International Journal of PharmTech Research
CODEN (USA): IJPRIF, ISSN: 0974-4304
Vol.9, No.4, pp 399-412, 2016

Global Chaos Control of the Generalized Lotka-Volterra Three-Species System via Integral Sliding Mode Control

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, India

Abstract: Since the recent research has shown the importance of biological control in many biological systems appearing in nature, this research paper investigates research in the dynamic and chaotic analysis of the generalized Lotka-Volterra three-species biological system, which was studied by Samardzija and Greller (1988). The generalized Lotka-Voterra biological system consists of two predator and one prey populations. This paper depicts the phase portraits of the 3-D generalized Lotka-Volterra system when the system undergoes chaotic behaviour. Next, this paper derives biological control law via integral sliding mode control (ISMC) for achieving output regulation of the states of the generalized Lotka-Volterra three-species biological system so as to track constant reference signals (set-point controls). All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been shown using MATLAB to illustrate all the main results for the three-species generalized Lotka-Volterra biological system and its output regulation.
Keywords: Chaos, chaotic systems, output regulation, biology, biological system, Lotka-Volterra system, etc.

1. Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The classical chaotic systems are due to Lorenz, who discovered chaos while studying a 3-D weather model in 1963 [3], and Rossler, who discovered chaos, while he was studying chemical reactions in 1976 [4]. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], intelligent control [101-110], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [111-128], biology [129-160], memristors [161-163], electrical circuits [164], etc.

One of the famous examples of simple biological models is the two-species predator-prey model developed by Lotka and Volterra [165]. Lotka-Voltera system describes the interaction of a two-species predator-prey model and id consists of a system of two nonlinear ordinary differential equations. This is a very popular model and it has many applications of interacting two-species systems. However, this model also has limitations such as it ignores many important factors such as interactions between another species of the same ecosystem, interactions with the environment etc. Thus, three-species models of biological species have more importance. Arneodo et al. [166] have shown that one can obtain chaotic behaviour for three species in an ecosystem. Three species predator-prey models typically consist of one-prey and two predators, and in this research work, we investigate such a three-species biological generalized Lotka-Volterra system investigated by Samardzija and Greller [167].

An agricultural ecosystem comprises a dynamic web of biological relationships among crop plants or trees, herbivores, predators, preys, disease organisms, etc. Organisms in an ecosystem interact in many ways through competition. These organisms constantly evolve and depend on each other and thereby they create a diverse, complex and dynamic environment.

This paper depicts the phase portraits of the 3-D generalized Lotka-Volterra system [167] when the system undergoes chaotic behaviour. Next, this paper derives biological control law via integral sliding mode control (ISMC) for achieving output regulation of the states of the generalized Lotka-Volterra three-species biological system so as to track constant reference signals (set-point controls). All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the three-species generalized Lotka-Volterra biological system and its output regulation.

2. Generalized Lotka-Volterra Three-Species Biological System

Samardzija and Greller (1988, [167]) derived a generalized Lotka-Volterra three-species biological system, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = x_1 - x_1 x_2 + c x_1^2 - a x_1^2 x_3 \\ \dot{x}_2 = -x_2 + x_1 x_2 \\ \dot{x}_3 = -b x_3 + a x_1^2 x_3 \end{cases} \tag{1}$$

In (1), x_1 is the prey population, x_2, x_3 are predator populations and a, b, c are positive constants.

In [145], it was shown that the three-species biological system (1) is *chaotic* when we take

$$a = 2.9851, \quad b = 3, \quad c = 2 \tag{2}$$

For numerical simulations, we take the initial conditions as $x_1(0) = 1.2$, $x_2(0) = 1.2$ and $x_3(0) = 1.2$.

The 3-D phase portrait of the generalized Lotka-Volterra system (1) is depicted in Figure 1. The 2-D projections of the generalized Lotka-Volterra systems (2) on the coordinate planes are depicted in Figures 2-3.

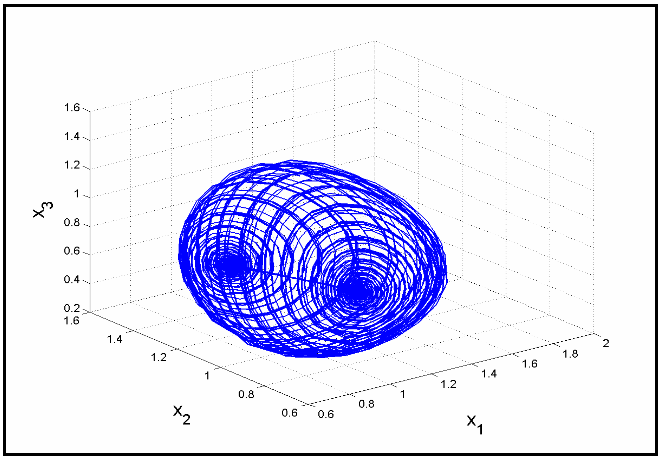

Figure 1. The 3-D phase portrait of the generalized Lotka-Volterra chaotic system

