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Stability Analysis of Combined Harvested Prey- Predator System Involving Intra-specific Competition with Active Control

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Abstract: In this article, we propose and study a three dimensional continuous time prey-predator model where the predator is exposed to the risk of disease with Holling type II functional response and we introduced combined harvesting to all the populations. The model consists of prey, susceptible predator and infected predator. We assumed that the infected predator do not predate the prey species. In this work we establish the local asymptotic stability of various equilibrium points to understand the dynamics of the model and also the global stability of the positive equilibrium solution are discussed by constructing a suitable Lyapunov function. Also the active feedback controls are introduced in this model and analysed. Finally, numerical simulations are given to illustrate the analytical results with the help of different sets of parameters.
Keywords: Prey-predator, Combined Harvesting, Global stability, Lyapunov function.

1. Introduction

A simple differential equations prey-predator model to describe the population dynamics of two interacting species was first proposed by an Italian mathematician Vito Volterra and the same differential equations was also derived by Alfred Lotka, a chemist. One of the earliest prey-predator models which are based on sound mathematical logic is the Lotka-Volterra model, which forms the basis of many models used in population dynamics. There are four factors in Lotka-Volterra model such as growth rate of prey, predation rate, mortality rate of predator and conversion rate to change prey biomass into predator population as well as prey population, which grows logistically.

Eco-epidemiological modelling provides challenges in both applied mathematics and theoretical ecology. Anderson and May(1986) [1], were the first who merged ecology and epidemiology and formulated a prey-predator model where the prey species were infected by some infectious diseases. Further, in recent years eco-epidemiological system with disease in predator become most interesting part of research among all mathematical models. Such systems governed mainly by continuous time models and these studies investigates stability, boundedness and persistence. Krishnapada Das et al. and Prasenjet Das et al. [2-3] studied the prey-predator system with disease in the predator population and discussed the chaos in this system. Pierre Auger et al., Ezio et al., Pallav et al., and so many authors have studied the prey- predator system with disease in predator [4-6]. Many authors have explored the population dynamics in eco-epidemiological systems ; (see for example [7-8]).

Now a days the study of Holling type functional responses like Holling type II, III and IV functional responses in population dynamics has attracted very much attention. To mode the phenomena of predator, Holling (1959b, 1965) [9-10], suggested three different kinds of functional response for different kinds of species. The functional responses depend on the prey density. The form of functional response $\varphi(x)$ have been developed during various different processes of energy transfer in prey-predator systems, which were proposed by different backgrounds and based on experimental data. These three forms are $\varphi(x)=\mu x$, $\varphi(x)=\frac{\mu x}{(a+x)}$ and $\varphi(x)=\frac{\mu x^2}{(a+x^2)}$ (sigmoidal), which are monotonic function with respect to x, where μ is the maximum predation rate and a is the half saturation rate, that is, if we consider the fact that in general, a single individual can feed only until the stomach is full, a saturation function indicate the intake of food. In addition, the form $\varphi(x)=\frac{\mu x}{(a+x^2)}$ is called Holling type IV functional response, which is non-monotonic function with respect to x. The qualitative analysis of prey-predator systems have been done by several papers [11-14].

Harvesting policy is obviously one of the major problems in ecology, eco-epidemiology, economics etc. The harvest of population species are mostly practiced in agriculture, fishery, forestry and population management. To control the oscillations which arise in eco-epidemiological systems, here we study the role of harvesting in an eco-epidemiological system where the prey, susceptible predator and infected predator are subjected to combined harvesting [15-16]. So many authors work on the analysis of prey-predator system with harvesting [17-18]. However, not much work has been dealt with stability analysis of three species continuous time models involving intra-specific competition and mortality with nonlinear feedback controls. The interaction between organisms or species in ecology is called competition. Due to limited resources like food, water, space etc.., competition between the species affect the community structure. Intra-specific competition is a particular form of competition in which the members of the same species competes for the same resources in an ecosystem [19].

The subject of control of the dynamical system is growing rapidly in many different fields such as ecological models, biological systems, aerospace science, structural engineering and economics. The nonlinear feedback controls, adaptive control etc... on prey-predator system has been studied by many authors [20-21]. In this work, the harvested prey-predator system monotonic functional response and with prey intra-specific competition is shown to control the chaos in the system.

This paper is organized as follows: In section 2: we have given the basic model and modified it by introducing non-selective harvesting in all the populations, intra-specific competition in prey and mortality rate in susceptible predator. In section 3: we prove for the boundedness of the non-dimensionlized model. In section 4: we find out the existence of the equilibrium points. In section 5: Local stability analysis for the trivial, axial, disease free and interior equilibrium points are presented. In section 6: Global stability analysis for the coexistent equilibrium point by constructing suitable Lyapunov function is presented. In section 7: The asymptotic stability of the total system (7) with the active feedback controls by using suitable Lyapunov function is presented. In section 8: Numerical simulations are carried out to support our analytical results. Finally, the last section 9, is devoted to the conclusion and remarks.

2. The Mathematical Model of the System

In this section, we study the dynamics of the continuous time three species prey-predator populations in which we will use the mathematical tools and biological assumptions for modelling the three species prey-predator system which consists of one prey and two predators.

2.1 The Basic Model and Assumptions

In this section, we will describe the three species continuous time prey-predator system which consists of prey, susceptible predator and infected predator. Such a system can be describe by the following set of non-linear differential equations:

In this system we assume that the susceptible predator only predates on prey. Such system can be described by the following set of nonlinear differential equations:

$$\begin{aligned}\frac{dN}{dt}&=rN\left(1-\frac{N}{K}\right)-g(N,S)\\ \frac{dS}{dt}&=mg(N,S)-QSI \qquad (1)\\ \frac{dI}{dt}&=QSI-d_2I\end{aligned}$$

The coefficients r,k,μ,a,β,d_1 and d_2 in model (1) are all positive constants and their ecological interpretation are as follows:

$N(t)$: the number of the prey population at time t,
$S(t)$: the number of the susceptible predator population at time t,
$I(t)$:the number of the infected predator population at time t,
r : represents the intrinsic growth rate of prey
K : denotes the carrying capacity of prey;
m : uptake constant of predator
Q : effective transmission rate of disease
d_2: death rate of infected predator

We assume that the infected predators do not predate prey which is very much realistic in real situation and only susceptible predators predates on prey.

