International Journal of ChemTech Research
CODEN (USA): IJCRGG ISSN: 0974-4290
Vol.9, No.2, pp 297-304, 2016

Anti-synchronization of Duffing Double-Well Chaotic Oscillators via Integral Sliding Mode Control

Sundarapandian Vaidyanathan[1]*

[1]R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, INDIA

*E-mail: sundarvtu@gmail.com

Abstract: Chaos theory has a manifold variety of applications in science and engineering. There are many systems in nature with several stable states, which are separated by energy barriers. When the system can move along the stable states, its dynamics can become quite complex. A simple mechanical model that depicts some of these complex dynamical features is the famous Duffing double-well oscillator (1918). This paper gives a summary description of the Duffing double-well chaotic oscillator. Next, new control results are obtained for the global chaos anti-synchronization of the identical Duffing double-well chaotic oscillators via integral sliding mode control (ISMC). MATLAB plots have been shown to illustrate the phase portraits of the Duffing double-well chaotic oscillator and the global chaos anti-synchronization of Duffing double-well chaotic oscillators via integral sliding mode control.

Keywords: Chaos, chaotic systems, chaos control, anti-synchronization, Duffing oscillator, mechanical system, oscillators, stable states, nonlinear oscillations, sliding mode control.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-4].

Chaos theory has a lot of applications in science and engineering [5]. Chaos theory has applications in dynamo systems [6-12], memristors [13-16], nonlinear oscillators [17-30], Tokamak systems [31-32], finance system [33], cellular neural networks [34-39], chemical reactors [40-50], neurology [51-58], population biology systems [59-67], etc.

A simple mechanical model that depicts some of these complex dynamical features is the famous Duffing double-well oscillator ([68], 1918). This paper gives a summary description of the Duffing double-well chaotic oscillator. Next, new results are obtained for the global chaos synchronization of the identical Duffing double-well chaotic oscillators via integral sliding mode control (ISMC). Sliding mode control is a popular control technique used in the control and synchronization of chaotic systems [69-73]. MATLAB plots have been shown to illustrate the phase portraits of the Duffing double-well chaotic oscillator and the global chaos synchronization of Duffing double-well chaotic oscillators via integral sliding mode control.

2. Duffing double-well chaotic oscillator

Duffing double-well chaotic oscillator [68] is described by the 2-D dynamics

$$\begin{cases} \dot{x} = y \\ \dot{y} = x - x^3 - ay + F\cos(\omega t) \end{cases} \tag{1}$$

The system (1) is *chaotic* when the system parameters are chosen as

$$F = 0.7, \quad a = 0.5, \quad \omega = 1 \tag{2}$$

For numerical simulations, we take the initial conditions

$$x(0) = 0.5, \quad y(0) \quad -0.5 \tag{3}$$

The 2-D phase portrait of the Duffing double-well chaotic oscillator (1) is depicted in Figure 1.