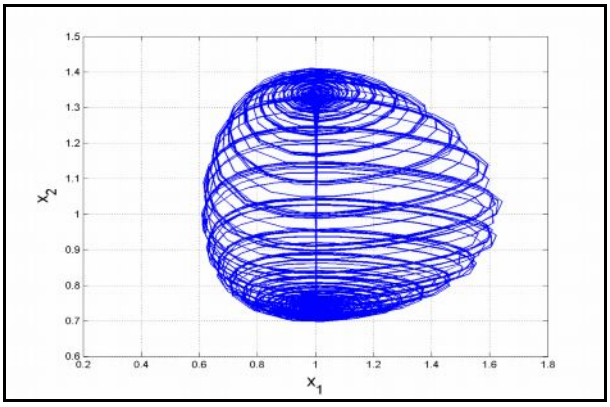

Figure 2. The 2-D projection of the generalized Lotka-Volterra system on (x_1, x_2) plane

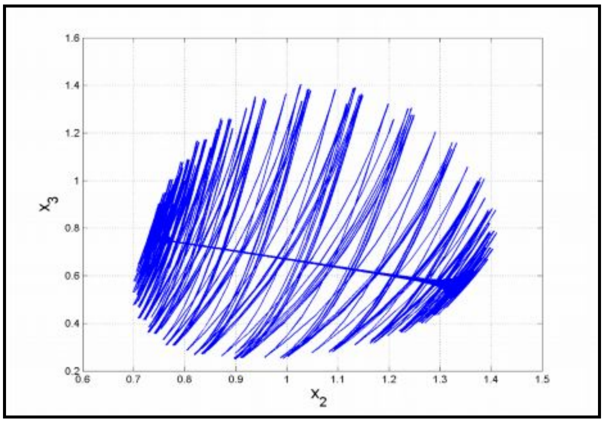

Figure 3. The 2-D projection of the generalized Lotka-Volterra system on (x_2, x_3) plane

3. Control of the generalized Lotka-Volterra three-species system via integral sliding mode control

The chaotic behaviour of the generalized Lotka-Voterra three-species biological system [167] is an example of the explosive route to chaos and it is attributed to the non-transversal saddle connection type bifurcation. Also, it is observed that the chaotic solution of the generalized Lotka-Volterra biological system (1) portrays a fractal torus in the 3-D phase space.

In this section, we use integral sliding mode control for controlling the states of the generalized Lotka-Volterra three-species biological system so as to track constant reference signals (set-point controls).

Thus, we consider the generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_1 - x_1 x_2 + c x_1^2 - a x_1^2 x_3 + u_1 \\ \dot{x}_2 = -x_2 + x_1 x_2 + u_2 \\ \dot{x}_3 = -b x_3 + a x_1^2 x_3 + u_3 \end{cases} \tag{3}$$

In (3), x_1, x_2, x_3 are the states, a, b, c are constant, positive, parameters of the system and u_1, u_2, u_3 are the controls to be determined.

Our design goal is to find controls u_1, u_2, u_3 so as to regulate the states of the system (3) so as to track constant reference signals (set-point controls).

Thus, we define the regulation errors as follows.

$$\begin{cases} e_1(t) = x_1(t) - \alpha_1 \\ e_2(t) = x_2(t) - \alpha_2 \\ e_3(t) = x_3(t) - \alpha_3 \end{cases} \tag{4}$$

In (4), $\alpha_1, \alpha_2, \alpha_3$ are constant reference signals or set-point controls.

Then the regulation error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = (e_1+\alpha_1) - (e_1+\alpha_1)(e_2+\alpha_2) + c(e_1+\alpha_1)^2 - a(e_1+\alpha_1)^2(e_3+\alpha_3) + u_1 \\ \dot{e}_2 = -(e_2+\alpha_2) + (e_1+\alpha_1)(e_2+\alpha_2) + u_2 \\ \dot{e}_3 = -b(e_3+\alpha_3) + a(e_1+\alpha_1)^2(e_3+\alpha_3) + u_3 \end{cases} \tag{5}$$

Based on the sliding mode control theory [168], the integral sliding surface of each error variable $e_i, \ (i=1,2,3)$ is defined as follows:

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)\left[\int_0^t e_i(\tau)d\tau\right] = e_i + \lambda_i \int_0^t e_i(\tau)d\tau, \quad (i=1,2,3) \tag{6}$$

The derivative of each equation in (6) yields

$$\dot{s}_i = \dot{e}_i + \lambda_i e_i, \quad (i=1,2,3) \tag{7}$$

The Hurwitz condition is satisfied if $\lambda_i > 0$ for $i=1,2,3.$

Based on the exponential reaching law [146], we set

$$\dot{s}_i = -\eta_i \operatorname{sgn}(s_i) - k_i s_i, \quad (i=1,2,3) \tag{8}$$

where $\eta_i, k_i, (i=1,2,3)$ are positive constants.

Comparing the equations (7) and (8), we get

$$\begin{cases} \dot{e}_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{e}_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{e}_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{9}$$

Using Eq. (5), we can rewrite Eq. (9) as follows.

$$\begin{cases} x_1 - x_1x_2 + cx_1^2 - ax_1^2x_3 + u_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ -x_2 + x_1x_2 + u_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ -bx_3 + ax_1^2x_3 + u_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{10}$$

From Eq. (10), the control laws are obtained as follows:

$$\begin{cases} u_1 = -x_1 + x_1x_2 - cx_1^2 + ax_1^2x_3 - \lambda_1 e_1 - \eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ u_2 = x_2 - x_1x_2 - \lambda_2 e_2 - \eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ u_3 = bx_3 - ax_1^2x_3 - \lambda_3 e_3 - \eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{11}$$

Next, we state and prove the main result of this section.

