



## Rheology of SFRC beams Strengthened with GFRP Laminates under Low Cycle Compressive Loads

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**Abstract:** Engineering structures are inevitably witnessing cyclic loads in the form of seismic loads. Due to this pulsating action, structural elements and frames undergo deformations resulting in stiffness reduction; formation of cracks followed by failure in the structural integrity of the elements itself. Hence efforts are made by researchers in all possible quarters to understand the behaviour of structural elements subjected to fatigue conditions. This paper presents an explicit experimental investigation on Steel Fibre Reinforced Concrete (SFRC) beams strengthened with Glass Fibre Reinforced Plastic (GFRP) laminates subjected to cyclic loading. The experimental program consists of six strengthened beams with steel fibre and one control beam without fibre and strengthening. The beams are tested under low cycle repeated compressive loading. The test results showed an enhanced performance of beams in terms of strength, deformation, ductility characteristics and crack resistance. The load-deflection and crack patterns are analyzed for loading-unloading-reloading nature of repeated compressive loads and the maximum number of loading cycle obtained is 14. The experimental results are validated with multi-linear regression equations. To substantiate this, fitness values and root mean square error for the predicted regression results are well within the limits.

**Keywords:** Cyclic loads, Stiffness, Energy absorption, SFRC, GFRP laminate, Regression, Fitness, RMS error.

### 1. Introduction

The use of fibre reinforced polymer (FRP) composite laminates in strengthening bridge structures [12] has become popular, primarily due to the higher strength-weight ratio and easier installation of the material compared to other conventional materials like steel plates. The application of fibre-reinforced plastics in infrastructure rehabilitation and retrofitting offers great potential for efficient increase of load carrying capacity and restoration of system integrity in reinforced concrete structures. Consequently, the use of FRP bonded to deteriorated and damaged reinforced concrete structures is rapidly gaining popularity worldwide; high strength FRPs have been used to retrofit [6] concrete members such as columns, slabs, beams and girders in structures such as bridges, parking decks, smoke stacks and buildings. Studies of FRP composites bonded to the tension face of concrete beam elements by researchers have demonstrated that theoretical gains in flexural strength [4] can be significant. An important advantage of FRP composites that make them suitable for retrofitting applications is their durability against environmental exposure. The FRP laminate and epoxy adhesive used in retrofitting applications have excellent corrosion resistance compared to conventional construction materials. Integrity and durability of the bond between concrete and the composite laminate [1] is of utmost importance in a retrofitted system.

Advanced cement based materials and improved concrete construction techniques provide opportunities for the design of structures to resist severe loads resulting from earthquakes, impact, fatigue, and blast environments. Conventional concrete cracks easily. When concrete is reinforced with random dispersed fibres, we get favourable behaviour for repeated loads. Fibres prevent micro cracks from widening. Addition of fibres makes components ductile and tough. Research carried out in various parts of the world has established that addition of fibres improves the static flexural strength, fatigue, ductility, and fracture toughness of the material.

**M.Maalej, W.H. Goh and P. Paramasivam (2001)** studied the local failure of debonding or ripping of concrete cover in FRP plated RC beams caused due to high interfacial shear and normal stress concentrations. Predictive models for finding the interfacial shear stress were reviewed and evaluated using experimental data. Finally, the most critical parameters governing the interfacial shear strength and stress as determined by the models were also examined.

**Patrick L. Minnaugh, Kent A. Harries (2009)** investigated into the identification of deleterious effects of fatigue loading on the bond behavior of SFRP. These specimens were paired with unretrofit and CFRP-retrofit companion specimens for direct comparison of results. The SFRP specimens were tested at various fatigue load levels ranging from service load level (2 million cycles) to an extreme load level in order to find out the fatigue induced failure of the internal reinforcing steel.

**Kaushal Parikh, C.D.Modhera (2012)** carried out the application of glass fiber reinforced polymer sheet on seventeen small scaled beams. The main experimental parameters include arrangement of GFRP sheet, preload level at the time of strengthening and numbers of layers of GFRP. Arrangement of FRP was evaluated by using ductility and toughness criteria. The results indicated that number of layers and arrangement of GFRP, as well as preload level have more influence on the stiffness, toughness and ductility of the strengthened beam, both at post cracking and post yielding stage, than that on the yielding and flexural strength of strengthened beam.

**Ravikant Shrivastava, Uttamasha Gupta, U B Choubey (2013)** made an experimental investigation on flexural behaviour of Reinforced Concrete (RC) beams strengthened using Fiber Reinforced Polymer (FRP) under cyclic loading. It was observed that flexural strengthening of RC beams provides additional strength but with brittle mode of failure and at cost of ductility. FRP strengthened beams after FRP rupture showed behaviour of unstrengthened beams with yielded steel. In no case end span debonding has been noticed, extending FRP to supports effectively mitigated concrete cover delamination. Strengthening using FRP was found to be more effective in case of under reinforced RC beams having lower amount of steel. Distribution of FRP over the tension face provided more effectiveness and better configuration. The permissible load capacity of FRP strengthened beams was decided using load-deflection stability point curves and was concluded that maximum load be reduced in case of cyclic loading.

In the present study, uni-directional Glass Fibre Reinforced Polymer (GFRP) bonded Steel Fibre Reinforced Concrete (SFRC) beams [3] were subjected to cyclic loads [2] of different amplitudes and their deformation characteristics are evaluated. An analytical approach using mathematical modelling of regression equations is also presented to calculate the deformation parameters of external GFRP bonded SFRC beams under cyclic loading.