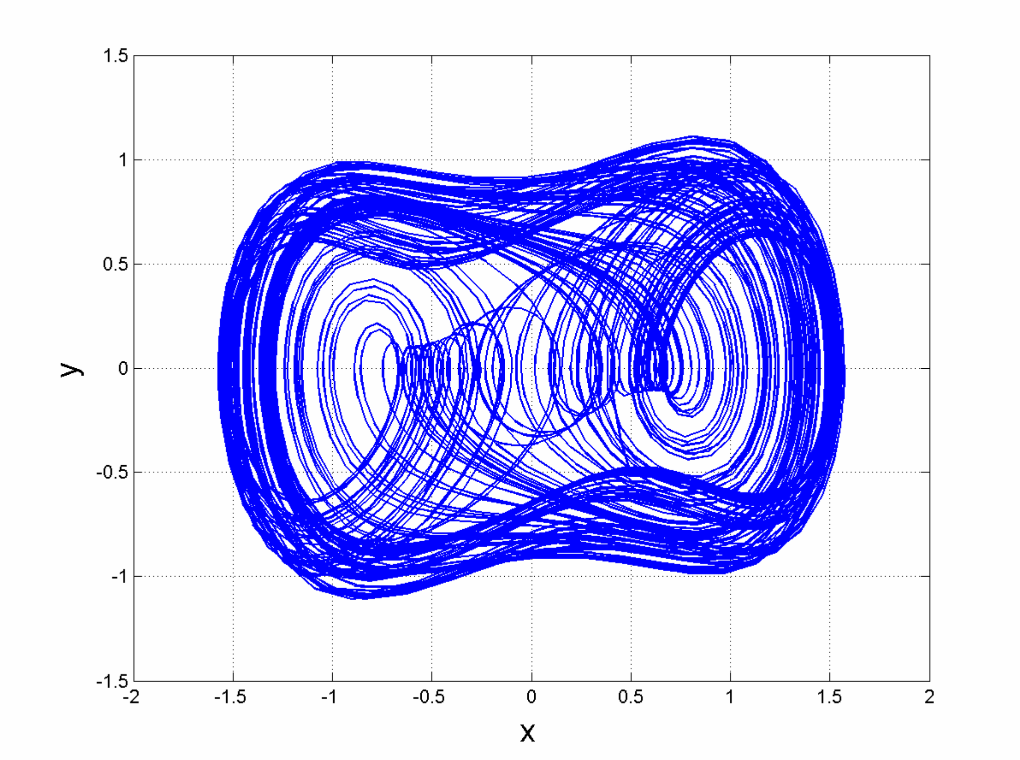

Figure 1. The 2-D phase portrait of the Duffing double-well chaotic oscillator

3. Anti-synchronization of the identical Duffing double-well chaotic oscillators

In this section, we use integral sliding mode control (ISMC) to achieve global chaos anti-synchronization of the identical novel Duffing double-well chaotic oscillators. We use Lyapunov stability theory to prove the main result derived in this section for the global chaos synchronization of the Duffing double-well chaotic oscillators.

As the master system, we consider the Duffing double-well chaotic oscillator given by

$$\begin{cases} \dot{x}_1 = y_1 \\ \dot{y}_1 = x_1 - x_1^3 - a y_1 + F\cos(\omega t) \end{cases} \tag{4}$$

where x_1, y_1 are the states and a, F, ω are constant positive parameters.

As the slave system, we consider the Duffing double-well chaotic oscillator given by

$$\begin{cases} \dot{x}_2 = y_2 + u_x \\ \dot{y}_2 = x_2 - x_2^3 - a y_2 + F\cos(\omega t) + u_y \end{cases} \tag{5}$$

where x_2, y_2 are the states and u_x, u_y are integral sliding mode controls to be determined.

The anti-synchronization error between the systems (4) and (5) is defined by

$$\begin{cases} e_x(t) = x_2(t) + x_1(t) \\ e_y(t) = y_2(t) + y_1(t) \end{cases} \tag{6}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x = e_y + u_x \\ \dot{e}_y = e_x - ae_y - x_2^3 - x_1^3 + 2F\cos(\omega t) + u_y \end{cases} \tag{7}$$

Based on the sliding mode control theory [74], the integral sliding surface of each error variable is defined as follows:

$$\begin{cases} s_x = \left[\dfrac{d}{dt} + \lambda_x\right]\left[\displaystyle\int_0^t e_x(\tau)d\tau\right] = e_x + \lambda_x \displaystyle\int_0^t e_x(\tau)d\tau \\ s_y = \left[\dfrac{d}{dt} + \lambda_y\right]\left[\displaystyle\int_0^t e_y(\tau)d\tau\right] = e_y + \lambda_y \displaystyle\int_0^t e_y(\tau)d\tau \end{cases} \tag{8}$$

The derivative of each equation in (8) yields

$$\begin{cases} \dot{s}_x = \dot{e}_x + \lambda_x e_x \\ \dot{s}_y = \dot{e}_y + \lambda_y e_y \end{cases} \tag{9}$$

The Hurwitz condition is satisfied if λ_x and λ_y are positive constants.

