

Parameter Extraction of Hodgkin-Huxley Type Circuit Model of Neuron using Advanced Algorithm

Rashmi Deka*, Jiten Ch. Dutta

Department of Electronics and Communication Engg. Tezpur University, Tezpur, Assam, India

Abstract : Hodgkin and Huxleys' conductance based model of biological neuron can accurately reproduce the waveform of the membrane voltage as well as the spike timing in response to injected currents. Based on this model, a simple electronic circuit is proposed as an analog of postsynaptic membrane of neuron. The simulated sodium and potassium conductances and the electronic action potential of this circuit are compared with Hodgkin-Huxley(HH) model. For finding the good model parameter set to membrane voltage recordings, we have first tested three algorithms, namely Genetic Algorithm(GA), Particle Swarm optimization(PSO) and Firefly algorithm(FA) on originally recorded action potential of squid giant axon by Hodgkin and Huxley and found that FA is more efficient method for parameter extraction of action potential. Due to this reason, FA has been used to extract the parameters of the action potential generated by our proposed electronic circuit model. Parameter values are compared with voltage clamped experimental data of Hodgkin and Huxley and found that this analog circuit could be very useful for simulating different types of action potential.

Keywords: Hodgkin-Huxley (HH) model; Firefly Algorithm; Particle Swarm Optimization; Genetic Algorithm.

Introduction

Hodgkin and Huxley in 1952 [1]-[4] used voltage-clamp methods to obtain extensive quantitative experimental results and proposed a system of four dimensional differential equations containing non-linear functions which is called Hodgkin-Huxley model (HH) of neuron[5]. Due to the inclusion of many biological phenomena of membrane through conductances, this model can reproduce electrophysiological measurements to a high degree of accuracy. HH model consists of a set of differential equation of the form given below:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^p (V - V_K) + \bar{g}_{Na} m^q h^r (V - V_{Na}) + \bar{g}_l (V - V_l) \quad (1)$$

$$\frac{dm}{dt} = \alpha_m(V)(1 - m) - \beta_m(V)m \quad (2)$$

$$\frac{dh}{dt} = \alpha_h(V)(1 - h) - \beta_h(V)h \quad (3)$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n \quad (4)$$

Where C_M & V represent the membrane capacitance and membrane voltage respectively. I is the total current, g_K and g_{Na} are the maximal conductance of potassium and sodium channels respectively. V_K , V_{Na} and V_l are respectively the displacement from the resting potential of potassium, sodium and leakage ions. Values m , n , h are activation and inactivation form of different type of channels. p , q and r are integer values.

α_m , β_m , α_h , β_h , α_n , β_n are the relevant rate constants dependent on membrane voltage.

Fitness Function

Fitness function, also called cost function is required to fit a model to reference data. We choose here to define fitness functions, making use of only rate constants as used by Hodgkin and Huxley:

$$\alpha_n = a (V + 10) / (\exp \frac{V + 10}{10} - 1) \quad (5)$$

$$\beta_n = b \exp(V / 80) \quad (6)$$

$$\alpha_m = c(V + 25) / (\exp \frac{V + 25}{10} - 1) \quad (7)$$

$$\beta_m = d \exp(V / 18) \quad (8)$$

$$\alpha_h = e \exp(V / 20) \quad (9)$$

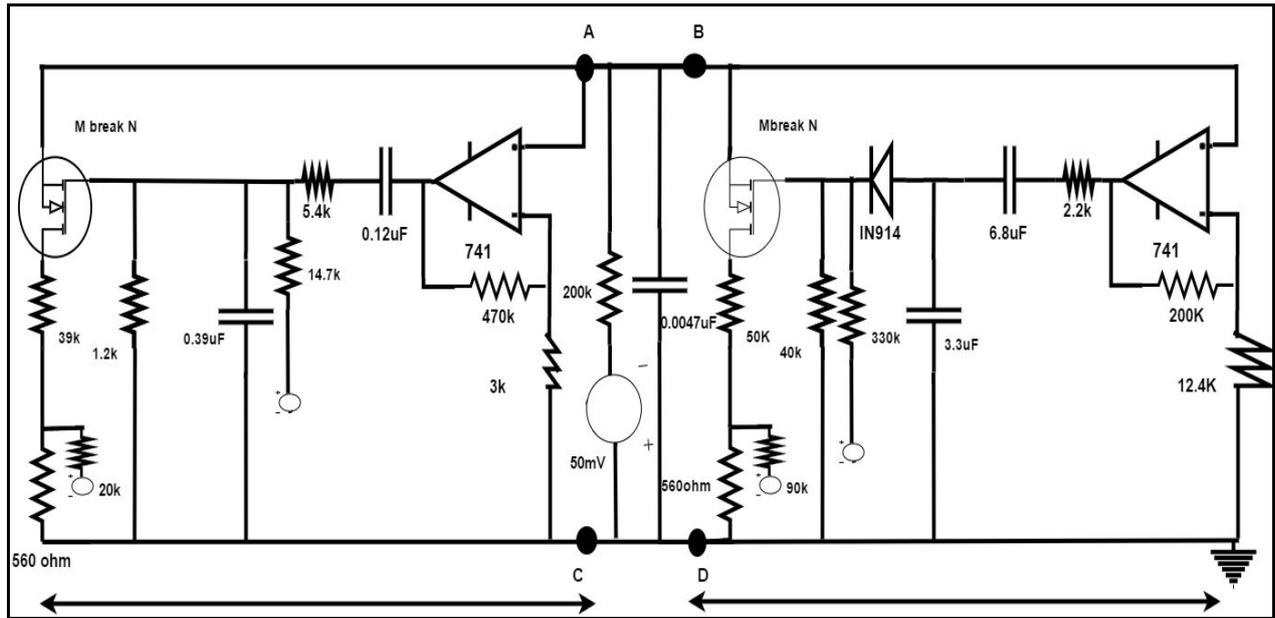
$$\beta_h = f / (\exp \frac{V + 30}{10} + 1) \quad (10)$$