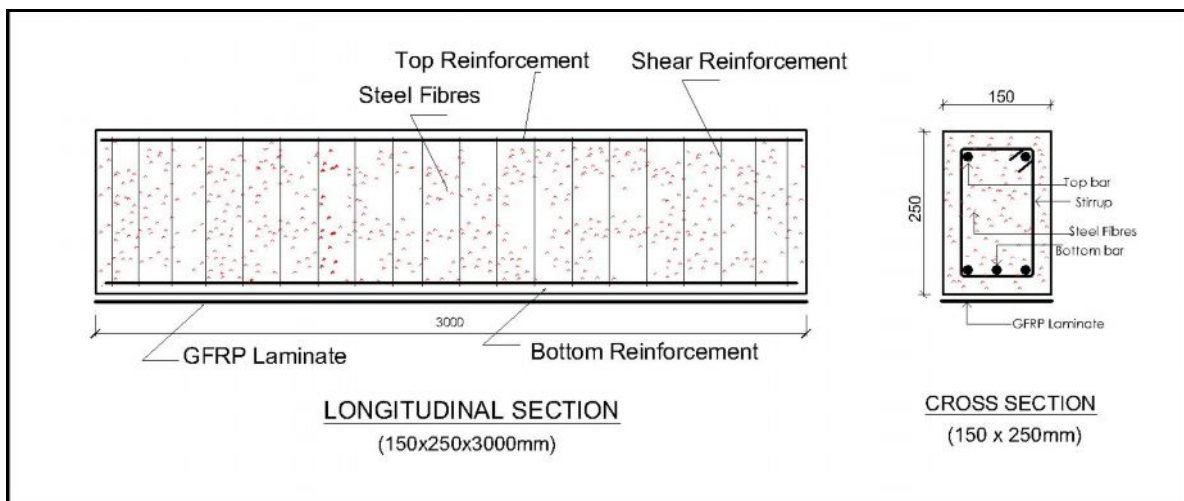
## 2. Experimental Programme

In this study, concrete of grade M20 was used and it was designed as per the BIS Standards. The mix was designed with a water-cement ratio (w/c) of 0.50. Hook end steel fibre with volume fractions (V<sub>f</sub>) of 0.5%, 1.0% and 1.5% were added to the concrete by weight of cement. The concrete mix proportion used in the test program is presented in the Table 1.

**Table 1. Constituents of Concrete Mix**

Grade of concrete	Cement	Fine Aggregate	Coarse Aggregate	Water	Steel Fibre
	kg/m <sup>3</sup>	kg/m <sup>3</sup>	kg/m <sup>3</sup>	kg/m <sup>3</sup>	kg/m <sup>3</sup>
M20	383	546	1187	192	0.5% = 1.92
					1.0% = 3.83
					1.5% = 5.75

The experimental programme was conducted to study the flexural performance of the GFRP strengthened steel fibre reinforced concrete beams [8] in conjunction with the conventional RC beam. The research work consisted of casting a total of 7 rectangular beams of cross-section 150mm x 250mm and 3m long. The beams were made of concrete of strength 27.11 MPa and provided with HYSD bars of yield strength 445.63 MPa. The beams were tested under four-point bending over a simple span of 2.8m. All the beams were designed for the under-reinforced condition with percentage of tension steel,  $P_t=1.14\%$ . The variables considered for the study include the steel fibre volume fraction 'Vf' and GFRP laminate thickness 't' bonded at the beam bottom from end-to-end. For all the test beams, the study parameters included ultimate load, number of cycles, mid span deflection, stiffness, energy absorption, and crack details [3]. In all the 7 beam test specimens, 6 steel fibre reinforced concrete beams were flexurally strengthened (soffit of the beam only) with GFRP laminates and 1 beam was used as control specimen. All the GFRP strengthened beams and control beam were tested under cyclic loading until failure load [4]. The details of beam are furnished in Table 2 and a typical reinforcement detail is shown in Fig.1.



**Fig. 1. Reinforcement Details of Test Beam**

**Table 2. Details of Test Specimens**

Beam ID	Beam Type	Steel Fibre Volume Fraction 'Vf'	GFRP Laminate Thickness 't'	Bottom Steel	Top Steel	2L - 8φ Stirrups Spacing
		%	mm			mm c/c
NSC	Control Beam	0	0	3-12φ	2-10φ	200
NSF-P3	GFRP Strengthened SFRC Beam	0.5	3	3-12φ	2-10φ	200
NSF-P5	GFRP Strengthened SFRC Beam	0.5	5	3-12φ	2-10φ	200
NSF-Q3	GFRP Strengthened SFRC Beam	1.0	3	3-12φ	2-10φ	200
NSF-Q5	GFRP Strengthened SFRC Beam	1.0	5	3-12φ	2-10φ	200
NSF-R3	GFRP Strengthened SFRC Beam	1.5	3	3-12φ	2-10φ	200
NSF-R5	GFRP Strengthened SFRC Beam	1.5	5	3-12φ	2-10φ	200

**Note:** GFRP orientation – Unidirectional, parallel to beam axis.

### Cyclic Test Procedure

Seven beams were tested under cyclic loading using a push pull jack arrangement. The specimens were tested under four point-bending in a loading frame of 50 Tons capacity in dynamic. The beams were simply supported at the ends with one end hinged and roller at the other end. The beams were supported with 100mm bearing at the ends, resulting in a test span of 2.8m. Two-point loading was applied through a spreader beam. The deflection measurements were recorded at the time of loading-unloading-reloading for each cycle. Crack widths, crack spacing, number of cracks and corresponding cycles were periodically measured during cyclic loading. All the above mentioned measurements were taken until the failure of the beam. The details of cyclic load test set-up of beams are in shown in Fig. 2.



Fig. 2. Cyclic Load Beam Test Set-up

### 3. Test Results and Discussions

The influence of cyclic loading on the behaviour of 3mm and 5mm thick GFRP laminate strengthened SFRC beams with various fibre volume fractions (0%, 0.5%, 1.0% and 1.5%) was investigated. Adequate data were obtained and presented in Table 3 with regard to ultimate load, number of cycles, deflection, stiffness, and energy absorption of GFRP laminate strengthened SFRC beams.