Based on the exponential reaching law [74], we set

$$\begin{cases} \dot{s}_x = -\eta_x \,\mathrm{sgn}(s_x) - k_x s_x \\ \dot{s}_y = -\eta_y \,\mathrm{sgn}(s_y) - k_y s_y \end{cases} \tag{10}$$

Comparing equations (9) and (10), we get

$$\begin{cases} \dot{e}_x + \lambda_x e_x = -\eta_x \,\mathrm{sgn}(s_x) - k_x s_x \\ \dot{e}_y + \lambda_y e_y = -\eta_y \,\mathrm{sgn}(s_y) - k_y s_y \end{cases} \tag{11}$$

Using Eq. (7), we can rewrite Eq. (11) as follows:

$$\begin{cases} e_y + u_x + \lambda_x e_x = -\eta_x \,\mathrm{sgn}(s_x) - k_x s_x \\ e_x - ae_y - x_2^3 - x_1^3 + 2F\cos(\omega t) + u_y + \lambda_y e_y = -\eta_y \,\mathrm{sgn}(s_y) - k_y s_y \end{cases} \tag{12}$$

From Eq. (12), the control laws are obtained as follows:

$$\begin{cases} u_x = -e_y - \lambda_x e_x - \eta_x \,\mathrm{sgn}(s_x) - k_x s_x \\ u_y = -e_x + ae_y + x_2^3 + x_1^3 - 2F\cos(\omega t) - \lambda_y e_y - \eta_y \,\mathrm{sgn}(s_y) - k_y s_y \end{cases} \tag{13}$$

Theorem 1. The Duffing double-well chaotic oscillators (4) and (5) are globally and asymptotically anti-synchronized for all initial conditions by the integral sliding mode controller (13), where the constants $\lambda_x, \lambda_y,\ \eta_x, \eta_y,\ k_x, k_y$ are all positive.

Proof. This result is proved using Lyapunov stability theory [75].

We consider the following quadratic Lyapunov function

$$V(s_x, s_y) = \frac{1}{2}\left(s_x^2 + s_y^2\right) \tag{14}$$

where s_x, s_y are as defined in (8).

The time-derivative of V is obtained as

$$\dot{V} = s_x \dot{s}_x + s_y \dot{s}_y \tag{15}$$

Substituting from Eq. (10) into (15), we get

$$\dot{V} = s_x[-\eta_x \,\mathrm{sgn}(s_x) - k_x s_x] + s_y[-\eta_y \,\mathrm{sgn}(s_y) - k_y s_y] \tag{16}$$

Simplifying Eq. (16), we obtain

$$\dot{V} = -\eta_x |s_x| - k_x s_x^2 - \eta_y |s_y| - k_y s_y^2 \tag{17}$$

Since $k_x, k_y > 0$ and $\eta_x, \eta_y > 0$, it follows from (17) that $\dot{V}$ is a negative definite function.

Thus, by Lyapunov stability theory [75], it follows that $(s_x, s_y) \to (0,0)$ as $t \to \infty$.

Hence, it is immediate that $(e_x, e_y) \to (0,0)$ as $t \to \infty$. This completes the proof. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (4) and (5), when the integral sliding mode controller (13) is implemented.

The parameter values of the Duffing double-well oscillators are taken as in the chaotic case, viz.

$$a = 0.5, \quad \omega = 1, \quad F = 0.7 \tag{18}$$

We take the sliding constants as

$$\lambda_x = \lambda_y = 0.1, \quad \eta_x = \eta_y = 0.1, \quad k_x = k_y = 30 \tag{19}$$

We take the initial conditions of the Duffing double-well chaotic oscillator (4) as

$$x_1(0) = 3.7, \quad y_1(0) = 12.8 \tag{20}$$

We take the initial conditions of the Duffing double-well chaotic oscillator (5) as

$$x_2(0) = 4.1, \quad y_2(0) = 16.4 \tag{21}$$

Figures 2-3 show the anti-synchronization of the Duffing double-well chaotic oscillators (4) and (5).

