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Global Chaos Control of the FitzHugh-Nagumo Chaotic Neuron Model via Integral Sliding Mode Control

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, India

Abstract: Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations. In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally. In this paper, we investigate the qualitative properties of the well-known FitzHugh-Nagumo (FHN) chaotic neuron model, which is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons. Next, new results are obtained for the output regulation of the FitzHugh-Nagumo (FHN) neuron model via integral sliding mode control (ISMC) method. MATLAB plots have been shown to illustrate the phase portraits of the FitzHugh-Nagumo (FHN) neuron model and the output regulation of the FHN neuron model.
Keywords: Chaos, chaotic systems, output regulation, biology, biological system, neuron, sliding mode control, FitzHugh-Nagumo system, bifurcations, nerve membranes, etc.

1. Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The classical chaotic systems are due to Lorenz, who discovered chaos while studying a 3-D weather model in 1963 [3], and Rossler, who discovered chaos, while he was studying chemical reactions in 1976 [4]. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], intelligent control [101-110], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [111-128], biology [129-160], memristors [161-163], electrical circuits [164], etc.

Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations.

In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Their mathematical model is referred to as *Hodgkin-Huxley equations* in the literature [165]. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally.

FitzHugh [166] and Nagumo [167] extended the Van der Pol equation in a planar field as a model for action potentials of neurons. FitzHugh-Nagumo (FHN) chaotic neuron model is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons.

This paper is organized as follows. Section 2 details the dynamics and properties of the FitzHugh-Nagumo chaotic neuron model. Section 3 details the output regulation of the FitzHugh-Nagumo (FHN) chaotic neuron model via integral sliding mode control method. Section 4 details the numerical simulations illustrating the main result derived in this research paper. Section 5 contains the main conclusions of this work.

2. FitzHugh-Nagumo Chaotic Neuron Model

FitzHugh-Nagumo (FHN) chaotic system is one of the most intensely studied systems in neuroscience. Many studies have been done on the significant and complex dynamical aspects of the FHN model including chaos, bifurcation, circuit design, noise effects and filtering, coupling, etc.

FitzHugh-Nagumo (FHN) chaotic neuron model [168, 169] is described by the 2-D dynamics

$$\begin{cases} \dot{x} = x(x-1)(1-\alpha x) - y + I_0(t) \\ \dot{y} = bx \end{cases} \tag{1}$$

In Eq. (1), $I_0(t)$ represents the external electrical stimulation

$$I_0(t) = \frac{a}{\omega}\cos(\omega t), \tag{2}$$

where a and ω are the amplitude (or strength) and frequency, respectively, of the applied field. Also, $\omega = 2\pi f$ (rad/s) and f (Hz) is the stimulus frequency.

It is known that the FHN system (1) is *chaotic*, when the parameter values are taken as

$$\alpha = 10, \quad b = 1, \quad a = 0.1, \quad f = 0.1271 \tag{3}$$

For numerical simulations, we take $x(0) = 0.2$ and $y(0) = 0.2$.

Figure 1 shows the $x-$ waveform of the FitzHugh-Nagumo system (1), while Figure 2 shows the $y-$waveform of the FitzHugh-Nagumo system (1). Figure 3 shows the chaotic phase portrait of the FHN system (1).