Table 3. Cyclic Test Results of Beams

Beam ID	Ultimate Load	Total Number of Cycles	Ultimate Deflection	Initial Stiffness	Final Stiffness	Total Energy Absorption
	kN	no's	mm	kN/mm	kN/mm	kNmm
NSC	32.5	3	1.5	32.60	21.67	42.69
NSF-P3	100.9	13	6.5	45.60	15.52	367.61
NSF-P5	100.9	13	6.5	45.60	15.52	386.15
NSF-Q3	100.9	14	7.0	45.60	14.41	410.51
NSF-Q5	100.9	14	7.0	45.60	14.41	421.74
NSF-R3	91.10	11	5.5	39.00	16.56	256.25
NSF-R5	110.60	11	5.5	45.60	20.11	309.60

#### Effect on Cyclic Test Parameters

In control beams without steel fibres, the ultimate load of 32.5 kN was reached within 3 cycles of loading. In 3mm and 5mm thick GFRP laminate strengthened beams, the ultimate load reached at 13 cycles for 0.5% Vf, 14 cycles for 1.0% Vf and 11 cycles for 1.5% Vf. The percentage increase in the number of loading cycles to reach the ultimate load of the strengthened beams when compared to control beam was 76.92% for

0.5%Vf, 78.57% for 1.0%Vf and 72.73% for 1.5% Vf. The variation of number of cycles for 3mm and 5 mm thick GFRP laminate strengthened beams corresponding to 0%, 0.5%, 1.0% and 1.5% fibre volume fraction is shown in Fig. 3.

The 3mm and 5mm thick GFRP laminate strengthened SFRC beams exhibited a higher resistance to deflection when it was examined for the ultimate load of the control beam. At ultimate load of 32.5kN of control beam, the strengthened beams measured a deflection of about 1.0 mm only and withstood a loading cycle up to 6. The increase in number of cycles of loading was 7 and the percentage variation in deflection was 66.67% between the control beam and strengthened beam. The variation of deflection for 3mm and 5 mm thick GFRP laminate strengthened beams corresponding to 0%, 0.5%, 1.0% and 1.5% fibre volume fraction is shown in Fig. 4.

The stiffness of the beam gets reduced as the load progresses till the failure of beams. The reduction in stiffness of beams was obtained by the percentage difference between initial and final stiffness values. The percentage reduction from initial stiffness to final stiffness of control beam was 33.52%. The percentage reduction from initial stiffness to final stiffness of 3mm and 5mm GFRP laminate strengthened beams were 65.96%(3mm & 5mm GFRP) for 0.5% Vf, 68.39% (3mm & 5mm GFRP) for 1.0%Vf and 55.89% (3mm) to 57.53% (5mm) for 1.5% Vf. The percentage reduction in stiffness from initial to final stages of loading for 3mm and 5 mm thick GFRP laminate strengthened beams corresponding to 0%, 0.5%, 1.0% and 1.5% fibre volume fraction is shown in Fig 5.

In control beams without steel fibres and GFRP laminate strengthening, the energy absorption capacity was 42.69kNmm. The energy absorption capacity of 3mm and 5mm thick GFRP laminate strengthened beams were 367.61kNmm & 386.15kNmm respectively for Vf=0.5%, 410.51kNmm &421.74kNmm respectively for Vf=1.0% and 256.25kNmm & 309.60kNmm respectively for Vf=1.5%. A comparison between 3mm and 5mm thick GFRP laminate revealed that the 5mm thick GFRP laminate exhibited a higher energy absorption capacity than the 3mm thick GFRP laminate. Also it was observed from the test results, that the energy absorption increased for Vf = 0.5 % & 1.0% and reduced for Vf=1.5%. Hence the addition of steel fibre beyond 1.0% Vf had no impact on the energy absorption capacity of beams. The variation of total energy absorption capacity of 3mm and 5 mm thick GFRP laminate strengthened beams corresponding to 0%, 0.5%, 1.0% and 1.5% fibre volume fraction is shown in Fig. 6.

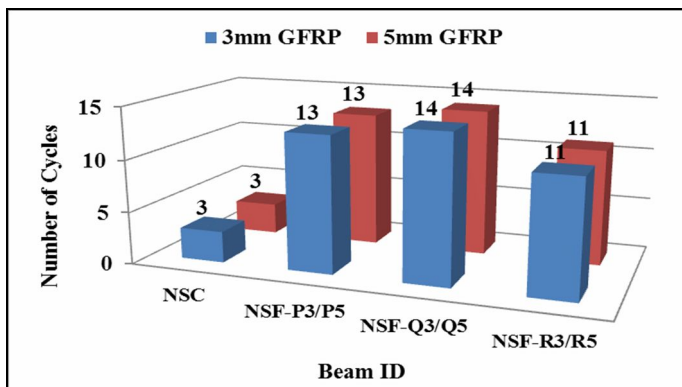


Fig. 3 Number of Cycles

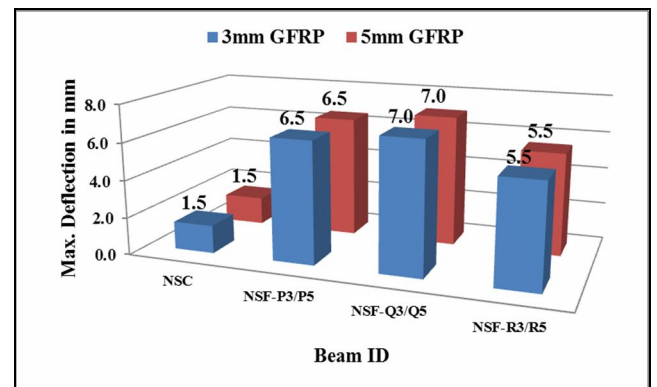
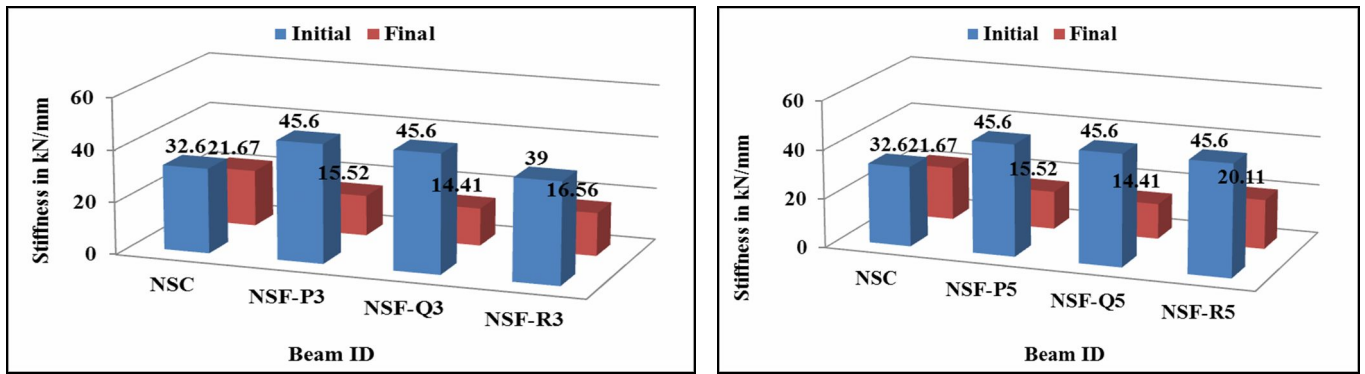