Theorem 1. The states x_1, x_2, x_3 of the generalized Lotka-Volterra three-species system (3) are regulated to track the constant reference signals $\alpha_1, \alpha_2, \alpha_3$ asymptotically as $t \to \infty$ for all initial conditions $\boldsymbol{x}(0) \in R^3$ by the integral sliding mode control law (11), where the constants λ_i, η_i, k_i are positive for $i=1,2,3.$

Proof. This result is proved using Lyapunov stability theory [169].

We consider the following quadratic Lyapunov function

$$V(s_1, s_2, s_3) = \frac{1}{2}\left(s_1^2 + s_2^2 + s_3^2\right) \tag{12}$$

where s_1, s_2, s_3 are as defined in Eq. (6). The time-derivative of V is obtained as

$$\dot{V} = s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 \tag{13}$$

Substituting from Eq. (8) into Eq. (13), we obtain

$$\dot{V} = s_1(-\eta_1 \operatorname{sgn}(s_1) - k_1 s_1) + s_2(-\eta_2 \operatorname{sgn}(s_2) - k_2 s_2) + s_3(-\eta_3 \operatorname{sgn}(s_1) - k_3 s_1) \tag{14}$$

Simplifying Eq. (14), we obtain

$$\dot{V} = -\eta_1 |s_1| - k_1 s_1^2 - \eta_2 |s_2| - k_2 s_2^2 - \eta_3 |s_3| - k_3 s_3^2 \tag{15}$$

Since $k_i > 0$ and $\eta_i > 0$ for $i = 1, 2, 3$, it follows from (15) that $\dot{V}$ is a negative definite function.

Thus, by Lyapunov's stability theory [169], it is immediate that the error $e_i(t) \to 0, \quad (i = 1, 2, 3)$ asymptotically as $t \to \infty$ for all initial conditions $\boldsymbol{e}(0) \in R^3$.

This completes the proof. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (3) when the integral sliding mode controller (11) is implemented.

The parameter values of the generalized Lotka-Volterra system (3) are taken as in the chaotic case, *viz.*

$$a = 2.9851, \; b = 3, \; c = 2 \tag{16}$$

We take the sliding constants as

$$\eta_i = \lambda_i = 0.1, \quad k_i = 30, \qquad (i = 1, 2, 3) \tag{17}$$

We take the initial conditions of the generalized Lotka-Volterra system (3) as

$$x_1(0) = 12.9, \quad x_2(0) = 14.1, \quad x_3(0) = 20.8$$

We take the constant reference signals as

$$\alpha_1 = 1, \quad \alpha_2 = 2, \quad \alpha_3 = 3 \tag{18}$$

Figures 4-6 show the output regulation of the generalized Lotka-Volterra system (3).

Figure 7 shows the time-history of the regulation errors e_1, e_2, e_3.