Hence we will consider these response functions as of Holling type II which is given below:

$g(N,S)=\dfrac{NS}{a+N}$ is Holling type II functional response for prey and susceptible predator.

In this

a : half saturation constant

Then the system (1) becomes:

$$\begin{aligned}\frac{dN}{dt}&=rN\left(1-\frac{N}{K}\right)-\frac{NS}{a+N}\\ \frac{dS}{dt}&=\frac{mNS}{a+N}-QSI \qquad (2)\\ \frac{dI}{dt}&=QSI-d_2I\end{aligned}$$

2.2 Assumption for the Modified System

Now to formulate the modified mathematical model of a prey-predator system with disease in predator population involving intra-specific competition in predator and non-selective harvesting in all the populations, we make the following assumptions:

A1. In the absence susceptible predator, prey population grow logistically with intrinsic growth rate $(r>0)$, carrying capacity $(K>0)$ and then we have

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) \qquad (3)$$

A2. In the presence of infection, the predator population is divided into two groups namely susceptible predator denoted by $S(t)$ and infected predator denoted by $I(t)$ at all time t, the total population is $X(t)=S(t)+I(t)$.

A3. The disease is spread among the predator population only and the disease is not genetically inherited. The infected predator populations do not recover or become immune. We assume that the disease transmission follows the simple law of mass action $QS(t)I(t)$ with Q as the transmission rate.

A4. The susceptible predator $S(t)$ is removed by the death rate $d_1(>0)$ (by natural death of susceptible predator) .

A5. We assume that the only the susceptible predator population consumes prey with Holling type II functional response function:

$$fg(N,S) = \frac{NS}{a+N}, (a>0) \qquad (4)$$

That is, is the Holling type II functional response for susceptible predator and its prey. In this a is the half saturation constant and $a>0$.

A6. We have considered that the prey population $N(t)$ experiences intra-specific competition ξ (>0) due to the limited number of food resources.

A7. The prey population $N(t)$, susceptible predator $S(t)$ and infected predator $I(t)$ are removed by combined harvesting rate Eq_1, Eq_2 and Eq_3 where $q_1(>0)$, $q_2(>0)$, $q_3(>0)$ and $E(>0)$ (non selective harvesting).

Therefore the modified of the model (2) becomes:

$$\begin{aligned}
\frac{dN}{dt} &= rN\left(1-\frac{N}{K}\right) - \frac{NS}{a+N} - \xi N^2 - p_1EN \\
\frac{dS}{dt} &= \frac{mNS}{a+N} - QSI - d_1S - p_2ES \qquad (5) \\
\frac{dI}{dt} &= QSI - d_2I - p_3EI
\end{aligned}$$

Where

ξ : the prey`s crowding effect.

E: the combined external effort devoted to non-selective harvesting of prey by the external harvester (not by predator), and it is an external effort of susceptible predator and infected predator.

p_1 : the catchability coefficient of the prey

p_2 : the catchability coefficient of the susceptible predator

p_3 : the catchability coefficient of the infected predator.

We assume that the less effective predator shall be easier to harvest that is, $p_2 > p_1$,we also assume that the infected predator not become susceptible again and also the disease does not affect the ability of the infected predator attacking prey.

With initial data $x \ge 0,\ y \ge 0,\ z \ge 0$ and the coefficients $r,\ K,\ Q,\ a, m,\ d_1,\ d_2,\ \xi,\ E,\ p_1,\ p_2$ and p_3 in model (5) are all positive constants.

2.3. Nondimensionalization

Now to reduce the number of the system parameters we will transform the system (4) to the nondimensional form by using the following transformation of the variables:

$$x = \frac{N}{K},\quad y = \frac{S}{K},\quad z = \frac{I}{K},\quad t = r\tau \qquad (6)$$

The modified Holling type II prey predator with infected predator dynamics that is, using the transformation (6) the system (5) takes the nondimensional form:

$$\begin{aligned}
\frac{dx}{dt} &= x(1-x) - \frac{xy}{b+x} - \eta x^2 - \alpha_1 x\\
\frac{dy}{dt} &= \frac{\mu xy}{b+x} - \beta yz - \delta_1 y - \alpha_2 y\\
\frac{dz}{dt} &= \beta yz - \delta_2 z - \alpha_3 z
\end{aligned} \qquad (7)$$

where the relations between the nondimensional and dimensional parameters are given by:

$$\begin{aligned}
&\beta = \frac{QK}{r},\ b = \frac{a}{Kr},\ \mu = \frac{m}{r},\ \eta = \frac{\xi K}{r},\ \delta_1 = \frac{d_1}{r},\ \delta_2 = \frac{d_2}{r},\ \delta_3 = \frac{d_3}{r},\\
&\alpha_1 = \frac{Ep_1}{r},\ \alpha_2 = \frac{Ep_2}{r},\ \alpha_3 = \frac{Ep_3}{r}
\end{aligned} \qquad (8)$$

The system (7) is more simplicity than (8) for the mathematical study, since the number of system parameters has been reduced from 12 to 10 only.

Now we will analyze the system (7) with the following initial conditions:

$$x(0) > 0,\ y(0) > 0,\ z(0) > 0 \qquad (9)$$

The conditions (9) represent the conditions of positivity or biologically feasibility of the densities of prey, susceptible predator and infected predator populations respectively.