The values of the constants a , b , c , d , e and f are estimated for originally recorded membrane voltage by Hodgkin and Huxley using the three algorithms namely GA, PSO and FA and then compared with H-H data. It was found that FA is more efficient method for parameter extraction of action potential.

Neuron Circuit Model: The NEUROMOSFET

The experimental data of Hodgkin and Huxley show that sodium and potassium conductances have very low values in their resting state. When an electrical voltage is applied across the membrane, both conductances are increased and when sufficiently large potential is applied, they reach a saturation value beyond which they remain constant. Also conductances show time dependence. Its relaxation time for both are voltage dependent. The potassium conductance follows a sigmoid time course to reach a steady-state amplitude, but sodium conductance is only temporary.

To simulate the characteristics of these conductances, the basic circuit used in Fig.1 has been used. The MOSFET has been used to represent the conductances because FET can be used as a variable resistor at low drain voltage (V_{DS}). The drain to source conductance g_{DS} can be varied by the gate to source voltage (V_{GS}). It is found that its g_{DS} versus V_{GS} characteristic is very similar to that of the axon membrane conductances. One MOSFET is used for g_{Na} and one for g_K . The voltage across the membrane is represented by V_{DS} and the membrane current is represented by drain to source current (I_{DS}). The transconductance g_{DS} represents either g_{Na} or g_K . To simulate the delayed rise in potassium conductance, the diode (IN914) has been introduced in the potassium circuit. The delay can be adjusted by varying the two negative biases on the diode. In case of sodium conductance, the time constant for the rise of g_{DS} is to be made more rapid and, therefore, coupling capacitor is much smaller than the potassium circuit to provide a faster inactivation for the sodium conductance in accordance with Hodgkin-Huxley's experimental data. A more complete description of the circuit can be found in [6]. P Spice simulation results are given as below:



Sodium circuit

Potassium circuit

Fig.1: The proposed Neuron Electronic Circuit using MOSFET: NEUROMOSFET

The results obtained from this NEUROMOSFET circuit is shown in Figures.2 to Fig.4. Fig. 2 shows the action potential obtained from the circuit. Fig.3 and Fig. 4 show the simulated sodium and potassium conductances respectively.