Figure 4 shows the time-history of the anti-synchronization errors e_x, e_y.

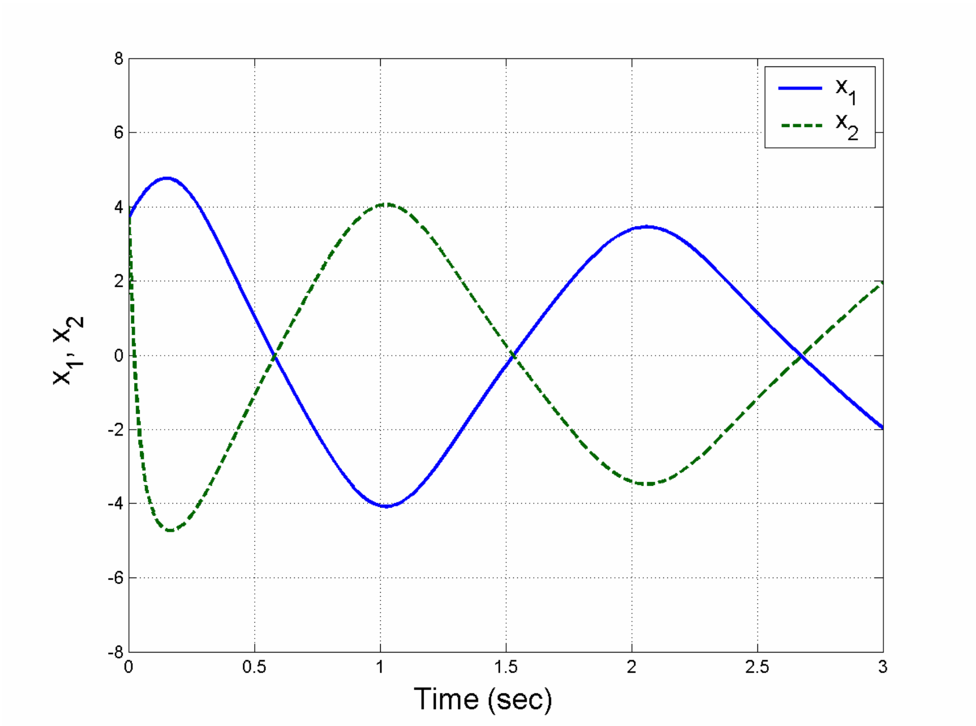

Figure 2. Anti-synchronization of the states x_1 and x_2

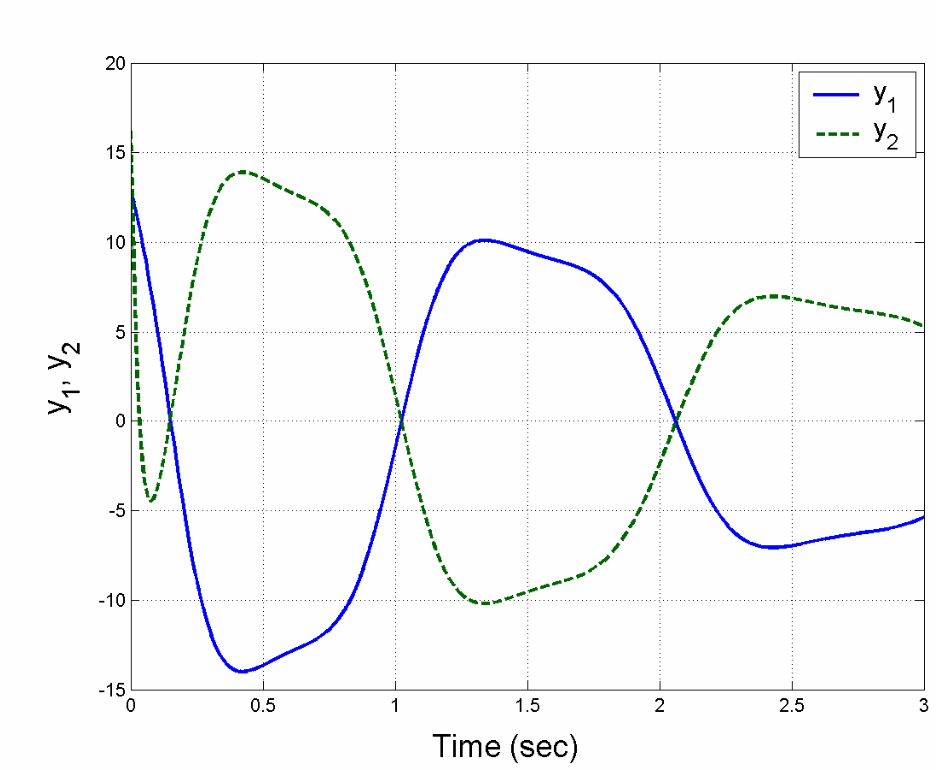

Figure 3. Anti-synchronization of the states y_1 and y_2

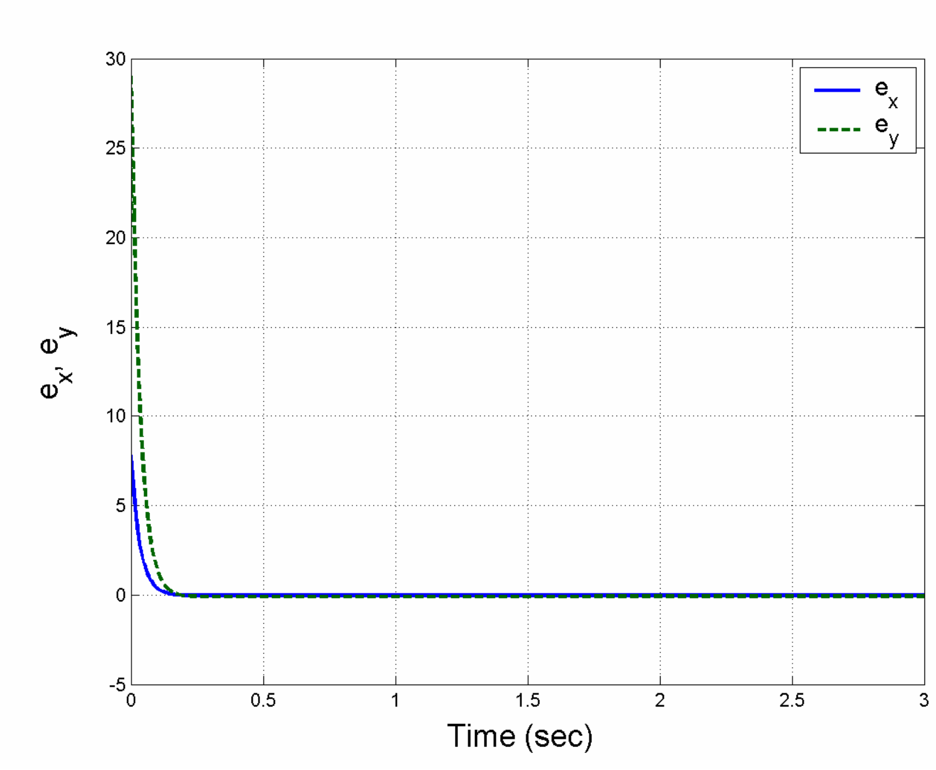

Figure 4. Time-history of the anti-synchronization errors e_x, e_y

5. Conclusions

In this paper, we first gave a summary description of the Duffing double-well chaotic oscillator. Next, new results were obtained for the global chaos anti-synchronization of the identical Duffing double-well chaotic oscillators via integral sliding mode control (ISMC). MATLAB plots were shown to illustrate all the main results derived in this research work for the Duffing double-well chaotic oscillator.