And also we observe that the right-hand side of the system of equations (7) is a smooth function of variables $(x,\ y,\ z)$ and the parameters $(\beta, b, \mu, \eta, \delta_1, \delta_2, \delta_3, \alpha_1, \alpha_2, \alpha_3)$, with the result that these quantities are non-negative. Hence the local existence and uniqueness properties hold in the positive octant. In the third equation of (7), if $z = 0$ gives $z(t) = 0$, which follows that $z = 0$ is an invariant subset, that is, $z = 0$ if and only if $z(t) = 0$ for some t. Thus $z(t) > 0$, for all t if $z(0) > 0$. The same argument follows for the second equation of (6) if $y = 0$ and for the first equation of (6) if $x = 0$. Hence $y(t) > 0$, for all t if $y(0) > 0$ and $x(t) > 0$, for all t if $x(0) > 0$.

3. Analysis of the Model

3.1 Existence and Dissipativeness

The model system (7) are continuous and have continuous partial derivatives on $\mathbf{R}_+^3 = \{(x,y,z) \in R^3 : x \geq 0,\ y \geq 0,\ z \geq 0\}$ with interaction functions $f_i\,(i=1,2,3)$. Hence the solution of the system (6) with non-negative initial condition exists and is unique, as the solution of the model system (6) initiating in the non-negative octant is bounded. And also, the system is said to be dissipative that is, all population are uniformly limited in time by the environments, if all population initiating in $\mathbf{R}_+^3$ are uniformly limited by their environment. The following theorem gives the boundedness of model system (7).

Theorem 1: All the non-negative solutions of the model system (7) that state in $\mathbf{R}_+^3$ are uniformly bounded .

Proof:

Let $x(t) > 0,\ y(t) > 0\ and\ \ z(t) > 0$ be any solution of the system with positive initial conditions.
Now we define the function

$$W(t) = x(t) + y(t) + z(t) \tag{10}$$

where $W : R_+ \to R_+$ is well defined and differentiable on some maximal interval.
Now, time derivative gives we get

$$\frac{dW}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} \tag{11}$$

$$= (1-2\eta)x + \eta(\eta - x^2) - (x-\eta)^2 - \Omega W \tag{12}$$

where $\delta = \min(1, \alpha_1, \alpha_2, \alpha_3, \delta_1, \delta_2)$

$$\frac{dW}{dt} + \Omega W \leq (1-2\eta) \tag{13}$$

$$\frac{dW}{dt} + \Omega W \leq (1-2\eta) = \varphi(say)$$

$$\frac{dW}{dt} + \Omega W = \varphi \tag{14}$$

Now applying the theory of differential inequality (Birkoff and Rota, 1982) [23], we obtain

$$0 \leq W(x,y,z) \leq \frac{\varphi}{\Omega} + W(x_0, y_0, z_0)e^{-\Omega t} \tag{15}$$

And for $t \to \infty$, we get

$$0 \leq W(x,y,z) \leq \frac{\varphi}{\Omega} \tag{16}$$

Hence, all the solutions of the system (7) that initiate in $\mathbf{R}_+^3$ are confined in the region B where,

$$B = \left\{(x,y,z) \in \mathbf{R}_+^3\ :\ 0 \leq W \leq \frac{\varphi}{\Omega} + \varepsilon,\ for\ any\ \ \varepsilon > 0\right\} \tag{17}$$

Which implies all species are uniformly bounded for any initial value in $\mathbf{R}_+^3$. And also according to the above theorem we assume that their exists $(\alpha_1, \alpha_2, \alpha_3) > 0$ such that-
$W(x_0, y_0, z_0) \subset \mathbb{R}_+^3\ \ \{(x,y,z) : 0 \leq x \leq \alpha_1, 0 \leq y \leq \alpha_2, 0 \leq z \leq \alpha_3\},\ \ \forall (x_0, y_0, z_0) \geq 0$ where $\Omega(x_0, y_0, z_0) \geq 0$ is the omega limit set of the orbit initiating at (x_0, y_0, z_0). Thus the model system (7) is uniformly limited in time by their environment. This is the complete proof.

4. Existence of Equilibria

The existence and stability condition for them as follows:

(1) The trivial equilibrium point $E_T(0,0,0)$ always exists.

(2) The axial equilibrium point $E_A(\bar{x},0,0)$ always exists as the prey population grows the carrying capacity in the absence of predation, in this $\bar{x} = \frac{1-\alpha_1}{1+\eta}$.

The predator population dies in the absence of prey. Therefore, points $(0,\alpha_1,0)$ and $(0,0,\alpha_2)$ with $j=1,2$ does not exists.

(3) In the absence of infected predator species the susceptible predator species can survive on its prey.

Hence the boundary equilibrium point $E_B(\bar{\bar{x}},\bar{\bar{y}},0)$ exists in the interior of positive quadrant of xy -plane, where $\bar{\bar{x}}$ and $\bar{\bar{y}}$ are given as follows:

$\bar{\bar{x}} = \frac{b(\delta_1+\alpha_2)}{\mu-\delta_1-\alpha_2}$ and $\bar{\bar{y}} = \frac{\delta_2+\alpha_3}{\beta}$

(4) Neither y nor z can survive in the absence of prey species x, hence there is no equilibrium point in yz plane. Due to the extinction scenario of susceptible predator, there is no equilibrium point in xz plane.