Fig. 4. Deflection of Beams



a. 3mm GFRP Laminate

b. 5mm GFRP Laminate

Fig. 5. Stiffness of Beams – Initial to Final

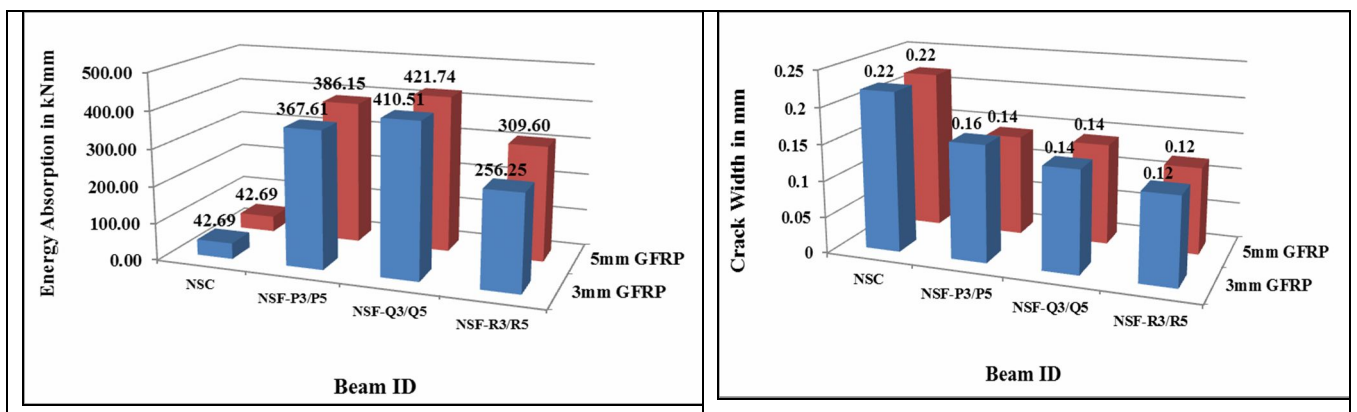
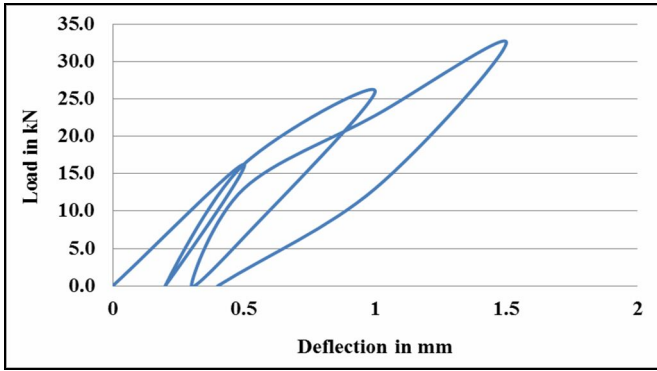


Fig. 6. Energy Absorption

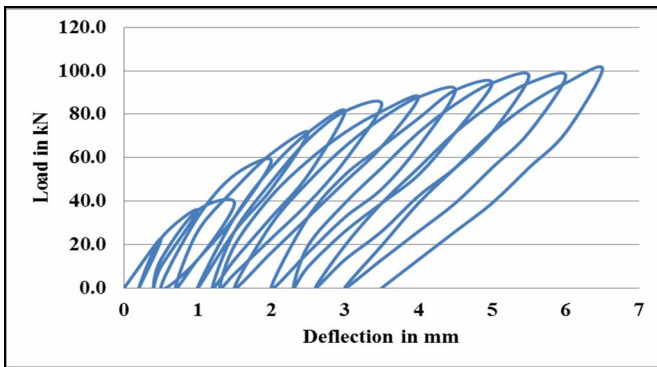
Fig. 7. Crack Width at Ultimate Stage

The loads Vs deflection behaviour of beams in loading-unloading-reloading stages pertaining to each load cycle was obtained in the form of loop curves and is shown in Fig. 8a to 8g. For each cycle of loading-unloading-reloading, the deflection increased while loading, then dropped to zero or minimum residual value while unloading and then attained a maximum value while reloading. All these individual load-deflection cycles are assembled from initial to ultimate stages of loading till failure to form the envelope curve of deflection under cyclic loadings. From observations, it was found that the envelope of the loop curve that deflection gradually increases with increase in load levels till it reached the ultimate load of failure. The deflection Vs number of cycles for beams with 3mm and 5mm GFRP laminate thickness exhibited a linear upward variation with increase in load levels and are shown in Fig. 9a & 9b. The stiffness Vs number of cycles for beams with 3mm and 5mm GFRP laminate thickness showed a non-linear downward variation as shown in Fig. 10a & 10b.