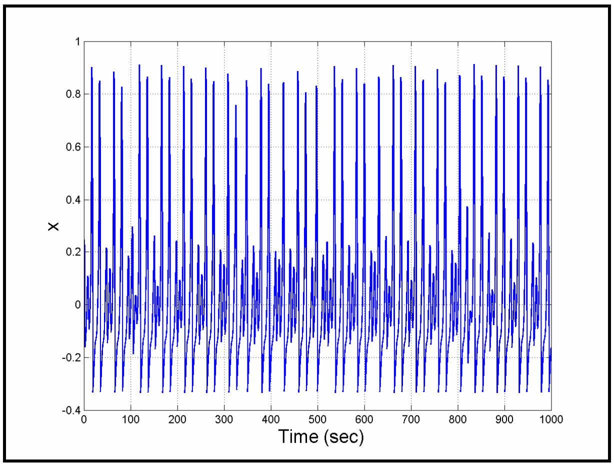

Figure 1. $x-$ **waveform of the FitzHugh-Nagumo chaotic neuron model**

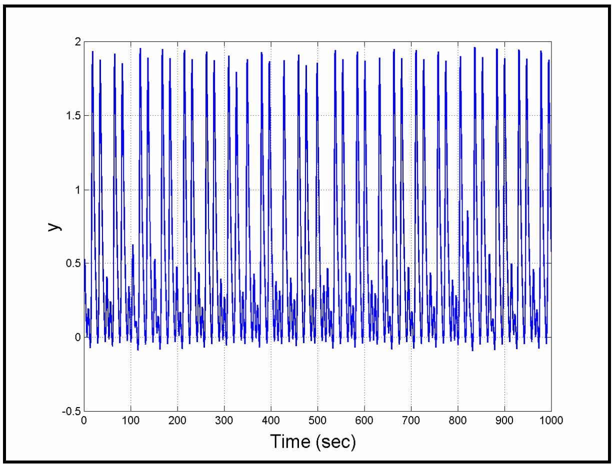

Figure 2. $y-$ **waveform of the FitzHugh-Nagumo chaotic neuron model**

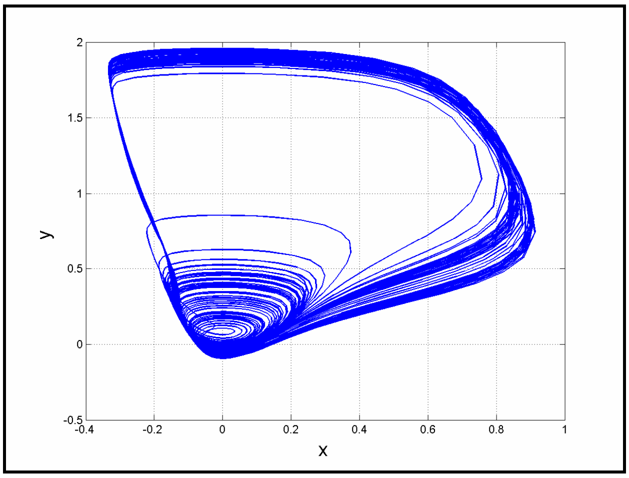

Figure 3. Chaotic phase portrait of the FitzHugh-Nagumo chaotic neuron model

3. Integral sliding mode control (ISMC) design for the FitzHugh-Nagumo chaotic neuron model

In this section, we derive new results for the output regulation of the FitzHugh-Nagumo (FHN) chaotic neuron model. The main control result is derived via integral sliding mode control theory [170] and established via Lyapunov stability theory [171].

We consider the FitzHugh-Nagumo (FHN) chaotic neuron model given by the 2-D dynamics

$$\begin{cases} \dot{x} = x(x-1)(1-\alpha x) - y + I_0(t) + u_x \\ \dot{y} = bx + u_y \end{cases} \tag{4}$$

where α, b are constant system parameters, and the external electrical stimulation $I_0(t) = \dfrac{a}{\omega}\cos(\omega t)$ is known.

The design goal is to find feedback controls u_x, u_y so that the controlled states $x(t)$ and $y(t)$ of the FitzHugh-Nagumo (FHN) chaotic neuron model (4) track the reference signals $r_x(t)$ and $r_y(t)$, respectively.

We define the tracking errors as follows:

$$\begin{cases} e_x(t) = x(t) - r_x(t) \\ e_y(t) = y(t) - r_y(t) \end{cases} \tag{5}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_x = x(x-1)(1-\alpha x) - y - \dot{r}_x + I_0(t) + u_x \\ \dot{e}_y = bx - \dot{r}_y + u_y \end{cases} \tag{6}$$

Based on the sliding mode control theory [170], the integral sliding surface of e_x and e_y is defined as follows:

$$\begin{cases} s_x = \left(\dfrac{d}{dt} + \lambda_x \right) \left[\displaystyle\int_0^t e_x(\tau) d\tau \right] = e_x + \lambda_x \displaystyle\int_0^t e_x(\tau) d\tau \\ s_y = \left(\dfrac{d}{dt} + \lambda_y \right) \left[\displaystyle\int_0^t e_y(\tau) d\tau \right] = e_y + \lambda_y \displaystyle\int_0^t e_y(\tau) d\tau \end{cases} \tag{7}$$

The derivative of each equation in (7) yields

$$\begin{cases} \dot{s}_x = \dot{e}_x + \lambda_x e_x \\ \dot{s}_y = \dot{e}_y + \lambda_y e_y \end{cases} \tag{8}$$

The Hurwitz condition is satisfied if $\lambda_x > 0$ and $\lambda_y > 0$.