(5) The positive equilibrium point $E^*(x^*,y^*,z^*)$ exists in the interior of the first octant if and only if there is a positive solution to the following algebraic non-linear system:

$$\left.\begin{aligned}
f_1(x,y,z) &= 1-x-\frac{y}{b+x}-\eta x-\alpha_1 = 0\\
f_2(x,y,z) &= \frac{\mu x}{b+x}-\beta z-\delta_1-\alpha_2 = 0\\
f_3(x,y,z) &= \beta y-\delta_2-\alpha_3 = 0
\end{aligned}\right\} \tag{18}$$

In $E^*(x^*,y^*,z^*)$,

$$x^* = \frac{(1-\alpha_1-b-\eta b)+\sqrt{(\alpha_1+b+\eta b-1)^2-4(1+\eta)\left(\alpha_1 b-b+\left(\frac{\delta_2+\alpha_3}{\beta}\right)\right)}}{2(1+\eta)}$$

$y^* = \frac{\delta_2+\alpha_3}{\beta}$ and

$$z^* = \frac{1}{\beta}\left[\frac{\mu x^*}{b+x^*}-\delta_1-\alpha_2\right]$$

The Jacobian matrix $J(x,y,z)$ associated with model system (7) evaluated at (x,y,z) is given by

$$J_{(x,y,z)} = \begin{bmatrix} 1-2x-\frac{by}{(b+x)^2}-2\eta x-\alpha_1 & -\frac{x}{b+x} & 0 \\ \frac{\mu by}{(b+x)^2} & \frac{\mu x}{b+x}-\beta z-\delta_1-\alpha_2 & -\beta y \\ 0 & \beta z & \beta y-\delta_2-\alpha_3 \end{bmatrix} \tag{19}$$

5. Stability Analysis Of Boundary And Positive Equibruim

Theorem 2:

The trivial equilibrium point E_T is locally asymptotically stable if $\alpha_1 > 1$ and is unstable otherwise.

Proof:

The Jacobian matrix $J(E_T)$ at the equilibrium point E_T is

$$J(E_T)=\begin{bmatrix} 1-\alpha_1 & 0 & 0 \\ 0 & -\delta_1-\alpha_2 & 0 \\ 0 & 0 & -\alpha_3 \end{bmatrix} \tag{20}$$

Since λ_2, λ_3 are negative, hence E_T is asymptotically stable in the x_2x_3 direction and since $\lambda_1 = 1-\alpha_1$ E_T is stable $\lambda_1 < 0$. Hence the theorem.

Theorem 3:

The axial equilibrium point E_A is locally asymptotically stable if $\dfrac{1-\alpha_1}{2(1+\eta)} < \bar{x} < \dfrac{b(\delta_1+\alpha_2)}{\mu-\delta_1-\alpha_2}$ and is unstable otherwise.

Proof:

The Jacobian matrix $J(E_A)$ at the equilibrium point E_A is

$$J(E_A)=\begin{bmatrix} 1-2\bar{x}-2\eta\bar{x}-\alpha_1 & -\dfrac{\bar{x}}{b+\bar{x}} & 0 \\ 0 & \dfrac{\mu\bar{x}}{b+\bar{x}}-\delta_1-\alpha_2 & 0 \\ 0 & 0 & -\delta_2-\alpha_3 \end{bmatrix} \tag{21}$$

where

$\bar{x}=\dfrac{1-\alpha_1}{1+\eta}$, in this λ_3 is negative. Hence is locally asymptotically stable if $\lambda_1, \lambda_2 < 0$. Hence the theorem.

Theorem 4:

The axial equilibrium point E_B is locally asymptotically stable if $S_1 > 0$, $S_2 > 0$ and $S_1S_3 - S_2 > 0$ is unstable otherwise.

Proof:

The Jacobian matrix $J(E_B)$ at the equilibrium point E_B is

$$J(E_B)=\begin{bmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix} \tag{22}$$

where

$$b_{11} = 1 - 2\overline{\overline{x}} - \frac{b\overline{\overline{y}}}{\left(b + \overline{\overline{x}}\right)^2} - 2\eta\overline{\overline{x}} - \alpha_1, \quad b_{12} \quad -\frac{\overline{\overline{x}}}{b + \overline{\overline{x}}}$$

$$b_{21} = \frac{\mu b\overline{\overline{y}}}{\left(b + \overline{\overline{x}}\right)^2}, \quad b_{22} \quad \frac{\mu x}{b + \overline{\overline{x}}} - \delta_1 - \alpha_2, \quad b_{23} \quad -\beta\overline{\overline{y}}$$

and

$$b_{33} = \beta\overline{\overline{y}} - \delta_2 - \alpha_3$$

The characteristic equation of the above Jacobian matrix is

$$\theta^3 + S_1\theta^2 + S_2\theta + S_3 = 0 \tag{23}$$

where

$$S_1 = -\left(b_{11} + b_{22} + b_{33}\right),$$
$$S_2 = b_{11}b_{22} + b_{11}b_{33} + b_{22}b_{33} - b_{12}b_{21},$$
$$S_3 = b_{33}\left(b_{12}b_{21} - b_{11}b_{22}\right)$$

If $b_{11} + b_{22} < 0, b_{33} < 0$ and $b_{12}b_{21} < b_{11}b_{22}$, then it easy to check that $S_1 > 0,\ S_2 > 0$ and $S_1S_3 - S_2 > 0$. Using Routh- Hurwitz criteria, it is clear that the system (7) is stable at the boundary equilibrium point E_B if the conditions $S_1 > 0,\ S_2 > 0$ and $S_1S_3 - S_2 > 0$ hold. Hence the disease free system is locally stable under these conditions. But the disease free equilibrium E_B is unstable if at least one of these conditions is violated.

Theorem 5:

The interior equilibrium point E^* is locally asymptotically stable if $\omega_1 > 0,\ \omega_2 > 0$ and $\omega_1\omega_3 - \omega_2 > 0$ is unstable otherwise.