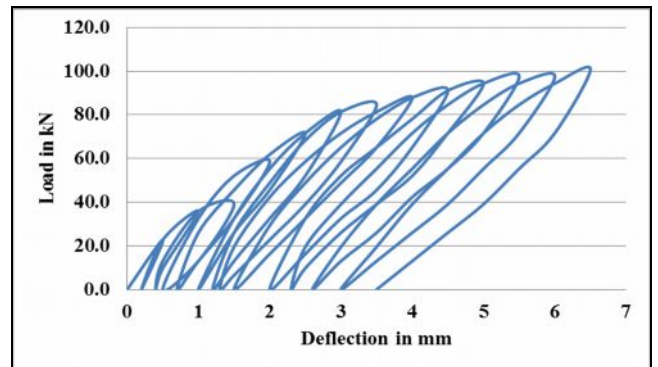
The energy absorbed by the beams when subjected to cyclic loading was computed from the area under the loop curve obtained as a result of load-deflection behaviour shown in Fig.8a to 8g. The area under the individual loop curve pertaining to each cycle of loading were added cumulatively to arrive at the total energy absorbed by a beam in withstanding the effect of cyclic loading. The energy absorption Vs number of cycles for beams with 3mm and 5mm GFRP laminate thickness increased with increase in load levels and are shown in Fig. 11a & 11b.