References

1. Azar, A. T., and Vaidyanathan, S., Chaos Modeling and Control Systems Design, Studies in Computational Intelligence, Vol. 581, Springer, New York, USA, 2015.
2. Azar, A. T., and Vaidyanathan, S., Computational Intelligence Applications in Modeling and Control, Studies in Computational Intelligence, Vol. 575, Springer, New York, USA, 2015.
3. Vaidyanathan, S., and Volos, C., Advances and Applications in Chaotic Systems, Studies in Computational Intelligence, Vol. 636, Springer, New York, USA, 2016.
4. Vaidyanathan, S., and Volos, C., Advances and Applications in Nonlinear Control Systems, Studies in Computational Intelligence, Vol. 635, Springer, New York, USA, 2016.
5. Azar, A.T., and Vaidyanathan, S., Advances in Chaos Theory and Intelligent Control, Studies in Fuzziness and Soft Computing, Vol. 337, Springer, New York, USA, 2016.
6. Vaidyanathan, S., Adaptive synchronization of Rikitake two-disk dynamo chaotic systems, International Journal of ChemTech Research, 2015, 8 (8), 100-111.
7. Vaidyanathan, S., Anti-synchronization of Rikitake two-disk dynamo chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 393-405.
8. Vaidyanathan, S., State regulation of Rikitake two-disk dynamo chaotic system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 374-386.
9. Vaidyanathan, S., Hybrid chaos synchronization of Rikitake two-disk dynamo chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (11), 12-25.
10. Vaidyanathan, S., Adaptive control of Rikitake two-disk dynamo system, International Journal of ChemTech Research, 2015, 8 (8), 121-133.
11. Vaidyanathan, S., Volos, C.K., and Pham, V.-T., Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium, Journal of Engineering Science and Technology Review, 2015, 8 (2), 232-244.
12. Vaidyanathan, S., Pham, V.-T., and Volos, C.K., A 5-D hyperchaotic Rikitake dynamo system with hidden attractors, European Physical Journal: Special Topics, 2015, 224 (8), 1575-1592.
13. Pham, V.-T., Vaidyanathan, S., Volos, C.K., Jafari, S., Kuznetsov, N.V. and Hoang, T.M., A novel memristive time-delay chaotic system without equilibrium points, European Physical Journal: Special Topics, 2016, 225 (1), 127-136.
14. Pham, V.-T., Jafari, S., Vaidyanathan, S., Volos, C. and Wang, X., A novel memristive neural network with hidden attractors and its circuitry implementation, Science China Technological Sciences, 2015, 59 (3), 358-363.
15. Volos, C.K., Kyprianidis, I.M., Stouboulos, I.N., Tlelo-Cuautle, E. and Vaidyanathan, S., Memristor: A new concept in synchronization of coupled neuromorphic circuits, Journal of Engineering Science and Technology Review, 2015, 8 (2), 157-173.
16. Pham, V.-T., Volos, C.K., Vaidyanathan, S., Le, T.P., and Vu, V.Y., A memristor-based hyperchaotic system with hidden attractors: Dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8 (2), 205-214.
17. Pham, V.-T., Volos, C.K. and Vaidyanathan, S., Multi-scroll chaotic oscillator based on a first-order delay differential equation, Studies in Computational Intelligence, 2015, 581, 59-72.
18. Volos, C.K., Pham, V.-T., Vaidyanathan, S., Kyprianidis, I.M., and Stouboulos, I.N., Synchronization phenomena in coupled Colpitts circuits, Journal of Engineering Science and Technology Review, 2015, 8 (2), 142-151.
19. Vaidyanathan, S., A novel coupled Van der Pol conservative chaotic system and its adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 79-94.
20. Vaidyanathan, S., Global chaos control of Mathieu-Van der Pol system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 406-417.
21. Pham, V.T., Vaidyanathan, S., Volos, C.K., and Jafari, S., Hidden attractors in a chaotic system with an exponential nonlinear term, European Physical Journal: Special Topics, 2015, 224 (8), 1507-1517.
22. Sampath, S., Vaidyanathan, S., Volos, C.K., and Pham, V.-T., An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8 (2), 1-6.
23. Vaidyanathan, S., Azar, A.T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23 (3), 267-277.
24. Vaidyanathan, S., Global chaos synchronization of Mathieu-Van der Pol chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (10), 148-162.
25. Vaidyanathan, S., Output regulation of the forced Van der Pol chaotic oscillator via adaptive control