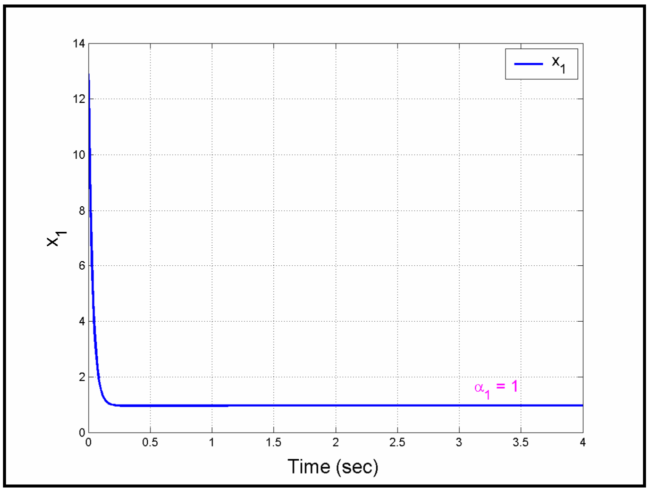

Figure 4. Output regulation of the state x_1

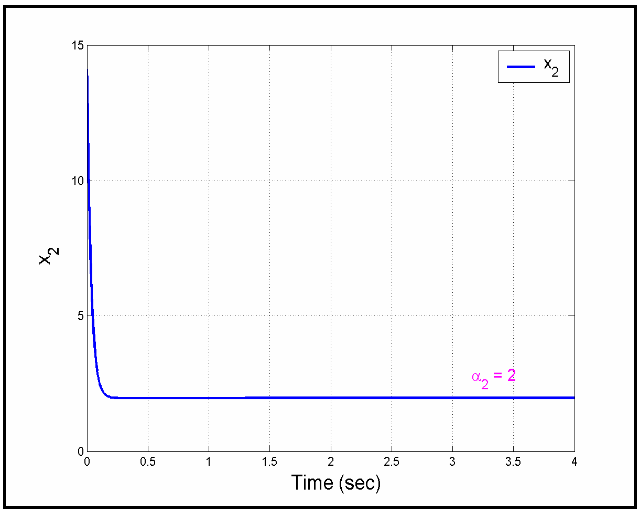

Figure 5. Output regulation of the state x_2

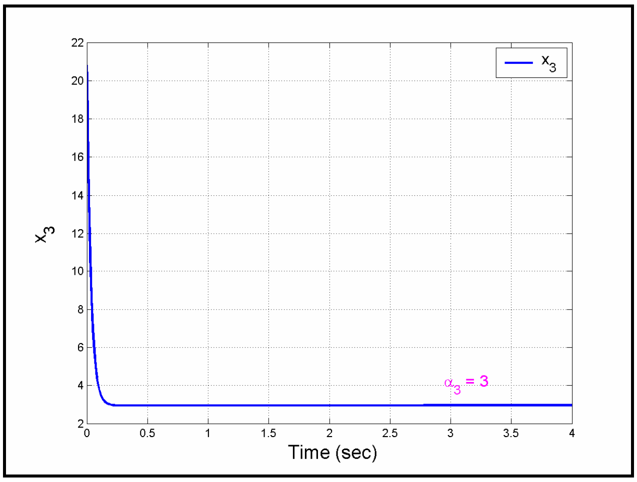

Figure 6. Output regulation of the state x_3

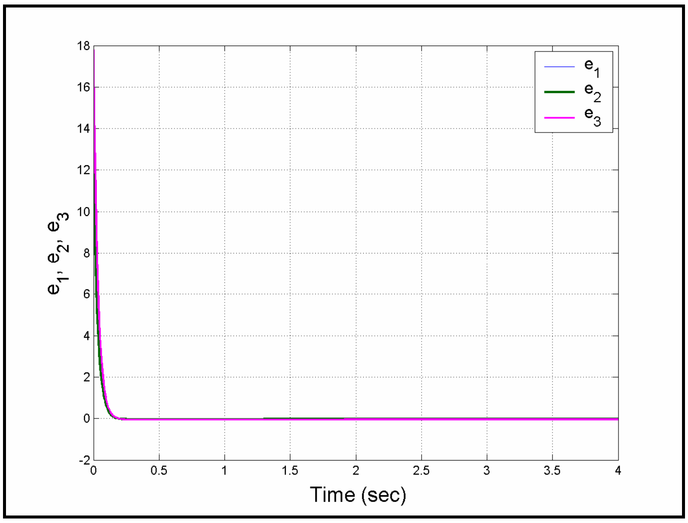

Figure 7. Time-history of the output regulation errors e_1, e_2, e_3

5. Conclusions

In this paper, new results have been derived for the analysis and adaptive synchronization of the three-species generalized Lokta-Volterra biological systems discovered by Samardzija and Greller (1988). After a description and dynamic analysis of the chaotic 3-D three species Samardzija-Greller model, we have designed a biological feedback controller for the global and asymptotic state regulation of the states of the 3-D three species Samardzija-Greller model to constant reference signals (set-point controls) via integral sliding mode control. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

References

1. Azar, A. T., and Vaidyanathan, S., Chaos Modeling and Control Systems Design, Studies in Computational Intelligence, Vol. 581, Springer, New York, USA, 2015.
2. Azar, A. T., and Vaidyanathan, S., Computational Intelligence Applications in Modeling and Control, Studies in Computational Intelligence, Vol. 575, Springer, New York, USA, 2015.
3. Lorenz, E. N., Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, 1963, 20, 130-141.
4. Rössler, O. E., An equation for continuous chaos, Physics Letters A, 1976, 57, 397-398.
5. Arneodo, A., Coullet, P., and Tresser, C., Possible new strange attractors with spiral structure, Communications in Mathematical Physics, 1981, 79, 573-579.
6. Sprott, J. C., Some simple chaotic flows, Physical Review E, 1994, 50, 647-650.
7. Chen, G., and Ueta, T., Yet another chaotic attractor, International Journal of Bifurcation and Chaos, 1999, 9, 1465-1466.
8. Lü, J., and Chen, G., A new chaotic attractor coined, International Journal of Bifurcation and Chaos, 2002, 12, 659-661.
9. Cai, G., and Tan, Z., Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain Systems, 2007, 1, 235-240.
10. Tigan, G., and Opris, D., Analysis of a 3D chaotic system, Chaos, Solitons and Fractals, 2008, 36, 1315-1319.
11. Sundarapandian, V., and Pehlivan, I., Analysis, control, synchronization and circuit design of a novel chaotic system, Mathematical and Computer Modelling, 2012, 55, 1904-1915.
12. Sundarapandian, V., Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, 2013, 6, 45-52.
13. Vaidyanathan, S., A new six-term 3-D chaotic system with an exponential nonlinearity, Far East Journal of Mathematical Sciences, 2013, 79, 135-143.
14. Vaidyanathan, S., Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters, Journal of Engineering Science and Technology Review, 2013, 6, 53-65.
15. Vaidyanathan, S., A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East Journal of Mathematical Sciences, 2014, 84, 219-226.
16. Vaidyanathan, S., Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2014, 22, 41-53.
17. Vaidyanathan, S., and Madhavan, K., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system, International Journal of Control Theory and Applications, 2013, 6, 121-137.
18. Vaidyanathan, S., Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities, European Physical Journal: Special Topics, 2014, 223, 1519-1529.
19. Vaidyanathan, S., Volos, C., Pham, V. T., Madhavan, K., and Idowu, B. A., Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities, Archives of Control Sciences, 2014, 24, 257-285.
20. Vaidyanathan, S., Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control, International Journal of Modelling, Identification and Control, 2014, 22, 207-217.
21. Vaidyanathan, S., and Azar, A.T., Analysis and control of a 4-D novel hyperchaotic system, Studies in Computational Intelligence, 2015, 581, 3-17.
22. Vaidyanathan, S., Volos, C., Pham, V.T., and Madhavan, K., Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation, Archives of Control Sciences, 2015, 25, 135-158.
23. Vaidyanathan, S., Volos, C., and Pham, V.T., Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation,

