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Hybrid Chaos Synchronization of the FitzHugh-Nagumo Chaotic Neuron Models via Adaptive Control Method

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Abstract: Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations. In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally. In this paper, we investigate the qualitative properties of the well-known FitzHugh-Nagumo (FHN) chaotic neuron model, which is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons. Next, new results are obtained for the hybrid chaos synchronization of the identical FitzHugh-Nagumo (FHN) neuron model using adaptive control method. The main control result of this work is established using Lyapunov stability theory. MATLAB plots have been shown to illustrate the phase portraits of the FitzHugh-Nagumo (FHN) neuron model and the adaptive hybrid chaos synchronization of the FHN neuron model.
Keywords: Chaos, chaotic systems, synchronization, neurons, FitzHugh-Nagumo system, hybrid synchronization, adaptive control, stability.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction.

Recently, many 3-D chaotic systems have been announced in the literature such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations.

In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Their mathematical model is referred to as *Hodgkin-Huxley equations* in the literature [143]. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally.

FitzHugh [144] and Nagumo [145] extended the Van der Pol equation in a planar field as a model for action potentials of neurons. FitzHugh-Nagumo (FHN) chaotic neuron model is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system, and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically. In the hybrid synchronization of chaotic systems, one set of the states are completely synchronized while the other set of the states are anti-synchronized.

In this paper, we derive new results for the hybrid chaos synchronization of the identical forced FitzHugh-Nagumo chaotic neuron models [146-147].

This paper is organized as follows. Section 2 details the dynamics and properties of the FitzHugh-Nagumo chaotic neuron model. Section 3 describes our new results for the anti-synchronization of the identical FitzHugh-Nagumo (FHN) chaotic neuron models using adaptive control method. The main control result derived in this work is established using Lyapunov stability theory. Section 4 details the numerical simulations illustrating the main result derived in this research paper. Section 5 contains the main conclusions of this work.

2. FitzHugh-Nagumo Chaotic Neuron Model

FitzHugh-Nagumo (FHN) chaotic system is one of the most intensely studied systems in neuroscience. Many studies have been done on the significant and complex dynamical aspects of the FHN model including chaos, bifurcation, circuit design, noise effects and filtering, coupling, etc.

FitzHugh-Nagumo (FHN) chaotic neuron model [146, 147] is described by the 2-D dynamics

$$\begin{cases} \dot{x} = x(x-1)(1-\alpha x) - y + I_0(t) \\ \dot{y} = bx \end{cases} \tag{1}$$

In Eq. (1), $I_0(t)$ represents the external electrical stimulation

$$I_0(t) = \frac{a}{\omega}\cos(\omega t), \tag{2}$$

where a and ω are the amplitude (or strength) and frequency, respectively, of the applied field.

Also, $\omega = 2\pi f$ (rad/s) and f (Hz) is the stimulus frequency.

It is known that the FHN system (1) is *chaotic*, when the parameter values are taken as

$$\alpha = 10, \quad b = 1, \quad a = 0.1, \quad f = 0.1271 \tag{3}$$

For numerical simulations, we take $x(0) = 0.2$ and $y(0) = 0.2$.

Figure 1 shows the $x_1 -$ waveform of the FitzHugh-Nagumo system (1).

Figure 2 shows the x_2 – waveform of the FitzHugh-Nagumo system (1).
Figure 3 shows the chaotic phase portrait of the FHN system (1).