method, International Journal of PharmTech Research, 2015, 8 (6), 106-116.
26. Vaidyanathan, S., Global chaos synchronization of the forced Van der Pol chaotic oscillators via adaptive control method, International Journal of PharmTech Research, 2015, 8 (6), 156-166.
27. Vaidyanathan, S., Global chaos synchronization of novel coupled Van der Pol conservative chaotic systems via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 95-111.
28. Vaidyanathan, S., and Volos, C., Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system, Archives of Control Sciences, 2015, 25 (3), 333-353.
29. Vaidyanathan, S., A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters, Journal of Engineering Science and Technology Review, 2015, 8 (2), 106-115.
30. Vaidyanathan, S., A novel 3-D jerk chaotic system with three quadratic nonlinearities and its adaptive control, Archives of Control Sciences, 2016, 26 (1), 5-33.
31. Vaidyanathan, S., Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 818-827.
32. Vaidyanathan, S., Dynamics and control of Tokamak system with symmetric and magnetically confined plasma, International Journal of ChemTech Research, 2015, 8 (6), 795-803.
33. Tacha, O.I., Volos, Ch.K., Kyprianidis, I.M., Stouboulos, I.N., Vaidyanathan, S., and Pham, V.-T., Analysis, adaptive control and circuit simulation of a novel nonlinear finance system, Applied Mathematics and Computation, 2016, 276, 200-217.
34. Vaidyanathan, S., 3-cells cellular neural network (CNN) attractor and its adaptive biological control, International Journal of PharmTech Research, 2015, 8 (4), 632-640.
35. Vaidyanathan, S., Hybrid chaos synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 61-73.
36. Vaidyanathan, S., Synchronization of 3-cells cellular neural network (CNN) attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (5), 946-955.
37. Vaidyanathan, S., Global chaos control of 3-cells cellular neural network attractor via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 211-221.
38. Vaidyanathan, S., Global chaos synchronization of 3-cells cellular neural network attractors via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 118-130.
39. Vaidyanathan, S., Anti-synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 26-38.
40. Vaidyanathan, S., Anti-synchronization of Brusselator chemical reaction systems via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 759-768.
41. Vaidyanathan, S., Adaptive synchronization of novel 3-D chemical chaotic reactor systems, International Journal of ChemTech Research, 2015, 8 (7), 159-171.
42. Vaidyanathan, S., A novel chemical chaotic reactor system and its adaptive control, International Journal of ChemTech Research, 2015, 8 (7), 146-158.
43. Vaidyanathan, S., Adaptive synchronization of chemical chaotic reactors, International Journal of ChemTech Research, 2015, 8 (2), 612-621.
44. Vaidyanathan, S., Adaptive control design for the anti-synchronization of novel 3-D chemical chaotic reactor systems, International Journal of ChemTech Research, 2015, 8 (11), 654-668.
45. Vaidyanathan, S., Anti-synchronization of Brusselator chemical reaction systems via integral sliding mode control, International Journal of ChemTech Research, 2015, 8 (11), 700-713.
46. Vaidyanathan, S., Integral sliding mode control design for the global chaos synchronization of identical novel chemical chaotic reactor systems, International Journal of ChemTech Research, 2015, 8 (11), 684-699.
47. Vaidyanathan, S., A novel chemical chaotic reactor system and its output regulation via integral sliding mode control, International Journal of ChemTech Research, 2015, 8 (11), 669-683.
48. Vaidyanathan, S., Global chaos synchronization of chemical chaotic reactors via novel sliding mode control, International Journal of ChemTech Research, 2015, 8 (7), 209-221.
49. Vaidyanathan, S., Analysis, control and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities, Kyungpook Mathematical Journal, 2015, 55 (3), 563-586.
50. Vaidyanathan, S., Adaptive control of a chemical chaotic reactor, International Journal of PharmTech Research, 2015, 8 (3), 377-382.
51. Vaidyanathan, S., Adaptive chaotic synchronization of enzymes-substrates system with ferroelectric behaviour in brain waves, International Journal of PharmTech Research, 2015, 8 (5), 964-973.
52. Vaidyanathan, S., Hybrid chaos synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 48-60.
53. Vaidyanathan, S., Adaptive backstepping control of enzymes-substrates system with ferroelectric