Archives of Control Sciences, 2014, 24, 409-446.
24. Vaidyanathan, S., A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control, International Journal of Control Theory and Applications, 2013, 6, 97-109.
25. Vaidyanathan, S., Analysis, properties and control of an eight-term 3-D chaotic system with an exponential nonlinearity, International Journal of Modelling, Identification and Control, 2015, 23, 164-172.
26. Vaidyanathan, S., Azar, A.T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23, 267-277.
27. Vaidyanathan, S., Qualitative analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with a quartic nonlinearity, International Journal of Control Theory and Applications, 2014, 7, 1-20.
28. Vaidyanathan, S., Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities, International Journal of Control Theory and Applications, 2014, 7, 35-47.
29. Vaidyanathan, S., and Pakiriswamy, S., A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control, Journal of Engineering Science and Technology Review, 2015, 8, 52-60.
30. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, adaptive control and adaptive synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 181-191.
31. Vaidyanathan, S., Rajagopal, K., Volos, C.K., Kyprianidis, I.M., and Stouboulos, I.N., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW, Journal of Engineering Science and Technology Review, 2015, 8, 130-141.
32. Pham, V.T., Volos, C.K., Vaidyanathan, S., Le, T.P., and Vu, V.Y., A memristor-based hyperchaotic system with hidden attractors: Dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8, 205-214.
33. Pham, V.T., Volos, C.K., and Vaidyanathan, S., Multi-scroll chaotic oscillator based on a first-order delay differential equation, Studies in Computational Intelligence, 2015, 581, 59-72.
34. Vaidyanathan, S., Volos, C.K., Kyprianidis, I.M., Stouboulos, I.N., and Pham, V.T., Analysis, adaptive control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 24-36.
35. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium, Journal of Engineering Science and Technology Review, 2015, 8, 232-244.
36. Sampath, S., Vaidyanathan, S., Volos, C.K., and Pham, V.T., An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 1-6.
37. Vaidyanathan, S., A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters, Journal of Engineering Science and Technology Review, 2015, 8, 106-115.
38. Vaidyanathan, S., Pham, V.-T., and Volos, C. K., A 5-D hyperchaotic Rikitake dynamo system with hidden attractors, European Physical Journal: Special Topics, 2015, 224, 1575-1592.
39. Pham, V.-T., Vaidyanathan, S., Volos, C. K., and Jafari, S., Hidden attractors in a chaotic system with an exponential nonlinear term, European Physical Journal: Special Topics, 2015, 224, 1507-1517.
40. Vaidyanathan, S., Hyperchaos, qualitative analysis, control and synchronisation of a ten-term 4-D hyperchaotic system with an exponential nonlinearity and three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2015, 23, 380-392.
41. Vaidyanathan, S., and Azar, A. T., Analysis, control and synchronization of a nine-term 3-D novel chaotic system, Studies in Computational Intelligence, 2015, 581, 19-38.
42. Vaidyanathan, S., and Volos, C., Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system, Archives of Control Sciences, 2015, 25, 333-353.
43. Vaidyanathan, S., Analysis, control and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities, Kyungpook Mathematical Journal, 2015, 55, 563-586.

44. Pehlivan, I., Moroz, I. M., and Vaidyanathan, S., Analysis, synchronization and circuit design of a novel butterfly attractor, Journal of Sound and Vibration, 2014, 333, 5077-5096.
45. Pham, V. T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristic neural network, Optoelectronics and Advanced Materials – Rapid Communications, 2014,
46. Freedman, H.I., Deterministic Mathematical Models in Population Ecology, Marcel Dekker, New York, USA, 1980.
47. Sundarapandian, V., Output regulation of Van der Pol oscillator, Journal of the Institution of Engineers (India): Electrical Engineering Division, 88, 20-24, 2007.
48. Sundarapandian, V., Output regulation of the Lorenz attractor, Far East Journal of Mathematical Sciences, 2010, 42, 289-299.
49. Vaidyanathan, S., and Rajagopal, K., Anti-synchronization of Li and T chaotic systems by active nonlinear control, Communications in Computer and Information Science, 2011, 198, 175-184.
50. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control, Communications in Computer and Information Science, 2011, 198, 10-17.
51. Vaidyanathan, S., Output regulation of the unified chaotic system, Communications in Computer and Information Science, 2011, 198, 1-9.
52. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control, 2011, 198, 84-93.
53. Vaidyanathan, S., Hybrid chaos synchronization of Liu and Lu systems by active nonlinear control, Communications in Computer and Information Science, 2011, 204, 1-10.
54. Sarasu, P., and Sundarapandian, V., Active controller design for generalized projective synchronization of four-scroll chaotic systems, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 26-33.
55. Vaidyanathan, S., and Rasappan, S., Hybrid synchronization of hyperchaotic Qi and Lu systems by nonlinear control, Communications in Computer and Information Science, 2011, 131, 585-593.
56. Vaidyanathan, S., and Rajagopal, K., Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 55-61.
57. Vaidyanathan, S., Output regulation of Arneodo-Coullet chaotic system, Communications in Computer and Information Science, 2011, 133, 98-107.
58. Sarasu, P., and Sundarapandian, V., The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control, International Journal of Soft Computing, 2011, 6, 216-223.
59. Vaidyanathan, S., and Pakiriswamy, S., The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems, Communications in Computer and Information Science, 2011, 245, 231-238.
60. Sundarapandian, V., and Karthikeyan, R., Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control, Journal of Engineering and Applied Sciences, 2012, 7, 254-264.
61. Vaidyanathan, S., and Pakiriswamy, S., Generalized projective synchronization of double-scroll chaotic systems using active feedback control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 84, 111-118.
62. Pakiriswamy, S., and Vaidyanathan, S., Generalized projective synchronization of three-scroll chaotic systems via active control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 85, 146-155.
63. Karthikeyan, R., and Sundarapandian, V., Hybrid chaos synchronization of four-scroll systems via active control, Journal of Electrical Engineering, 2014, 65, 97-103.
64. Vaidyanathan, S., Azar, A. T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23, 267-277.
65. Yassen, M. T., Chaos synchronization between two different chaotic systems using active control, Chaos, Solitons and Fractals, 2005, 23, 131-140.
66. Jia, N., and Wang, T., Chaos control and hybrid projective synchronization for a class of new chaotic systems, Computers and Mathematics with Applications, 2011, 62, 4783-4795.

67. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control, Communication in Computer and Information Science, 2011, 205, 193-202.
68. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control, International Journal of Soft Computing, 2011, 6, 111-118.
69. Vaidyanathan, S., Adaptive controller and synchronizer design for the Qi-Chen chaotic system, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunication Engineering, 2012, 85, 124-133.
70. Sundarapandian, V., Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lectures on Electrical Engineering, 2013, 131, 319-327.
71. Vaidyanathan, S., Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, 2013, 177, 1-10.
72. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control, Communications in Computer and Information Science, 2011, 205, 193-202.
73. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of Lü and Pan systems by adaptive nonlinear control, European Journal of Scientific Research, 2011, 64, 94-106.
74. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 18-25.
75. Sundarapandian, V., and Karthikeyan, R., Adaptive anti-synchronization of uncertain Tigan and Li systems, Journal of Engineering and Applied Sciences, 2012, 7, 45-52.
76. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of three-scroll chaotic systems via adaptive control, European Journal of Scientific Research, 2012, 72, 504-522.
77. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control, International Journal of Soft Computing, 2012, 7, 28-37.
78. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of two-scroll systems via adaptive control, International Journal of Soft Computing, 2012, 7, 146-156.
79. Sarasu, P., and Sundarapandian, V., Adaptive controller design for the generalized projective synchronization of 4-scroll systems, International Journal of Systems Signal Control and Engineering Application, 2012, 5, 21-30.
80. Vaidyanathan, S., Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control, International Journal of Control Theory and Applications, 2012, 5, 41-59.
81. Vaidyanathan, S., and Pakiriswamy, S., Generalized projective synchronization of six-term Sundarapandian chaotic systems by adaptive control, International Journal of Control Theory and Applications, 2013, 6, 153-163.
82. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback, Malaysian Journal of Mathematical Sciences, 2013, 7, 219-246.
83. Suresh, R., and Sundarapandian, V., Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback, Far East Journal of Mathematical Sciences, 2013, 73, 73-95.
84. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback, Archives of Control Sciences, 2012, 22, 343-365.
85. Rasappan, S., and Vaidyanathan, S., Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design, Kyungpook Mathematical Journal, 2014, 54, 293-320.
86. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback, Arabian Journal for Science and Engineering, 2014, 39, 3351-3364.
87. Vaidyanathan, S., Idowu, B. A., and Azar, A. T., Backstepping controller design for the global chaos synchronization of Sprott's jerk systems, Studies in Computational Intelligence, 2015, 581, 39-58.
88. Vaidyanathan, S., Volos, C. K., Rajagopal, K., Kyprianidis, I. M., and Stouboulos, I. N., Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, Journal of Engineering Science and Technology Review, 2015, 8, 74-82.