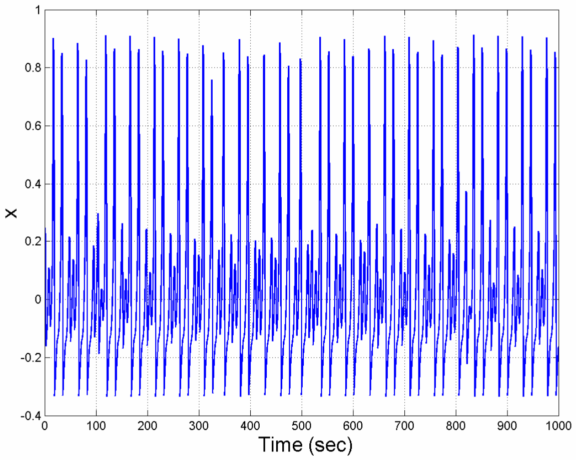

Figure 1. x **– waveform of the FitzHugh-Nagumo chaotic neuron model**

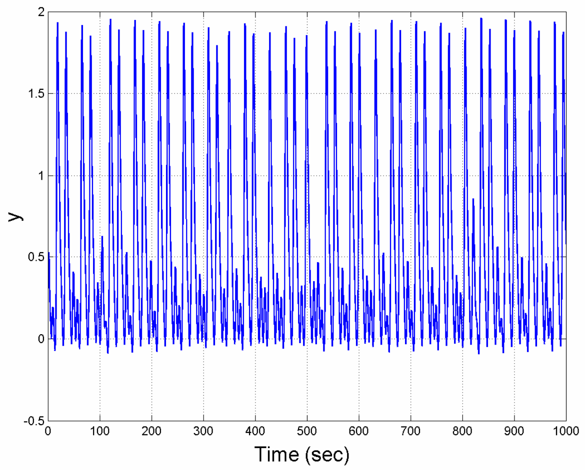

Figure 2. y **– waveform of the FitzHugh-Nagumo chaotic neuron model**

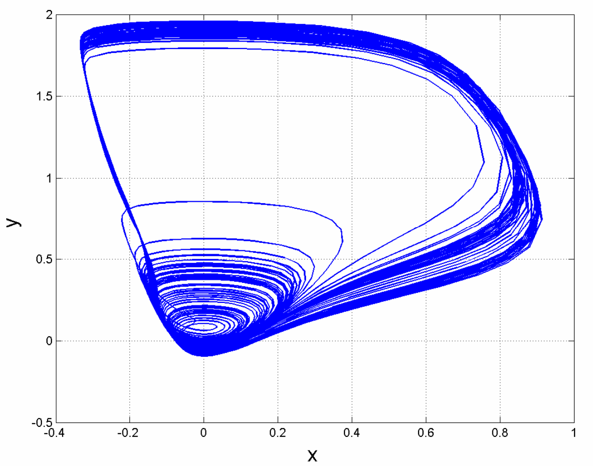

Figure 3. Chaotic phase portrait of the FitzHugh-Nagumo chaotic neuron model

3. Adaptive Hybrid Synchronization of the FitzHugh-Nagumo (FHN) Chaotic Neuron Models

In this section, we derive new results for the anti-synchronization of the FitzHugh-Nagumo (FHN) chaotic neuron models via adaptive control method. The main control result is established via Lyapunov stability theory [148].

As the master system, we consider the FitzHugh-Nagumo (FHN) chaotic neuron model given by

$$\begin{cases} \dot{x}_1 = x_1(x_1 - 1)(1 - \alpha x_1) - y_1 + I_0(t) \\ \dot{y}_1 = b x_1 \end{cases} \tag{4}$$

where α, b are unknown system parameters, and the external electrical stimulation $I_0(t) = \dfrac{a}{\omega}\cos(\omega t)$ is known.

As the slave system, we consider the FitzHugh-Nagumo (FHN) chaotic neuron model given by

$$\begin{cases} \dot{x}_2 = x_2(x_2 - 1)(1 - \alpha x_2) - y_2 + I_0(t) + u_x \\ \dot{y}_2 = b x_2 + u_y \end{cases} \tag{5}$$

where u_x, u_y are adaptive controls to be determined using estimates $\hat{\alpha}(t)$ and $\hat{b}(t)$ of the unknown parameters α and b, respectively.

We define the hybrid chaos synchronization errors as follows:

$$\begin{cases} e_x(t) = x_2(t) - x_1(t) \\ e_y(t) = y_2(t) + y_1(t) \end{cases} \tag{6}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_x = x_2(x_2 - 1) - x_1(x_1 - 1) - \alpha\left[x_2^2(x_2 - 1) - x_1^2(x_1 - 1)\right] - y_2 + y_1 + u_x \\ \dot{e}_y = b\left(x_1 + x_2\right) + u_y \end{cases} \tag{7}$$

We consider the adaptive control defined by

$$\begin{cases} u_x = -x_2(x_2 - 1) + x_1(x_1 - 1) + \hat{\alpha}(t)\left[x_2^2(x_2 - 1) - x_1^2(x_1 - 1)\right] + y_2 - y_1 - k_x e_x \\ u_y = -\hat{b}(t)\left(x_1 + x_2\right) - k_y e_y \end{cases} \tag{8}$$

where k_x and k_y are positive gain constants.