Proof:

The Jacobian matrix $J\left(E^*\right)$ at the equilibrium point E^* is

$$b_1 J\left(E^*\right) = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \tag{24}$$

where

$$a_{11} = 1 - 2x^* - \frac{by^*}{\left(b + x^*\right)^2} - 2\eta x^* - \alpha_1, \quad a_{12} \quad -\frac{x^*}{b + x^*}$$

$$a_{21} = \frac{\mu by^*}{\left(b + x^*\right)^2}, \quad a_{22} \quad \frac{\mu x^*}{b + x^*} - \beta z^* - \delta_1 - \alpha_2, \quad a_{23} \quad -\beta y^*$$

and

$$a_{32} = \beta z^*, \quad a_{33} \quad \beta y^* - \delta_2 - \alpha_3$$

The characteristic equation of the above Jacobian matrix is

$$\lambda^3 + \omega_1\lambda^2 + \omega_2\lambda + \omega_3 = 0 \tag{25}$$

where

$$\omega_1 = -\left(a_{11} + a_{22} + a_{33}\right),$$
$$\omega_2 = a_{11}a_{22} + a_{11}a_{33} + a_{23}a_{33} - a_{32}a_{23} - a_{12}a_{21},$$
$$\omega_3 = a_{11}a_{23}a_{32} + a_{33}\left(a_{12}a_{21} - a_{11}a_{22}\right)$$

If $a_{11}+a_{22}<0,\ a_{33}<0$ and $a_{12}a_{21}<a_{11}a_{22}$, then it easy to check that $\omega_1>0,\ \omega_2>0$ and $\omega_1\omega_3-\omega_2>0$. Using Routh- Hurwitz criteria, it is clear that the system (7) is stable at the interior equilibrium point E^* if the conditions $\omega_1>0,\ \omega_2>0$ and $\omega_1\omega_3-\omega_2>0$ hold.

6. Global Stablility Of $E^*\left(x^*,y^*,z^*\right)$

We have determined the conditions for global stability of interior equilibrium point $E^*\left(x^*,y^*,z^*\right)$ through the following theorem by constructing suitable Lyapunov function.

Theorem 6: Assume that the positive equilibrium point $E^*\left(x^*,y^*,z^*\right)$ is locally asymptotically stable then, it is a globally asymptotically stable in the interior of positive octant assuming the $\mu=1$.

Proof:

In order to prove the global stability, we define the following Lyapunov function

$$G(x,y,z)=G_1(x,y,z)+G_2(x,y,z)+G_3(x,y,z) \tag{26}$$

where $G_1=x-x^*-x^*\ln\dfrac{x}{x^*}$ (27)

$$G_2=y-y^*-y^*\ln\frac{y}{y^*} \tag{28}$$

$$G_3=z-z^*-z^*\ln\frac{z}{z^*} \tag{29}$$

Which implies G is a continuous function on integer $\mathbf{R}_+^3$, are positive constants to be determined. Now in order to investigate the global dynamics of the non-negative equilibrium point $E^*\left(x^*,y^*,z^*\right)$ of the model system (7) the derivative of G with respect to time along the solution of the system (26) is computed as

$$\frac{dG}{dt}=\frac{dG_1}{dt}+\frac{dG_2}{dt}+\frac{dG_3}{dt} \tag{30}$$

The time derivative of the above function will be

$$\dot G(t)=\left(x-x^*\right)\frac{\dot x}{x}+\left(y-y^*\right)\frac{\dot y}{y}+\left(z-z^*\right)\ \frac{\dot z}{z} \tag{31}$$

Using the set of equations (7) and (31) we obtain

$$\begin{aligned}\dot G(x,y,z)=&\left(x-x^*\right)\left\{1-x-\frac{y}{b+x}-\eta x-\alpha_1\right\}\\&+\left(y-y^*\right)\left\{\frac{\mu x}{b+x}-\beta z-\delta_1-\alpha_2\right\}\\&+\left(z-z^*\right)\ \left\{\beta y-\delta_2-\alpha_3\right\}\end{aligned} \tag{32}$$

$$\begin{aligned}=&\left(x-x^*\right)\left\{-\left(x-x^*\right)-\frac{\left(y-y^*\right)}{b+\left(x-x^*\right)}-\eta\left(x-x^*\right)\right\}\\&+\left(y-y^*\right)\left\{\frac{\mu\left(x-x^*\right)}{b+\left(x-x^*\right)}-\beta\left(z-z^*\right)\right\}\\&+\left(z-z^*\right)\ \left\{\beta\left(y-y^*\right)\right\}\end{aligned} \tag{33}$$

Now choosing $\mu=1$, we get

$$\dot{G}(x,y,z) = -\left(x-x^*\right)^2 - \eta\left(x-x^*\right)^2 \tag{34}$$

Which is a negative definite function. This shows that the interior equilibrium point $E^*\left(x^*,y^*,z^*\right)$ is globally asymptotically stable. Hence the Lyapunov theorem implies that $E^*\left(x^*,y^*,z^*\right)$ is globally asymptotically stable.

7. A Prey Predator Model with Vulnerable Infected predator consisting of Active feedback controls

In this section, we stabilize the chaos of prey-predator system with infected predator involving prey competition by using active control method. We constructed a suitable Lyapunov function, through this function we stabilize the system (7) by using Lyapunov stability theory [22].

Theorem 7: The prey-predator system with infection in predator, Holling type II functional response and involving competition in prey (7) and its slave system are globally and exponentially stable by using active feedback controls.

Proof:

Consider our system (7) with dynamics $\dot{x},\dot{y},\dot{z}$ as master system which consists of three states x, y, z.