89. Vaidyanathan, S., and Sampath, S., Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control, Communications in Computer and Information Science, 2011, 205, 156-164.
90. Sundarapandian, V., and Sivaperumal, S., Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, 2011, 6, 224-231.
91. Vaidyanathan, S., and Sampath, S., Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, 2012, 9, 274-279.
92. Vaidyanathan, S., Analysis and synchronization of the hyperchaotic Yujun systems via sliding mode control, Advances in Intelligent Systems and Computing, 2012, 176, 329-337.
93. Vaidyanathan, S., and Sampath, S., Sliding mode controller design for the global chaos synchronization of Coullet systems, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 84, 103-110.
94. Vaidyanathan, S., and Sampath, S., Hybrid synchronization of hyperchaotic Chen systems via sliding mode control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 85, 257-266.
95. Vaidyanathan, S., Global chaos control of hyperchaotic Liu system via sliding control method, International Journal of Control Theory and Applications, 2012, 5, 117-123.
96. Vaidyanathan, S., Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, 2012, 5, 15-20.
97. Vaidyanathan, S., Global chaos synchronisation of identical Li-Wu chaotic systems via sliding mode control, International Journal of Modelling, Identification and Control, 2014, 22, 170-177.
98. Vaidyanathan, S., and Azar, A. T., Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems, Studies in Computational Intelligence, 2015, 576, 527-547.
99. Vaidyanathan, S., and Azar, A. T., Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems, Studies in Computational Intelligence, 2015, 576, 549-569.
100. Vaidyanathan, S., Sampath, S., and Azar, A. T., Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system, International Journal of Modelling, Identification and Control, 2015, 23, 92-100.
101. Li, H., Liao, X., Li, C., and Li, C., Chaos control and synchronization via a novel chatter free sliding mode control strategy, Neurocomputing, 2011, 74, 3212-3222.
102. Inbarani, H. H., Bagyamathi, M., and Azar, A. T., A novel hybrid feature selection method on rough set and improved harmony search, Neural Computing and Applications, 2015, 26 (8), 1859-1880.
103. Anter, A. M., Hassanien, A. E., ElSoud, M.A., and Azar, A. T., Automatic liver parenchyma segmentation system from abdominal CT scans using hybrid techniques, International Journal of Biomedical Engineering and Technology, 2015, 17 (2), 148-167.
104. Azar, A. T., and Hassanien, A. E., Dimensionality reduction of medical big data using neural-fuzzy classifier, Soft Computing, 2015, 19 (4), 1115-1127.
105. Ding, S., Shi, Z., Chen, K., and Azar, A. T., Mathematical modeling and analysis of soft computing, Mathematical Problems in Engineering, 2015, art no. 578321.
106. Mekki, H., Boukhetala, D., and Azar, A. T., Sliding modes for fault tolerant control, Studies in Computational Intelligence, 2015, 576, 407-433.
107. Azar, A. T., and Serrano, F. E., Design and modeling of anti wind up PID controllers, Studies in Fuzziness and Soft Computing, 2015, 319, 1-44.
108. Azar, A. T., and Serrano, F. E., Adaptive sliding mode control of the furuta pendulum, Studies in Computational Intelligence, 2015, 576, 1-42.
109. Azar, A. T., and Serrano, F. E., Deadbeat control for multivariable systems with time varying delays, Studies in Computational Intelligence, 2015, 581, 97-132.
110. Azar, A. T., and Serrano, F. E., Robust IMC-PID tuning for cascade control systems with gain and phase margin specifications, Neural Computing and Applications, 2014, 25(5), 983-995.
111. Ding, S., Shi, Z., and Azar, A. T., Research and development of advanced computing technologies, Scientific World Journal, 2015, art. No. 239723.
112. Vaidyanathan, S., Adaptive synchronization of chemical chaotic reactors, International Journal of ChemTech Research, 2015, 8 (2), 612-621.
113. Vaidyanathan, S., Adaptive control of a chemical chaotic reactor, International Journal of PharmTech

Research, 2015, 8 (3), 377-382.
114. Vaidyanathan, S., Dynamics and control of Brusselator chemical reaction, International Journal of ChemTech Research, 2015, 8 (6), 740-749.
115. Vaidyanathan, S., Anti-synchronization of Brusselator chemical reaction systems via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 759-768.
116. Vaidyanathan, S., Dynamics and control of Tokamak system with symmetric and magnetically confined plasma, International Journal of ChemTech Research, 2015, 8 (6), 795-803.
117. Vaidyanathan, S., Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 818-827.
118. Vaidyanathan, S., A novel chemical chaotic reactor system and its adaptive control, International Journal of ChemTech Research, 2015, 8 (7), 146-158.
119. Vaidyanathan, S., Adaptive synchronization of novel 3-D chemical chaotic reactor systems, International Journal of ChemTech Research, 2015, 8 (7), 159-171.
120. Vaidyanathan, S., Global chaos synchronization of chemical chaotic reactors via novel sliding mode control method, International Journal of ChemTech Research, 2015, 8 (7), 209-221.
121. Vaidyanathan, S., Sliding mode control of Rucklidge chaotic system for nonlinear double convection, International Journal of ChemTech Research, 2015, 8 (8), 25-35.
122. Vaidyanathan, S., Global chaos synchronization of Rucklidge chaotic systems for double convection via sliding mode control, International Journal of ChemTech Research, 2015, 8 (8), 61-72.
123. Vaidyanathan, S., Anti-synchronization of chemical chaotic reactors via adaptive control method, International Journal of ChemTech Research, 2015, 8 (8), 73-85.
124. Vaidyanathan, S., Adaptive synchronization of Rikitake two-disk dynamo chaotic systems, International Journal of ChemTech Research, 2015, 8 (8), 100-111.
125. Vaidyanathan, S., Adaptive control of Rikitake two-disk dynamo system, International Journal of ChemTech Research, 2015, 8 (8), 121-133.
126. Vaidyanathan, S., State regulation of Rikitake two-disk dynamo chaotic system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 374-386.
127. Vaidyanathan, S., Anti-synchronization of Rikitake two-disk dynamo chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 393-405.
128. Vaidyanathan, S., Global chaos control of Mathieu-Van der Pol system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 406-417.
129. Vaidyanathan, S., Global chaos synchronization of Mathieu-Van der Pol chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (10), 148-162.
130. Garfinkel, A., Spano, M.L., Ditto, W.L., and Weiss, J.N., Controlling cardiac chaos, Science, 1992, 257, 1230-1235.
131. May, R.M., Simple mathematical models with very complicated dynamics, Nature, 261, 259-267.
132. Vaidyanathan, S., Adaptive backstepping control of enzymes-substrates system with ferroelectric behaviour in brain-waves, International Journal of PharmTech Research, 2015, 8 (2), 256-261.
133. Vaidyanathan, S., Adaptive biological control of generalized Lotka-Volterra three species biological system, International Journal of PharmTech Research, 2015, 8 (4), 622-631.
134. Vaidyanathan, S., 3-cells cellular neural network (CNN) attractor and its adaptive biological control, International Journal of PharmTech Research, 2015, 8 (4), 632-640.
135. Vaidyanathan, S., Adaptive synchronization of generalized Lotka-Volterra three species biological systems, International Journal of PharmTech Research, 2015, 8 (5), 928-937.
136. Vaidyanathan, S., Synchronization of 3-cells cellular neural network (CNN) attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (5), 946-955.
137. Vaidyanathan, S., Chaos in neurons and adaptive control of Birkhoff-Shaw strange chaotic attractor, International Journal of PharmTech Research, 2015, 8 (5), 956-963.
138. Vaidyanathan, S., Adaptive chaotic synchronization of enzymes-substrates system with ferroelectric behaviour in brain waves, International Journal of PharmTech Research, 2015, 8 (5), 964-973.
139. Vaidyanathan, S., Lotka-Volterra population biology models with negative feedback and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (5), 974-981.
140. Vaidyanathan, S., Chaos in neurons and synchronization of Birkhoff-Shaw strange chaotic attractors via adaptive control, International Journal of PharmTech Research, 2015, 8 (6), 1-11.
141. Vaidyanathan, S., Lotka-Volterra two species competitive biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (6), 32-44.