Substituting (8) into (7), we get the closed-loop control system as

$$\begin{cases} \dot{e}_x = -[\alpha - \hat{\alpha}(t)]\left[x_2^2(x_2 - 1) - x_1^2(x_1 - 1)\right] - k_x e_x \\ \dot{e}_y = [b - \hat{b}(t)]\left(x_1 + x_2\right) - k_y e_y \end{cases} \tag{9}$$

We define the parameter estimation errors as

$$\begin{cases} e_\alpha(t) = \alpha - \hat{\alpha}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{10}$$

Using (9), the closed-loop system (8) can be simplified as follows:

$$\begin{cases} \dot{e}_x = -e_\alpha\left[x_2^2(x_2 - 1) - x_1^2(x_1 - 1)\right] - k_x e_x \\ \dot{e}_y = e_b\left(x_1 + x_2\right) - k_y e_y \end{cases} \tag{11}$$

Differentiating (10) with respect to t, we get

$$\begin{cases} \dot{e}_\alpha(t) = -\dot{\hat{\alpha}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{12}$$

We consider the Lyapunov function defined by

$$V(e_x, e_y, e_\alpha, e_b) = \frac{1}{2}\left(e_x^2 + e_y^2 + e_\alpha^2 + e_b^2\right) \tag{13}$$

which is positive definite on R^4.

Differentiating V along the trajectories of (11) and (12), we obtain

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 + e_\alpha\left[-e_x\left[x_2^2(x_2-1) - x_1^2(x_1-1)\right] - \dot{\hat{\alpha}}\right] + e_b\left[e_y(x_1+x_2) - \dot{\hat{b}}\right] \tag{14}$$

In view of (14), we take the parameter update law as

$$\begin{cases} \dot{\hat{\alpha}} = -e_x\left[x_2^2(x_2-1) - x_1^2(x_1-1)\right] \\ \dot{\hat{b}} = e_y\left(x_1+x_2\right) \end{cases} \tag{15}$$

Next, we state and prove the main result of this section.

Theorem 1. The identical FitzHugh-Nagumo (FHN) chaotic neuron models (4) and (5) are globally and exponentially hybrid synchronized by the adaptive control law (8) and the parameter update law (15), where k_x, k_y are positive gain constants.

Proof. This result is a consequence of the Lyapunov stability theory [148].

The quadratic Lyapunov function V defined by (13) is positive definite on R^4. Substituting (15) into (14), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 \tag{16}$$

which is negative semi-definite on R^4.

Thus, using Barbalat's lemma [148], we conclude that the error dynamics (11) is globally exponentially stable.

This completes the proof. ■

4. Numerical Simulations

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with step-size $h = 10^{-6}$ to solve the FitzHugh-Nagumo (FHN) chaotic neuron system (4), when the adaptive control law (7) and the parameter update law (17) are implemented.

The external electrical stimulation is $I_0(t) = \dfrac{a}{\omega}\cos(\omega t)$, where $\omega = 2\pi f$.

We take the parameter values as in the chaotic case, *i.e.* $\alpha = 10$, $b = 1$, $a = 0.1$, $f = 0.1271$.

We take the positive gain constants as $k_x = 10$ and $k_y = 10$.

We take the initial conditions of the master system (4) as $x_1(0) = 6.4$ and $x_2(0) = 1.7$.

We take the initial conditions of the slave system (5) as $y_1(0) = 3.9$ and $y_2(0) = 4.2$.

We take the initial condition of the parameter estimates as $\hat{\alpha}(0) = 1.3$ and $\hat{b}(0) = 4.6$.

Figures 4 and 5 show the hybrid chaos synchronization of the FitzHugh-Nagumo neuron models (4) and (5).

Figure 6 shows the time-history of the hybrid chaos synchronization errors $e_x(t)$ and $e_y(t)$.