Now consider the slave system as follows:

$$\begin{aligned}
\dot{x}_s &= x_s\left(1-x_s\right) - \frac{x_s y_s}{b+x_s} - \eta x_s^{\,2} - \alpha_1 x_s + u_1 \\
\dot{y}_s &= \frac{\mu x_s y_s}{b+x_s} - \beta y_s z_s - \delta_1 y_s - \alpha_2 y_s + u_2 \\
\dot{z}_s &= \beta y_s z_s - \delta_2 z_s - \alpha_3 z_s + u_3
\end{aligned} \tag{35}$$

Now let us define the stabilization errors as:

$$\begin{aligned}
\gamma_1(t) &= x_s(t) - x(t) \\
\gamma_2(t) &= y_s(t) - y(t) \\
\gamma_3(t) &= z_s(t) - z(t)
\end{aligned} \tag{36}$$

The derivative of (36) along (35) and (7) is as follows:

$$\begin{aligned}
\dot{\gamma}_1(t) &= \gamma_1(t) - \left(x_s^{\,2} - x^2\right) - \left(\frac{x_s y_s}{b+x_s} - \frac{xy}{b+x}\right) - \eta\left(x_s^{\,2} - x^2\right) - \alpha_1\left(x_s - x\right) + u_1 \\
\dot{\gamma}_2(t) &= \mu\left(\frac{x_s y_s}{b+x_s} - \frac{xy}{b+x}\right) - \beta\left(y_s z_s - yz\right) - \delta_1\gamma_2 - \alpha_2\gamma_2 + u_2 \\
\dot{\gamma}_3(t) &= \beta\left(y_s z_s - yz\right) - \delta_2\gamma_3 - \alpha_3\gamma_3 + u_3
\end{aligned} \tag{37}$$

where u_1, u_2, u_3 are active feedback controllers, which is the function of the state variables given below:

$$\begin{aligned}
u_1 &= -\gamma_1 + \left(x_s^{\,2} - x^2\right) + \left(\frac{x_s y_s}{b+x_s} - \frac{xy}{b+x}\right) + \eta\left(x_s^{\,2} - x^2\right) + \alpha_1\left(x_s - x\right) - g_1\gamma_1 \\
u_2 &= -\mu\left(\frac{x_s y_s}{b+x_s} - \frac{xy}{b+x}\right) + \beta\left(y_s z_s - yz\right) + \delta_1\gamma_2 + \alpha_2\gamma_2 - g_2\gamma_2 \\
u_3 &= -\beta\left(y_s z_s - yz\right) + \delta_2\gamma_3 + \alpha_3\gamma_3 - g_3\gamma_3
\end{aligned} \tag{38}$$

where g_1, g_2 and g_3 are positive constants.

Substitute (38) in (37), we get the error dynamics,

$$\dot{\gamma}_1(t) = -g_1\gamma_1$$
$$\dot{\gamma}_2(t) = -g_2\gamma_2 \qquad (39)$$
$$\dot{\gamma}_3(t) = -g_3\gamma_3$$

Let us construct Lyapunov function which is given as follows:

$$V(\gamma_1, \gamma_2, \gamma_3) = \frac{1}{2}\gamma_1^2 + \frac{1}{2}\gamma_2^2 + \frac{1}{2}\gamma_3^2 \qquad (40)$$

Which is a positive definite function on $\mathbb{R}^3$.

Differentiating $V(\gamma_1, \gamma_2, \gamma_3)$ along the trajectories of the error dynamics, we get,

$$\dot{V}(\gamma_1, \gamma_2, \gamma_3) = \gamma_1\dot{\gamma}_1 + \gamma_2\dot{\gamma}_2 + \gamma_3\dot{\gamma}_3 \qquad (41)$$

Substitute the error dynamics (39) in (41), we get

$$\dot{V} = -g_1\gamma_1^2 - g_2\gamma_2^2 - g_3\gamma_3^2 \qquad (42)$$

Which is a negative definite function on $\mathbb{R}^3$.

Hence by Lyapunov stability theory [22], the error dynamics (39) and the modified Holling type II functional response system (7) is globally exponentially stable

8. Numerical Simulation

We perform the numerical simulations of system (7) with the following set of parameters and explaining their complex dynamical nature. The phase portraits and the corresponding time series graph are obtained for the system (7). Parameter values are takes as follows:

Fixed parameters $b = 0.3$, $\eta = 0.055$, $\mu = 1$, $\delta_1 = 0.05$, $\delta_2 = 0.05$, $\alpha_1 = 0.03$, $\alpha_2 = 0.03$, $\alpha_3 = 0.03$ and varying the disease transmission rate β.

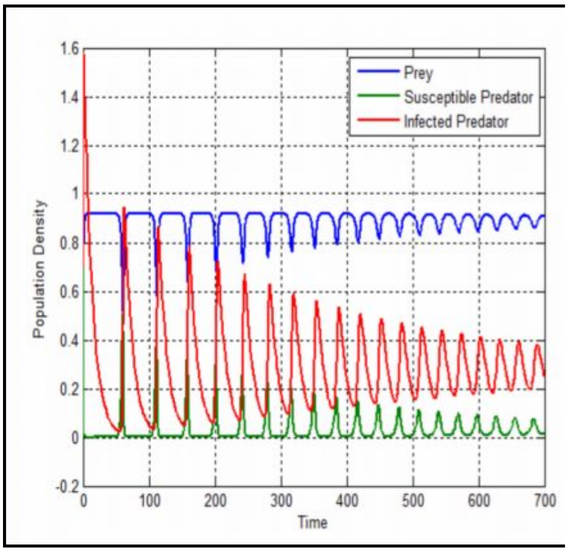

Figure 1 : The population variation with β=2.375 which shows the periodic oscillations