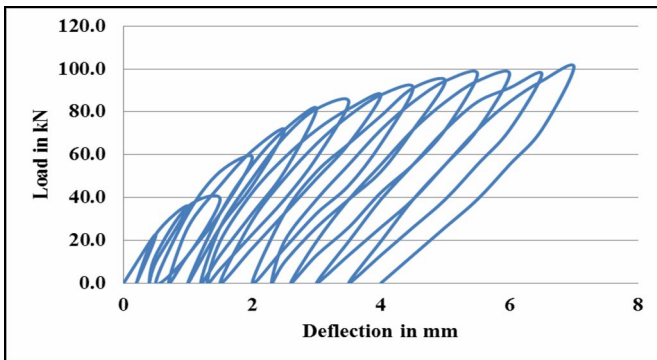
a. NSC Beam



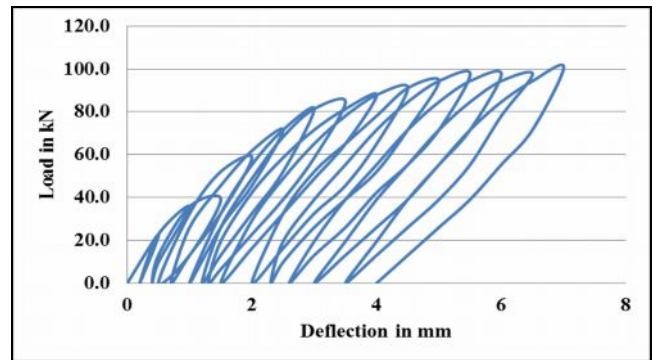
b. NSF-P3 Beam



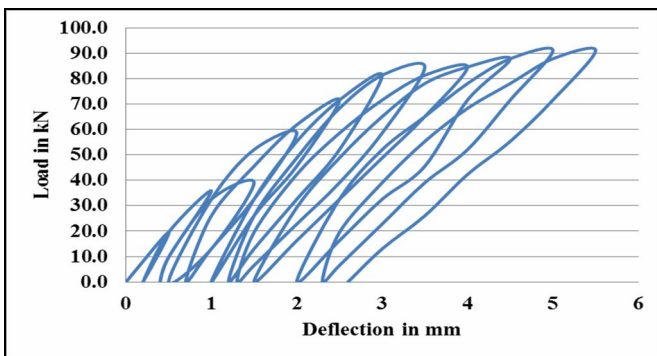
c. NSF-P5 Beam



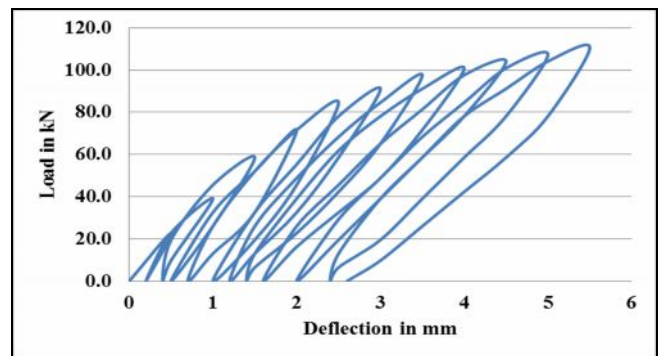
d. NSF-Q3 Beam



e. NSF-Q5 Beam

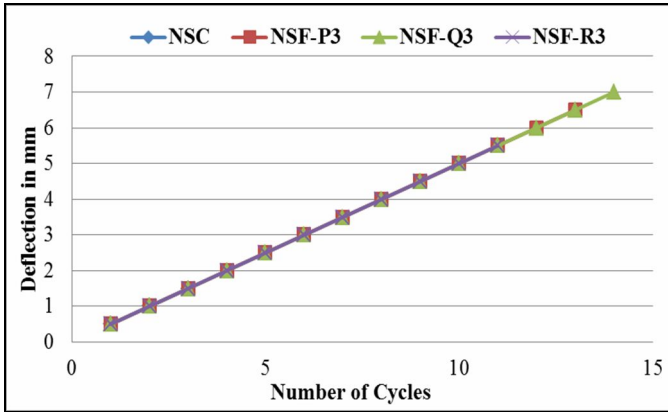


f. NSF-R3 Beam

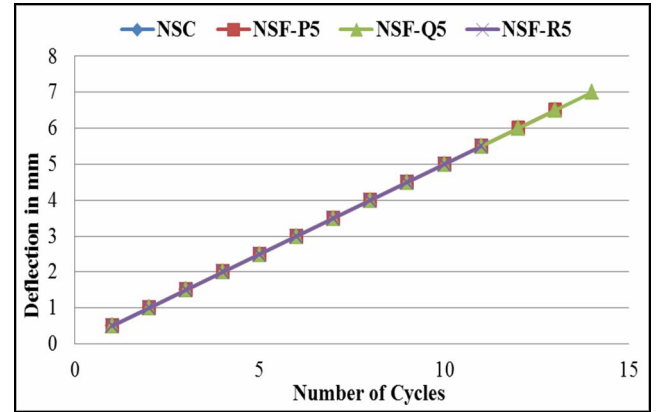


g. NSF-R5 Beam

Fig 8. Load Vs Deflection

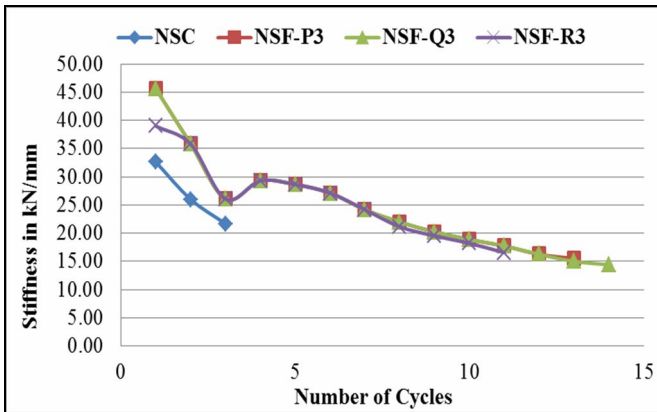


a. 3mm GFRP Laminate

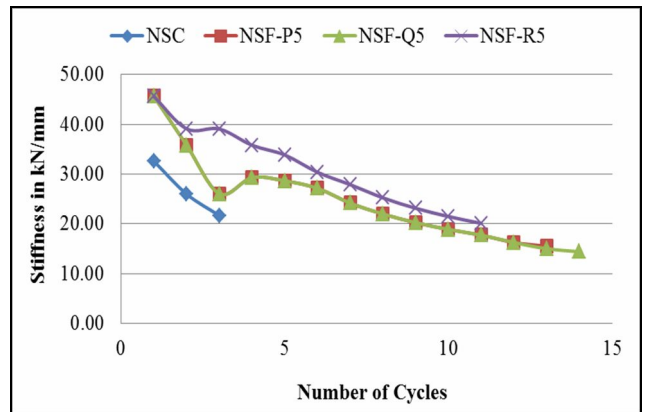


b. 5mm GFRP Laminate

Fig. 9. Deflection Vs Number of Cycles

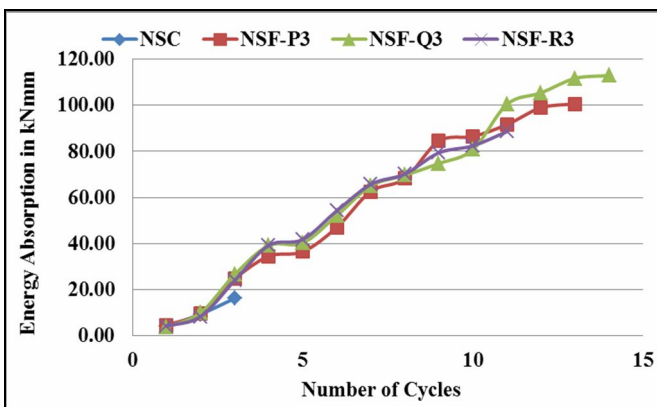


a. 3mm GFRP Laminate

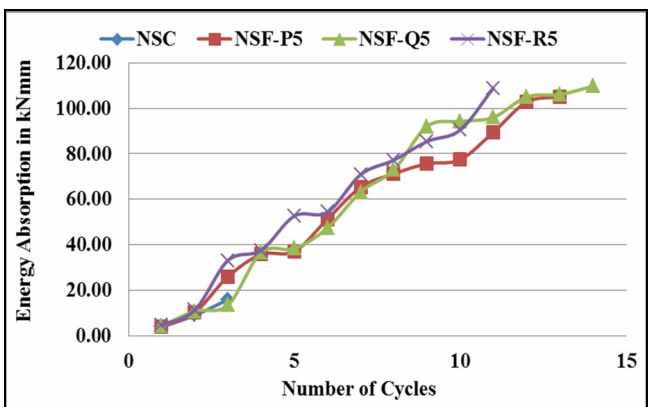


b. 5mm GFRP Laminate

Fig. 10. Stiffness Vs Number of Cycles



a. 3mm GFRP Laminate



b. 5mm GFRP Laminate

Fig. 11. Energy Absorption Vs Number of Cycles



### Effect on Crack Width

The beams experienced considerable flexural cracking and vertical deflection near to failure. Well distributed closely spaced cracking was observed. Flexural cracks were initiated in the constant moment region as the tensile strength of concrete was reached. The cracks propagated upwards as loading progressed but remained very narrow throughout the loading period. The crack details pertaining to crack width, crack spacing and numbers of cracks at ultimate load are presented in Table 4. The crack pattern and failure modes are shown in Fig. 12.

**Table 4. Crack Details at Ultimate Stage**

Beam ID	Ultimate Load	Crack Width	Crack Spacing	Number of Cracks
	kN	mm	mm	No's
NSC	32.5	0.22	84	20
NSF-P3	100.9	0.16	81	16
NSF-P5	100.9	0.14	78	15
NSF-Q3	100.9	0.14	81	15
NSF-Q5	100.9	0.14	80	13
NSF-R3	91.10	0.12	83	15
NSF-R5	110.60	0.12	98	13

The beams strengthened with 3mm and 5mm thick GFRP laminate had considerably reduced the effect of crack width at ultimate loads when compared to control beams. The percentage reduction in crack width for 3mm and 5mm thick GFRP laminate strengthened beams corresponding to 0.5%  $V_f$ , was 27.27% and 36.36% respectively when compared to control beam. The percentage reduction in crack width for both 3mm and 5mm thick GFRP laminate strengthened beams was 36.36% for  $V_f=1.0\%$  and 45.45% for  $V_f=1.5\%$  when compared to control beam. The variation of crack width at ultimate loads for 3mm and 5 mm thick GFRP laminate strengthened beams for 0%, 0.5%, 1.0% and 1.5% fibre volume fraction is shown in Fig 7.