142. Vaidyanathan, S., Coleman-Gomatam logarithmic competitive biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (6), 94-105.
143. Vaidyanathan, S., Output regulation of the forced Van der Pol chaotic oscillator via adaptive control method, International Journal of PharmTech Research, 2015, 8 (6), 106-116.
144. Vaidyanathan, S., Adaptive control of the FitzHugh-Nagumo chaotic neuron model, International Journal of PharmTech Research, 2015, 8 (6), 117-127.
145. Vaidyanathan, S., Global chaos synchronization of the forced Van der Pol chaotic oscillators via adaptive control method, International Journal of PharmTech Research, 2015, 8 (6), 156-166.
146. Vaidyanathan, S., Adaptive synchronization of the identical FitzHugh-Nagumo chaotic neuron models, International Journal of PharmTech Research, 2015, 8 (6), 167-177.
147. Vaidyanathan, S., Global chaos synchronization of the Lotka-Volterra biological systems with four competitive species via active control, International Journal of PharmTech Research, 2015, 8 (6), 206-217.
148. Vaidyanathan, S., Anti-synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 26-38.
149. Vaidyanathan, S., Active control design for the anti-synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 2015, 8 (7), 58-70.
150. Vaidyanathan, S., Anti-synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 71-83.
151. Vaidyanathan, S., Sliding controller design for the global chaos synchronization of enzymes-substrates systems, International Journal of PharmTech Research, 2015, 8 (7), 89-99.
152. Vaidyanathan, S., Sliding controller design for the global chaos synchronization of forced Van der Pol chaotic oscillators, International Journal of PharmTech Research, 2015, 8 (7), 100-111.
153. Vaidyanathan, S., Lotka-Volterra two-species mutualistic biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (7), 199-212.
154. Vaidyanathan, S., Active control design for the hybrid chaos synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 2015, 8 (8), 30-42.
155. Vaidyanathan, S., Hybrid chaos synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 48-60.
156. Vaidyanathan, S., Hybrid chaos synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 61-73.
157. Vaidyanathan, S., A novel coupled Van der Pol conservative chaotic system and its adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 79-94.
158. Vaidyanathan, S., Global chaos synchronization of novel coupled Van der Pol conservative chaotic systems via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 95-111.
159. Vaidyanathan, S., Global chaos synchronization of 3-cells cellular neural network attractors via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 118-130.
160. Vaidyanathan, S., Anti-synchronization of the generalized Lotka-Volterra three-species biological systems via adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 144-156.
161. Vaidyanathan, S., Global chaos control of 3-cells cellular neural network attractor via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 211-221.
162. Pham, V.-T., Volos, C. K., Vaidyanathan, S., and Vu, V. Y., A memristor-based hyperchaotic system with hidden attractors: dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8, 205-214.
163. Volos, C. K., Kyprianidis, I. M., Stouboulos, I. N., Tlelo-Cuautle, E., and Vaidyanathan, S., Memristor: A new concept in synchronization of coupled neuromorphic circuits, Journal of Engineering Science and Technology Review, 2015, 8, 157-173.
164. Pham, V.-T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristive neural network, Optoelectronics and Advanced Materials, Rapid Communications, 2014, 8, 1157-1163.
165. Volos, C. K., Pham, V.-T., Vaidyanathan, S., Kyprianidis, I. M., and Stouboulos, I. N., Synchronization phenomena in coupled Colpitts circuits, Journal of Engineering Science and

Technology Review, 2015, 8, 142-151.
166. Arneodo, A., Coullet, P., and Tresser, C., Occurrence of strange attractors in three-dimensional Volterra equation, Physics Letters A, 1980, 259-263.
167. Samardzija, N., and Greller, L.D., Explosive route to chaos through a fractal torus in a generalized Lotka-Volterra model, Bulletin of Mathematical Biology, 1988, 50, 465-491.
168. Slotine, J. and Li, W., Applied Nonlinear Control, Prentice Hall, New Jersey, USA, 1991.
169. Khalil, H.K., Nonlinear Systems, Prentice Hall, New Jersey, USA, 2001.

*****