Based on the exponential reaching law [170], we set

$$\begin{cases} \dot{s}_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ \dot{s}_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \end{cases} \tag{9}$$

where η_x, η_y, k_x, k_y are positive constants.

Comparing the equations (8) and (9), we get

$$\begin{cases} \dot{e}_x + \lambda_x e_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ \dot{e}_y + \lambda_y e_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \end{cases} \tag{10}$$

Using Eq. (6), we can rewrite Eq. (10) as follows.

$$\begin{cases} x(x-1)(1-\alpha x) - y - \dot{r}_x + I_0(t) + u_x + \lambda_x e_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ bx - \dot{r}_y + u_y + \lambda_y e_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \end{cases} \tag{11}$$

From Eq. (11), the control laws are obtained as follows:

$$\begin{cases} u_x = -x(x-1)(1-\alpha x) + y + \dot{r}_x - I_0(t) - \lambda_x e_x - \eta_x \operatorname{sgn}(s_x) - k_x s_x \\ u_y = -bx + \dot{r}_y - \lambda_y e_y - \eta_y \operatorname{sgn}(s_y) - k_y s_y \end{cases} \tag{12}$$

Next, we state and prove the main result of this section.

Theorem 1. The integral sliding mode control law (12) achieves output regulation of the FitzHugh-Nagumo (FHN) chaotic neuron model (4), where the constants λ_x, λ_y, η_x, η_y, k_x, k_y are all positive.

Proof. This result is proved using Lyapunov stability theory [171].

We consider the following quadratic Lyapunov function

$$V(s_x, s_y) = \frac{1}{2}\left(s_x^2 + s_y^2\right) \tag{13}$$

where s_x, s_y are as defined in Eq. (7). The time-derivative of V is obtained as

$$\dot{V} = s_x \dot{s}_x + s_y \dot{s}_y \tag{14}$$

Substituting from Eq. (9) into Eq. (14), we obtain

$$\dot{V} = s_x(-\eta_x \operatorname{sgn}(s_x) - k_x s_x) + s_y(-\eta_y \operatorname{sgn}(s_y) - k_y s_y) \tag{15}$$

Simplifying Eq. (15), we obtain

$$\dot{V} = -\eta_x |s_x| - k_x s_x^2 - \eta_y |s_y| - k_y s_y^2 \tag{16}$$

Since η_x, η_y, k_x, k_y are positive constants, it follows from (16) that $\dot{V}$ is a negative definite function.

Thus, by Lyapunov's stability theory [171], the proof is complete. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (4) when the integral sliding mode controller (12) is implemented.

The external electrical stimulation is $I_0(t) = \dfrac{a}{\omega}\cos(\omega t)$, where $\omega = 2\pi f$.

We take the parameter values as in the chaotic case, *i.e.*

$\alpha = 10, \ b = 1, \ a = 0.1, \ f = 0.1271.$

We take the reference signals as $r_x = \cos t$ and $r_y = \sin t$.

We take the sliding constants as

$\eta_x = 0.1, \ \eta_y = 0.1, \ \lambda_x = 0.1, \ \lambda_y = 0.1, \ k_x = 30, \ k_y = 30$

We take the initial conditions of the FHN chaotic system (4) as $x(0) = 7.4$ and $y(0) = 12.5$.

Figure 4 shows the output regulation of the $x-$waveform of the FHN system (4).

Figure 5 shows the output regulation of the $y-$waveform of the FHN system (4).

Figure 6 shows the time-history of the output regulation errors $e_x(t)$ and $e_y(t)$.

Since the output regulation errors converge to zero in less than 0.5 seconds, it is very clear that the integral sliding mode controller designed in this paper is very efficient for the control of the FHN chaotic system.