behaviour in brain waves, International Journal of PharmTech Research, 2015, 8 (2), 256-261.
54. Vaidyanathan, S., Sliding controller design for the global chaos synchronization of enzymes-substrates systems, International Journal of PharmTech Research, 2015, 8 (7), 89-99.
55. Vaidyanathan, S., Chaos in neurons and adaptive control of Birkhoff-Shaw strange chaotic attractor, International Journal of PharmTech Research, 2015, 8 (5), 956-963.
56. Vaidyanathan, S., Anti-synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 71-83.
57. Vaidyanathan, S., Adaptive control of the FitzHugh-Nagumo chaotic neuron model, International Journal of PharmTech Research, 2015, 8 (6), 117-127.
58. Vaidyanathan, S., Adaptive synchronization of the identical FitzHugh-Nagumo chaotic neuron models, International Journal of PharmTech Research, 2015, 8 (6), 167-177.
59. Vaidyanathan, S., Volos, C.K., Rajagopal, K., Kyprianidis, I.M. and Stouboulos, I.N., Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, Journal of Engineering Science and Technology Review, 2015, 8 (2), 74-82.
60. Vaidyanathan, S., Anti-synchronization of the generalized Lotka-Volterra three-species biological systems via adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 141-156.
61. Vaidyanathan, S., Adaptive synchronization of generalized Lotka-Volterra three-species biological systems, International Journal of PharmTech Research, 2015, 8 (5), 928-937.
62. Vaidyanathan, S., Adaptive biological control of generalized Lotka-Volterra three-species biological system, International Journal of PharmTech Research, 2015, 8 (4), 622-631.
63. Vaidyanathan, S., Active control design for the hybrid chaos synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 2015, 8 (8), 30-42.
64. Vaidyanathan, S., Global chaos synchronization of the Lotka-Volterra biological systems with four competitive species via active control, International Journal of PharmTech Research, 2015, 8 (6), 206-217.
65. Vaidyanathan, S, Active control design for the anti-synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 8 (7), 58-70.
66. Vaidyanathan, S., Anti-synchronization of 3-cells cellular neural network attractors via integral sliding mode control, International Journal of PharmTech Research, 2016, 9 (1), 193-205.
67. Vaidyanathan, S., Hybrid synchronization of the generalized Lotka-Volterra three-species biological systems via adaptive control, International Journal of PharmTech Research, 2016, 9 (1), 179-192.
68. Hilborn, R. C., Chaos and Nonlinear Dynamics, Oxford University Press, UK, 1994.
69. Vaidyanathan, S., and Sampath, S., Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control, Communications in Computer and Information Science, 2011, 205, 156-164.
70. Sundarapandian, V., and Sivaperumal, S., Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, 2011, 6 (5), 224-231.
71. Vaidyanathan, S., and Sampath, S., Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, 2012, 9 (3), 274-279.
72. Vaidyanathan, S., Global chaos control of hyperchaotic Liu system via sliding control method, International Journal of Control Theory and Applications, 2012, 5 (2), 117-123.
73. Vaidyanathan, S., Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, 2012, 5 (1), 15-20.
74. Slotine, J. and Li, W., Applied Nonlinear Control, Prentice Hall, New Jersey, USA, 1991.
75. Khalil, H. K., Nonlinear Systems, Third Edition, Prentice Hall, New Jersey, USA, 2002.

*****