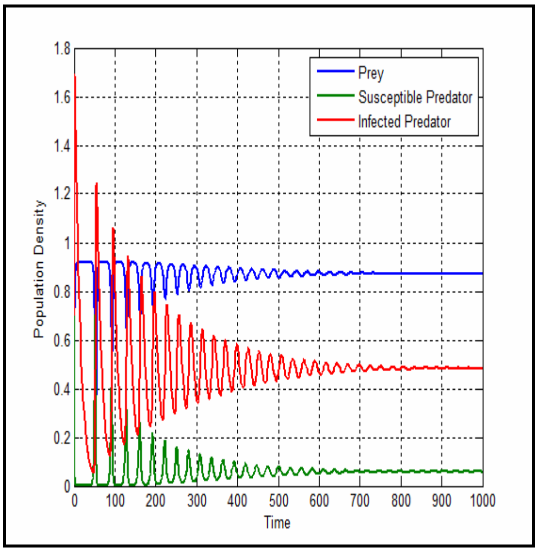

Figure 2: The population variation with β=1.375 which approaches the stable point (0.8732,0.0578,0.4833)

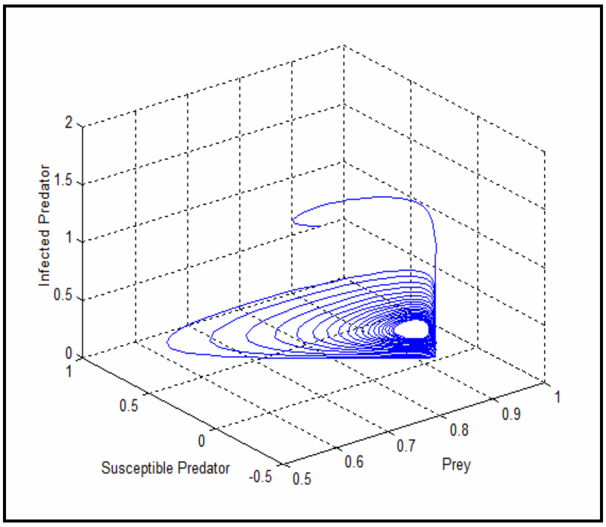

Figure 4: The phase portrait β=2.375 which approaches the stable point (0.9098,0.0131,0.2433)

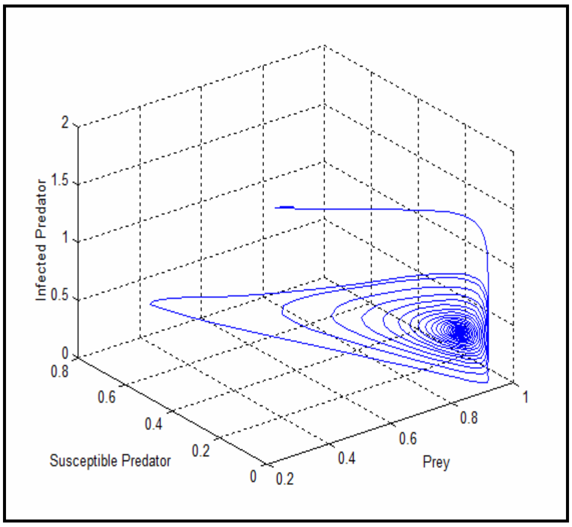

Figure 5: The phase portrait β=1.375 which approaches the stable point (0.8732,0.0578,0.4833)

When the disease transmission rate β=2.375, the periodic oscillations between prey, susceptible predator and infected predator shown in figure 1 and the corresponding phase portrait is shown in figure 4, which approaches the stable point.

When we reduce the disease transmission rate β=1.375, the population density approaches the stable point (0.8732,0.0578,0.4833) which shows in figure 2, in this we observe that the dynamic behaviour of the infected predator is reduced and the prey density is increased and the corresponding phase portrait is shown in figure 5.

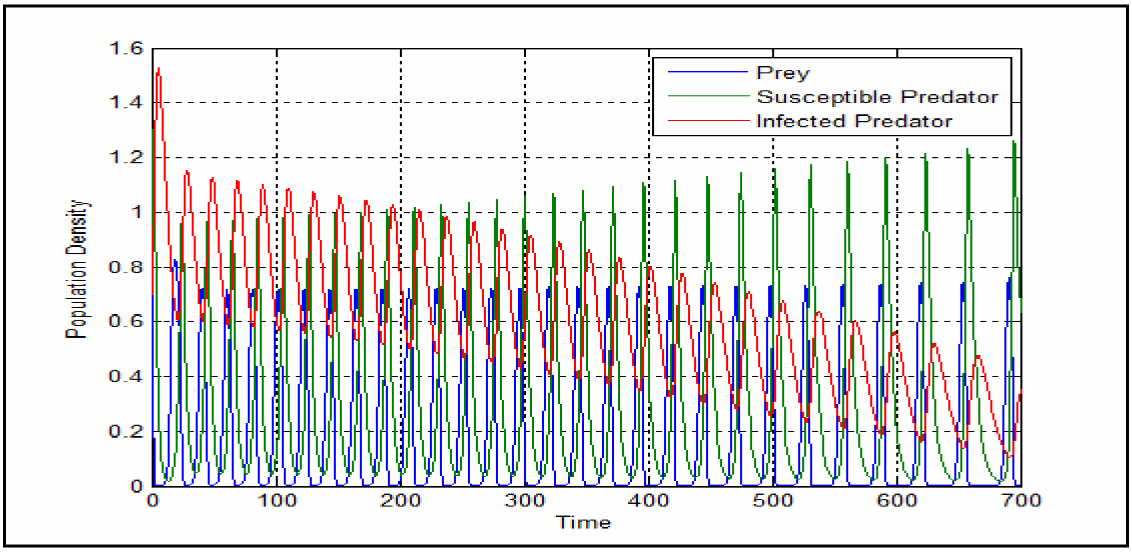

Figure 3: The dynamical behaviour of the population variation with β=0.255

When we again gradually reduce the disease transmission rate β=0.255, the complex behaviour of the prey and susceptible predator shown in figure 3 and the chaotic behaviour of the whole population density is shown in figure 6, and figure 7 shows the chaotic behaviour of the population density when β=0.275.