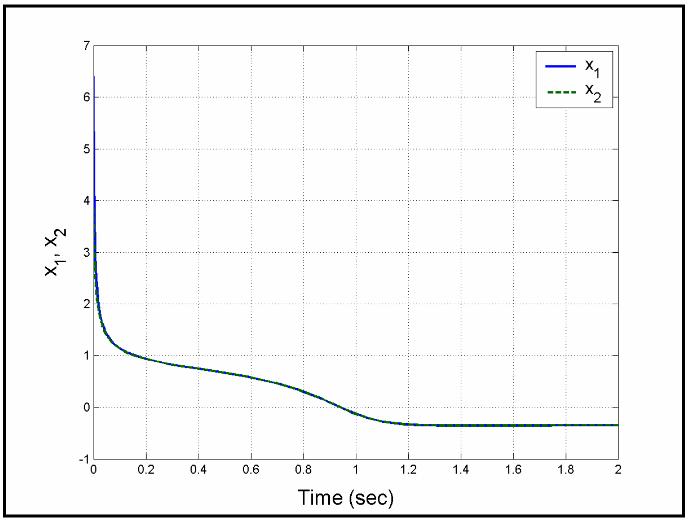

Figure 4. Hybrid synchronization of the states x_1 and x_2

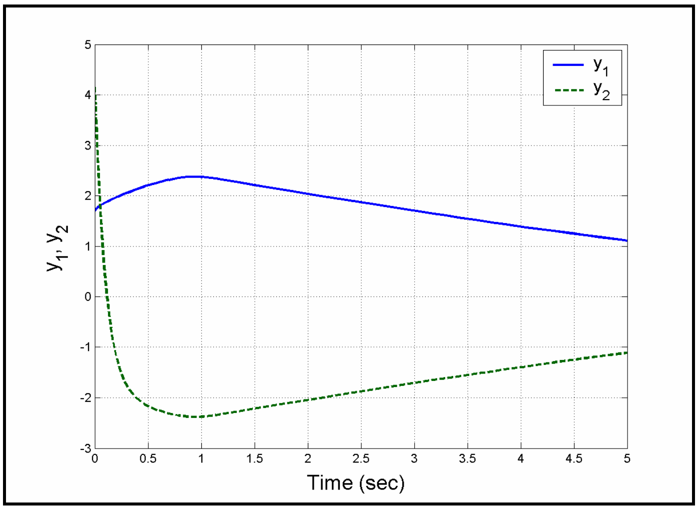

Figure 5. Hybrid synchronization of the states y_1 and y_2

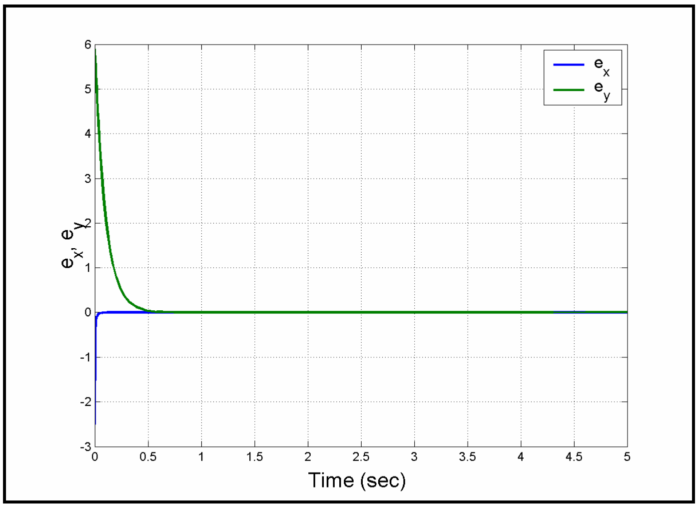

Figure 6. Time-history of the hybrid chaos synchronization errors $e_x(t), e_y(t)$

5. Conclusions

In this paper, we investigated the qualitative properties of the well-known FitzHugh-Nagumo (FHN) chaotic neuron model, which is a two-dimensional simplification of the Hodgkin-Huxley model of spike generation in squid giant axons. Next, we derived new results for the adaptive hybrid chaos synchronization of the identical FitzHugh-Nagumo (FHN) neuron models using Lyapunov stability theory. MATLAB plots were depicted to illustrate the phase portraits of the FitzHugh-Nagumo (FHN) neuron model and the adaptive hybrid chaos synchronization FHN neuron models.

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