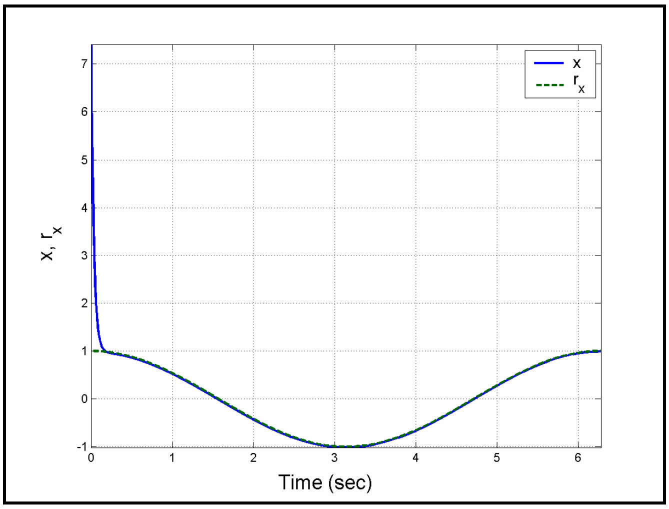

Figure 4. Output regulation of the $x-$waveform of the FitzHugh-Nagumo neuron system

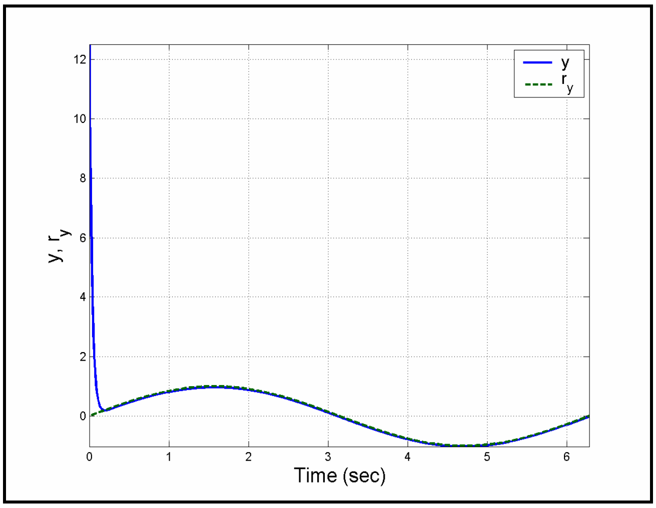

Figure 5. Output regulation of the $y-$waveform of the FitzHugh-Nagumo neuron system

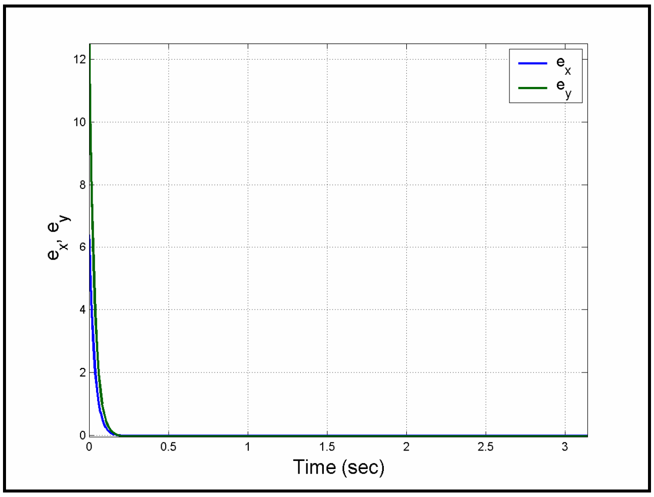

Figure 6. Time-history of the output regulation errors $e_x(t), e_y(t)$

5. Conclusions

In this paper, we investigated the qualitative properties of the well-known FitzHugh-Nagumo (FHN) chaotic neuron model, which is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons. Next, we derived new results for the output regulation of the FitzHugh-Nagumo (FHN) neuron model via integral sliding mode control (ISMC) method. MATLAB plots were depicted to illustrate the phase portraits of the FitzHugh-Nagumo (FHN) neuron model and the output regulation of the FHN neuron model.

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