**Fig. 12. Crack Patterns under Cyclic Loading**

### 3. Analytical Predictions

The study parameters under cyclic loading obtained by the laboratory tests are validated by means of developing a reliable mathematical modeling using regression equations. The regression equations are formed on the basis of multiple linear regressions using multiple independent predictor variables to fit the equation for dependent variables. In this analysis fibre volume fraction ( $V_f$ ), concrete strength ( $f_{ck}$ ) and thickness of GFRP laminate ( $t$ ) are considered as independent predictor variables whereas number of cycles, ultimate load, ultimate deflection, stiffness, total energy absorption and crack width are considered as dependent variables. The method employed to obtain these proposed regression equation is by method of least squares. The data obtained from experimental study are used for calibrating the values of unknown regression co-efficients in such a way that the difference between the predicted values and experimental values remain a minimum. The proposed regression equations for the study parameters are presented in Table 5. From the proposed equations the predicted values [5] are arrived which are correlated with the laboratory results for its accuracy and are presented in Table 6 and Fig. 13a to 13f.

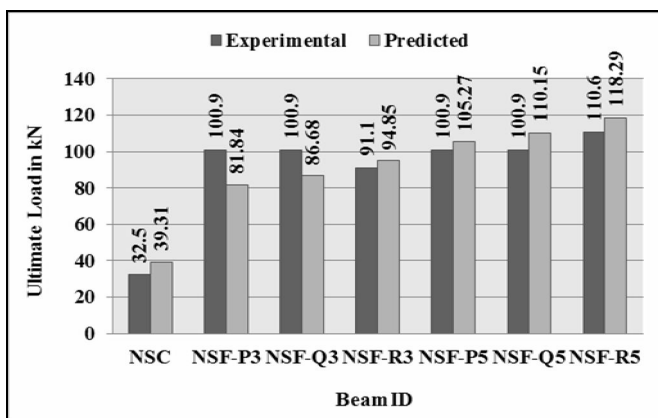
**Table 5. Proposed Regression Equations for Study Parameters**

Parameter	Regression Equation	RMS Error	Fitness
Total Number of Cycles	$104 + 23.5 (V_f) - 3.66 (f_{ck}) + 1.02 (t)$	2.880	0.532
Ultimate Load	$-61 - 10 (V_f) + 3.7 (f_{ck}) + 12.55 (t)$	14.424	0.94
Ultimate Deflection	$51.8 + 11.8 (V_f) - 1.83 (f_{ck}) + 0.512 (t)$	1.440	0.532
Ultimate Stiffness	$-111 - 30.0 (V_f) + 4.85 (f_{ck}) - 0.04 (t)$	2.877	0.503
Total Energy Absorption	$4598 + 1031 (V_f) - 166 (f_{ck}) + 30.9 (t)$	9.624	0.415
Crack Width at Ultimate Load	$0.579 + 0.046 (V_f) - 0.0135 (f_{ck}) - 0.01173 (t)$	0.012	0.435

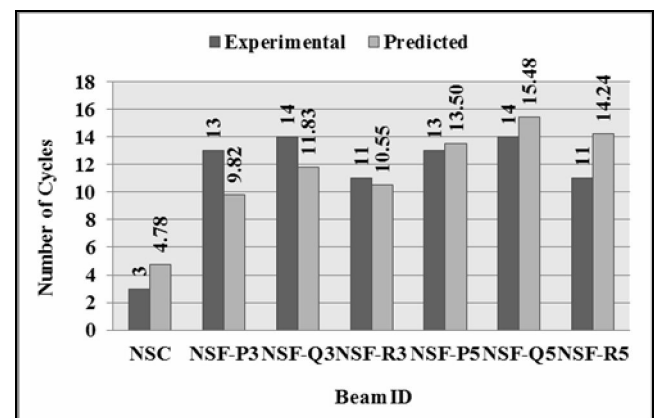
**Table 6. Comparison of Experimental and Predicted Results**

Sl. No.	Parameter	Unit	Beam ID	Experimental	Predicted	Prediction Error
1	Total Number of Cycles	No	NSC	3	4.77	-1.77
			NSF-P3	13	9.81	3.19
			NSF-P5	13	13.50	-0.5
			NSF-Q3	14	11.82	2.18
			NSF-Q5	14	15.48	-1.48
			NSF-R3	11	10.55	0.45
			NSF-R5	11	14.23	-3.23
2	Ultimate Load	kN	NSC	32.5	39.30	-6.8
			NSF-P3	100.9	81.83	19.07
			NSF-P5	100.9	105.27	-4.37

			NSF-Q3	100.9	86.67	14.23
			NSF-Q5	100.9	110.15	-9.25
			NSF-R3	91.1	94.85	-3.75
			NSF-R5	110.6	118.28	-7.68
3	Ultimate Deflection	mm	NSC	1.5	2.18	-0.68
			NSF-P3	6.5	4.73	1.77
			NSF-P5	6.5	6.58	-0.08
			NSF-Q3	7	5.77	1.23
			NSF-Q5	7	7.60	-0.6
			NSF-R3	5.5	5.15	0.35
			NSF-R5	5.5	7.00	-1.5
4	Ultimate Stiffness	kN/m m	NSC	21.67	20.48	1.19
			NSF-P3	15.52	18.31	-2.79
			NSF-P5	15.52	16.05	-0.53
			NSF-Q3	14.41	16.21	-1.8
			NSF-Q5	14.41	14.00	0.41
			NSF-R3	16.56	18.48	-1.92
			NSF-R5	20.11	16.21	3.9
5	Total Energy Absorption	kNmm	NSC	42.69	97.74	-55.05
			NSF-P3	367.61	362.72	-4.89
			NSF-P5	386.15	399.22	13.07
			NSF-Q3	410.51	336.66	73.85
			NSF-Q5	421.74	471.50	-49.76
			NSF-R3	256.25	261.20	-4.95
			NSF-R5	309.60	397.70	-88.1
6	Crack Width at Ultimate Load	mm	NSC	0.22	0.21	0.01
			NSF-P3	0.16	0.16	0
			NSF-P5	0.14	0.15	-0.01
			NSF-Q3	0.12	0.13	-0.01
			NSF-Q5	0.14	0.15	-0.01
			NSF-R3	0.14	0.13	0.01
			NSF-R5	0.12	0.11	0.01