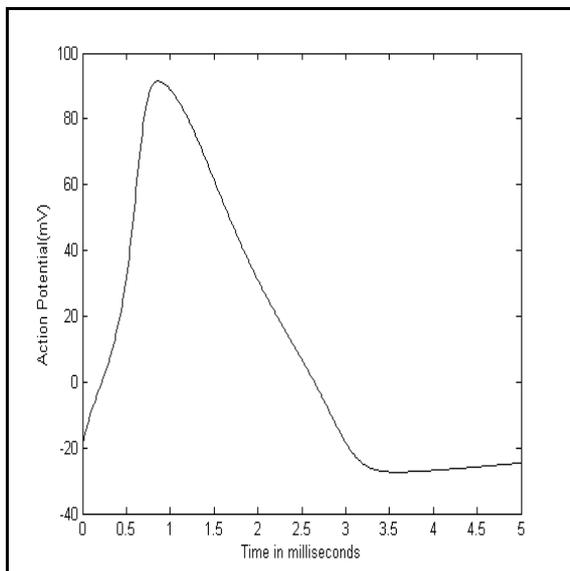


Fig.2: Action potential obtained from NEUROMOSFET (simulated in

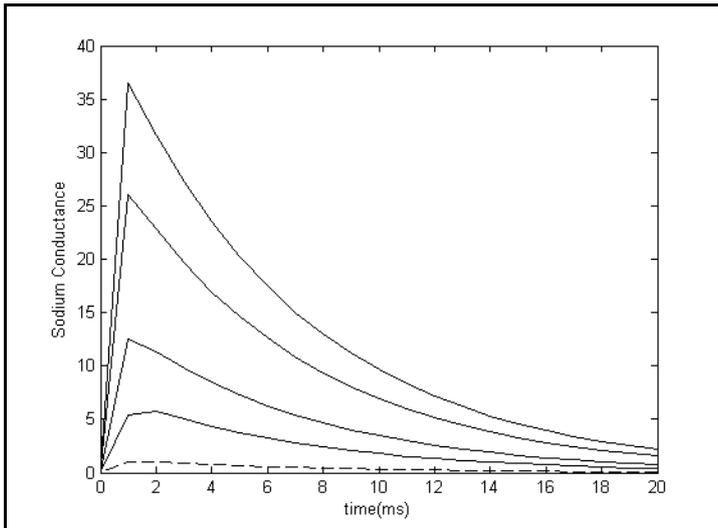


Fig.3: Sodium Conductance (µS) obtained from the sodium circuit at V_{GS} equal to 20 mV,40 mV,60 mV,80 mV, and 100 mV PSPICE).

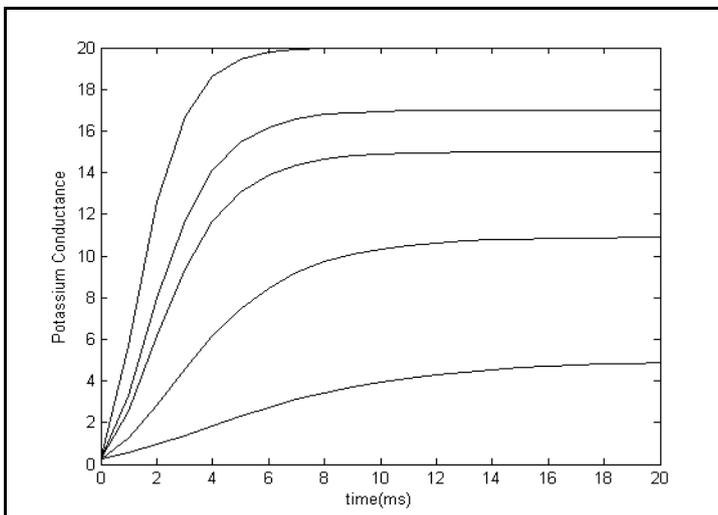


Fig.4: Potassium Conductance (µS) obtained from the potassium circuit at V_{GS} equal to 20 mV,40 mV,60 mV, 80 mV and 100 mV.

Firefly Algorithm

Firefly algorithm is based on the flashing patterns of fireflies and is a nature inspired algorithm. The fireflies are unisex and they are attracted to each other by its brightness. The less bright one moves towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases . If there is no brighter one than a particular firefly, it wil move randomly. The variation of attractiveness β with the distance r can be defined as [7]-[8]:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{11}$$

Where β_0 is the attractiveness at $r = 0$.

The movement of the firefly i is attracted to another brighter firefly j can be defined as [7]-[8]:

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2} (x_j^t - x_i^t) + \alpha_t \epsilon_i^t \tag{12}$$

Here the second term of RHS is the attraction of fireflies. Third term is for randomization with α_t and ε_1^t is a vector of random numbers obtained from Gaussian or uniform distribution. If $\gamma=0$, it takes the form of particle swarm optimization and if $\beta=0$, it reduces to random walk. $\beta=1$ is taken for most of the applications [7].

As described above, α_t is randomization parameter, the tuning of parameter is done in the iteration methods to produce diverse solutions. α_t is described as:

$$\alpha_t = \alpha_0 \delta^t, \quad (0 < \delta < 1) \quad (13)$$

α_0 is the initial random scaling factor and δ is a cooling factor ranging from 0.95 to 0.97.

FA is more efficient if α_0 is related to scaling variable. If the average scaling variable of problem is L , it is set to $0.01L$. 0.01 is taken because random walk requires number of steps to reach the desired solution before jumping too far. γ should also be related to scaling variable L and can be set as $\gamma = 1/\sqrt{L}$.

In most of the applications, population size is taken from 25 to 40.