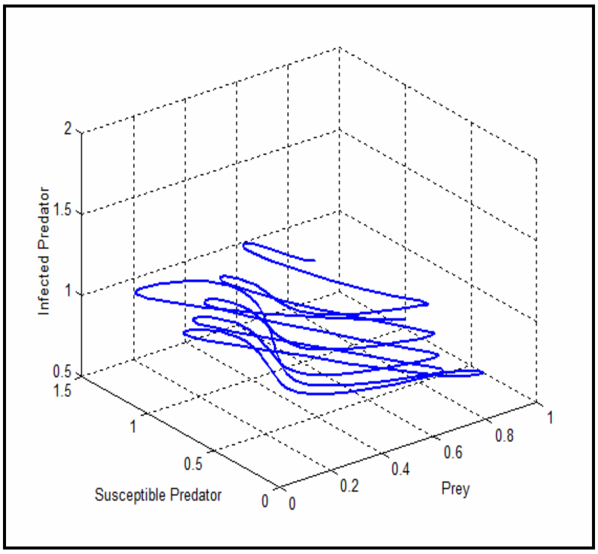

Figure 6 : The phase portrait β=0.275 which exhibits chaotic behaviour

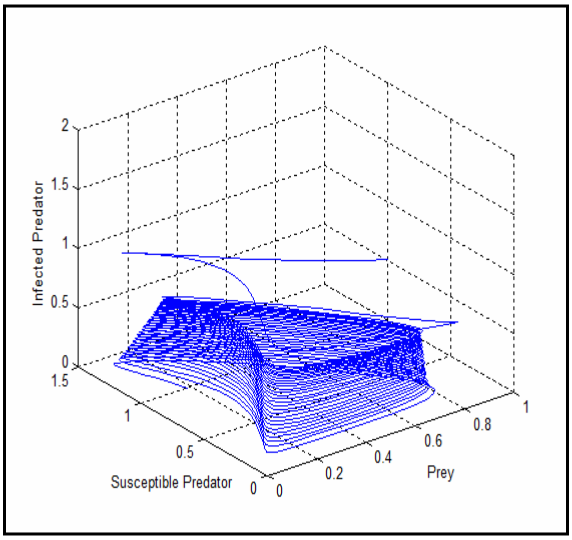

Figure 7: The phase portrait β=0.255which exhibits chaotic behaviour

Now, let us perform the numerical simulations of non linear control system (38) with the following set of parameters and explaining their stabilization. The time series graph and the corresponding phase portrait are obtained for the controlled system (38). Same parameter values are takes as follows:

Fixed parameters $b = 0.3$, $\eta = 0.055$, $\mu = 1$, $\delta_1 = 0.05$, $\delta_2 = 0.05$ and varying the disease transmission rate β and also harvesting rate α_1, α_2, α_3.

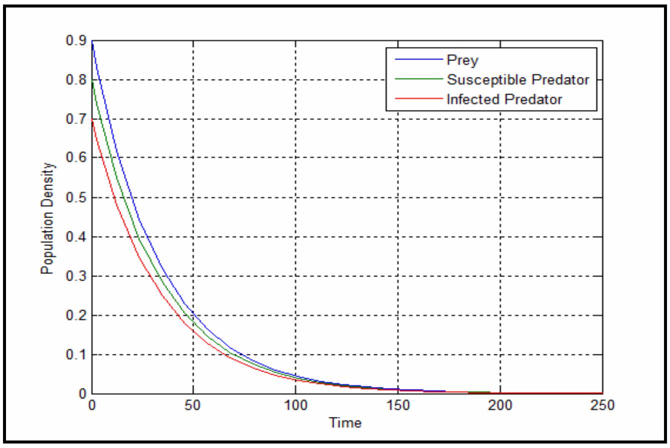

Figure 8: The population variation with $0.245 < \beta < 2.475$ **and** $0.01 < \alpha_1, \alpha_2, \alpha_3 < 0.033$ **which approaches the point (0,0,0)**

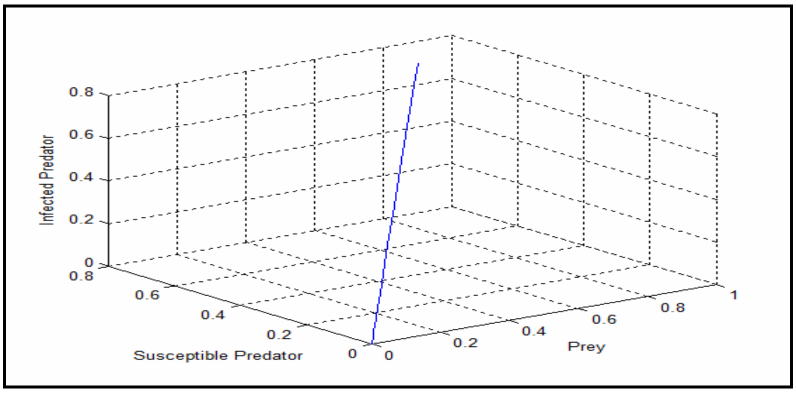

Figure 9: The phase portrait with $0.245 < \beta < 2.475$ **and** $0.01 < \alpha_1, \alpha_2, \alpha_3 < 0.033$ **which approaches the point (0,0,0)**

9. Conclusion

In this paper we have investigated the dynamical complexities of a harvesting prey-predator system with infected predator and Holling type II functional response involving prey competition. The boundedness of the trajectories and existence of equilibrium points are established. The local stability for non-negative equilibrium points has been analysed. The global stability has been analysed by constructing Lyapunov function. Also in this work, we have introduced the active feedback controls to the system and analysed. Finally, numerical simulations are carried out by using MATLAB software package. We have generated phase plots and time series diagrams. Also the comparison between the control and uncontrol system has been investigate.

AMS Subject Classification: 92D25; 92D30; 34C23; 34D23

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