a. Ultimate Load



b. Number of Cycles

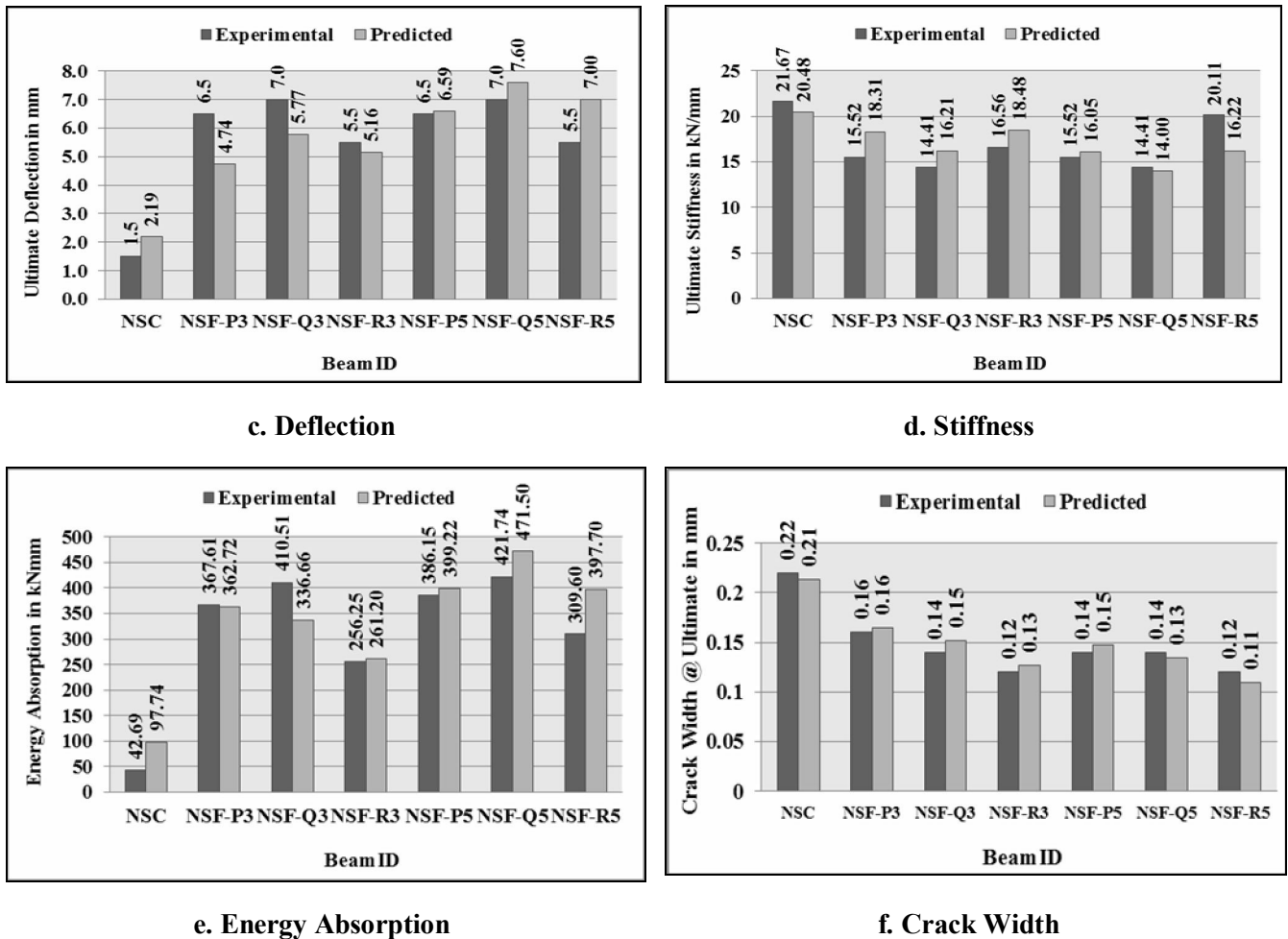


Fig.13. Experimental Vs Predicted

**Conclusion**

1. The SFRC beams strengthened with GFRP laminates displayed high ductility by its post failure behaviour only after the failure of tension reinforcement.
2. Concrete softening occurring during repeated loads tends to increase the stresses in the tensile steel reinforcement. The higher steel stresses resulting from concrete softening, will be smaller for a GFRP strengthened reinforced concrete beam, than for the conventionally reinforced concrete beam.
3. For the control beams, only fatigue failures in the tension steel reinforcement was observed. For SFRC beams strengthened with GFRP laminate, most specimens failed by fatigue of the steel bar. Some specimens had a fatigue failure of the bond between the GFRP laminate and the epoxy. No significant degradation in the GFRP laminates due to cyclic loading. A fatigue failure in the GFRP laminate occurred when the beam was subjected to a high load range.
4. Strengthening the conventional RC beams with GFRP laminate increased the endurance or fatigue limit to a higher level than that of the control beams.
5. The addition of steel fibre in reinforced concrete beams decreases the number of cracks and crack width with respect to baseline specimen.
6. The regression equations proposed in the present study closely predict the study parameters of ultimate load, deflection, stiffness, energy absorption and crack width of GFRP strengthened steel fibre reinforced concrete beams. The measure of fitness of regression shows that the multivariate linear regression can estimate the prediction values with reasonable levels of accuracy.

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