$ for some NS closed set NC .but $Ncl(A) \subseteq NC$

Hence the theorem proved.

Theorem4.2: Let (X,τ) be NTS and A be a neutrosophicsubset of X then A is a neutrosophic pre-closed sets if and only $C(A)$ is neutrosophic pre-open set in X.

Proof: Let A be a neutrosophic pre-closed set subset of X .

Clearly $Ncl(N\operatorname{int}(A)) \subseteq A$
Taking complement on both sides
$C(A) \subseteq C(Ncl(N\operatorname{int}(A)))$
$C(A) \subseteq N\operatorname{int}(Ncl(C(A)))$
Hence $C(A)$ is neutrosophic pre-open set
Conversely suppose that $C(A)$ is neutrosophic pre-open set i.e. $C(A) \subseteq N\operatorname{int}(Ncl(C(A)))$
Taking complement on both sides we get $Ncl(N\operatorname{int}(A)) \subseteq A,\ A$ neutrosophic pre-closed set

Hence the theorem proved.

Theorem4.3: Let (X,τ) be a neutrosophic topological spaces. Then intersection of two neutrosophic pre-closed set is also a neutrosophic pre-closed set.

Proof: let $A\, and\, B$ are neutrosophic pre-closed sets on (X,τ)

Then $Ncl(N\operatorname{int}(A)) \subseteq A,\ Ncl(N\operatorname{int}(B)) \subseteq B$
Consider $A \cap B \supseteq Ncl(N\operatorname{int}(A)) \cap Ncl(N\operatorname{int}(B))$

$$\supseteq Ncl\left(N\operatorname{int}(A)\bigcap N\operatorname{int}(B)\right)$$
$$\supseteq Ncl\left(N\operatorname{int}(A\cap B)\right)$$

$$Ncl\left(N\operatorname{int}(A\cap B)\right)\subseteq A\cap B$$

Hence $A\cap B$ is neutrosophic pre-closed set.

Remark4.1: The union of any two neutrosophic pre-closed sets need not be a neutrosophicclosed set on (X,τ).

Theorem 4.4: Let $\{A\}_{\alpha\in\Delta}$ be a collection of neutrosophic pre-closed sets on (X,τ) then $\underset{\alpha\in\Delta}{\cap} A_{\alpha\in\Delta}$ is neutrosophic pre-closed sets on (X,τ).

Proof: we have a neutrosophic set NC_α such that $NC_\alpha\left(N\operatorname{int}(A)\right)\subseteq A_\alpha\subseteq NC_\alpha$ for all $\alpha\in\Delta$

Then $\underset{\alpha\in\Delta}{\cap} NC_\alpha\left(N\operatorname{int}(A)\right)\subseteq\underset{\alpha\in\Delta}{\cap} A_\alpha\subseteq\underset{\alpha\in\Delta}{\cap} NC_\alpha$

$\underset{\alpha\in\Delta}{\cap} NCl_\alpha\left(N\operatorname{int}(A)\right)\subseteq\underset{\alpha\in\Delta}{\cap} A_\alpha$

Hence $\underset{\alpha\in\Delta}{\cap} A_\alpha$ is neutrosophic pre-closed set on (X,τ).

Theorem 4.5: Every neutrosophic closed set in the neutrosophic topological spaces (X,τ) is neutrosophic pre-closed set in (X,τ).

Proof: Let A be neutrosophic closed set means $A=Ncl(A)$ and also $N\operatorname{int}(A)\subseteq A$

From that,

$Ncl\left(N\operatorname{int}(A)\right)\subseteq Ncl(A), Ncl\left(N\operatorname{int}(A)\right)\subseteq A$, since $A=Ncl(A)$

Hence A is a neutrosophic pre-closed sets.

Theorem 4.6: Let A be a neutrosophic closed set in neutrosophic topological spaces (X,τ) and suppose $N\operatorname{int}(A)\subseteq B\subseteq A$ then B is neutrosophic pre-closed set on (X,τ)

Proof: Let A be a neutrosophic set in neutrosophic topological spaces (X,τ)

Suppose $N\operatorname{int}(A)\subseteq B\subseteq A$

There exist a neutrosophic closed set NC, such that $NC\left(N\operatorname{int}(A)\right)\subseteq B\subseteq A\subseteq NC$.

Then $B\subseteq NC$ and also $N\operatorname{int}(B)\subseteq B\subseteq NC$

Thus, $Ncl\left(N\operatorname{int}(B)\right)\subseteq B$

Hence B is neutrosophic pre-closed set on (X,τ).

Theorem4.7: Let X and Y are neutrosophic topological space such that X is neutrosophic product related to Y then the neutrosophic product $A\times B$ is a neutrosophic pre closed set of the neutrosophic product topological space $X\times Y$. Where neutrosophic pre closed set A of X and a neutrosophic pre closed set B of Y.

Proof: Let A and B are neutrosophic pre-closed set

$C_1(N\operatorname{int}(A)) \subseteq A \subseteq C_1 \; and \; C_2(N\operatorname{int}(A)) \subseteq A \subseteq C_2$

Form the above,

$C_1(N\operatorname{int}(A)) \times C_2(N\operatorname{int}(A)) \subseteq A \times B \subseteq C_1 \times C_2$

$(C_1 \times C_2)(N\operatorname{int}(A \times B)) \subseteq A \times B \subseteq C_1 \times C_2$

Hence $A \times B$ is neutrosophic pre-closed set in neutrosophic topological space $X \times Y$.

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