Results and Discussion

The values of the parameter of the action potential generated by the NEUROMOSFET circuit have been estimated using the FA optimization algorithm. The algorithm has been implemented in MATLAB 2010. Fig.5 shows the action potential for estimation of parameters discussed in above section for the NEUROMOSFET. The output signal of this circuit has been taken as reference signal. Table 1 shows the values obtained by implementing FA on the proposed circuit output and the values obtained from Hodgkin and Huxley' voltage clamp experiments. Analysis shows that the proposed NEUROMOSFET circuit can reproduce action potential of neuron satisfactorily.

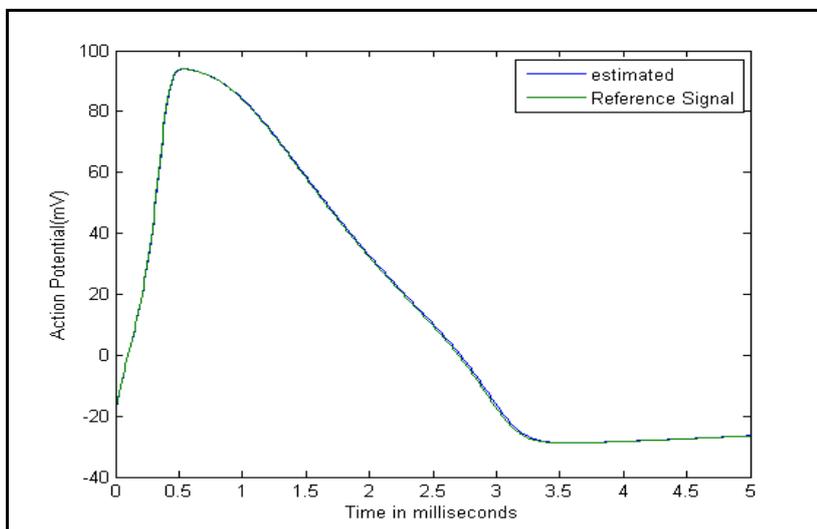


Fig.5: Estimation of parameters from reference potential

Table 1: Estimation of parameters using FA

Parameters	Values obtained from voltage-clamped experiments of Hodgkin and Huxley	Parameters estimated by advanced algorithm (FA)	Parameters	Values obtained from voltage-clamped experiments of Hodgkin and Huxley	Parameters estimated by advanced algorithm (FA)
$g_{Na}(\mu S/cm^2)$	120	120	$E_l(mV)$	-10.613	-10
$g_K(\mu S/cm^2)$	36	36	$C_M(\mu F)$	1	0.98
$g_l(\mu S/cm^2)$	0.3	0.5	a	0.01	0.01
$E_{Na}(mV)$	-115	-115	b	0.125	0.12
$E_K(mV)$	12	12	c	0.1	0.1
d	4	4	p	4	4
e	0.07	0.07	q	3	3
f	1	1	r	1	1

Conclusion

A simple analog containing a few electronic elements has been proposed (NEUROMOSFET) to simulate the action potential of excitable membrane. The parameters of the simulated action potential of the NEUROMOSFET circuit have been extracted by using FA with respect to HH model as reference. The estimated parameters have been compared with voltage clamped experimental data of Hodgkin and Huxley and found that the circuit can be used for simulating a wide variety of membrane conductances and different types of action potential. This may also play an important role in the field of neurology for simulation of receptor functions and electrical activity of neuron.

Acknowledgment

The authors like to extend their gratitude to the Tezpur University for providing facilities in the Laboratory.

Conflict of Interest

There is no conflict of interest

References

- Hodgkin AL, Huxley AF. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of physiology*. 1952 Aug 28;117(4):500-544.
- Hodgkin AL, Huxley AF. Currents carried by sodium and potassium ions through the membrane of the giant axon of *Loligo*. *The Journal of physiology*. 1952;116(4):449-472.
- Hodgkin AL, Huxley AF. The components of membrane conductance in the giant axon of *Loligo*. *The Journal of physiology*. 1952; 116:473–96.
- Hodgkin AL, Huxley AF. The dual effect of membrane potential on sodium conductance in the giant axon of *Loligo*. *The Journal of physiology*. 1952; 116:497–506.
- Wang J, Chen L, Fei X. Analysis and control of the bifurcation of Hodgkin-Huxley model. *Chaos, Solutions and Fractals*. 2007;31: 247-256
- Roy, G. A simple electronic analog of the squid axon membrane: The NEUROFET. *IEEE Transactions on Biomedical Engineering*. 1972: 60-63.
- Yang XS. Firefly Algorithm for Multimodal Optimization. *Stochastic Algorithms: Foundations and Applications*. 5th International Symposium, SAGA 2009, Sapporo, Japan. October 26-28, 2009.

8. Yang XS, He X. Firefly algorithm: recent advances and applications. International Journal of Swarm Intelligence.2013; 1(1) :36-50.
