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Active Control Design for the Anti-Synchronization of Lotka-Volterra Biological Systems with Four Competitive Species

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Abstract: Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. In the biological systems, there is great interest in the stud of Lotka-Volterra biological models, especially in the study of predator-prey models and competitive models. Such models have great interest in fields like agriculture and ecology. In 2005, Sprott, Wildenberg and Azizi found a high-dimensional Lotka-Volterra multi-species competitive model that exhibits spatiotemporal chaos. In 2006, Vano, Wildenberg, Anderson, Noel and Sprott discovered chaos in basic Lotka-Volterra models of four competing species. In this paper, we apply active control method to derive new control results for the global chaos anti-synchronization of Lotka-Volterra biological systems with four competing species discovered by Vano, Wildenberg, Anderson, Noel and Sprott. MATLAB plots have been shown to illustrate chaos in the Vano competitive biological system and also the global chaos synchronization of the Vano competitive biological systems.
Keywords: Chaos, chaotic systems, Lotka-Volterra system, population biology, active control, synchronization.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction.

Recently, many 3-D chaotic systems have been announced in the literature such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal of anti-synchronization is to device a feedback mechanism so that the trajectories of the master and slave systems are asymptotically equal in magnitude but opposite in sign. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the anti-synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-107], biology [108-125], memristors [126-128], electrical circuits [129], etc.

Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. In 2005, Sprott, Wildenberg and Azizi found a high-dimensional Lotka-Volterra multi-species competitive model that exhibits spatiotemporal chaos [130]. In 2006, Vano, Wildenberg, Anderson, Noel and Sprott discovered chaos in basic Lotka-Volterra models of four competing species [131]. In this paper, we discuss the global chaos synchronization of Lotka-Volterra biological systems with four competing species discovered by Vano, Wildenberg, Anderson, Noel and Sprott. MATLAB plots have been shown to illustrate chaos in the Vano competitive biological system and also the global chaos synchronization of the Vano competitive biological systems.

This paper is organized as follows. Section 2 details the dynamics and properties of the Vano competitive biological system. Section 3 describes our new results for the global chaos synchronization of the identical Vano competitive biological systems using active control method. Section 4 details the numerical simulations illustrating the main result derived in this research paper. Section 5 contains the main conclusions of this work.

2. Spatiotemporal Chaos in Lotka-Volterra Competitive Biological Systems

The Lotka-Volterra model [132] is widely used to study the dynamics of interacting species in ecology, biology and many applications. In [130], Sprott, Wildenberg and Azizi found a high-dimensional Lotka-Volterra multi-species model that exhibits spatiotemporal chaos. In [131], Sprott et al. considered the simplest form of such a system in which N species with population x_i (for $i=1$ to N) compete for a finite set of resources according to the model:

$$\boxed{\dot{x}_i = r_i x_i \left(1 - \sum_{j=1}^{N} a_{ij} x_j \right)} \tag{1}$$

Here, r_i represents the growth rate of species i and a_{ij} represents the extent to which the species j competes for the resources used by species i.

In [119], Vano, Wildenberg, Anderson, Noel and Sprott studied the occurrence of chaos in basic Lotka-Volterra models of four competing species. In [119], Vano *et al.* found chaos in a four-species competitive Lotka-Volterra model for values such as:

$$[r_i] = \begin{bmatrix} 1 \\ 0.72 \\ 0.53 \\ 1.27 \end{bmatrix} \quad \text{and} \quad [a_{ij}] = \begin{bmatrix} 1 & 1.09 & 1.52 & 0 \\ 0 & 1 & 0.44 & 1.36 \\ 2.33 & 0 & 1 & 0.47 \\ 1.21 & 0.51 & 0.35 & 1 \end{bmatrix} \tag{2}$$

It is noted that the four-species competing model (1) with parameter sets (2) is an example of a four-dimensional competitive chaotic system where each species i has its own distinct growth rate r_i.

The four-dimensional Vano competing chaotic model (1) with the parameter set (2) can be also written as follows.

$$\begin{cases} \dot{x}_1 = x_1(1 - x_1 - 1.09x_2 - 1.52x_3) \\ \dot{x}_2 = 0.72x_2(1 - x_2 - 0.44x_3 - 1.36x_4) \\ \dot{x}_3 = 1.53x_3(1 - 2.33x_1 - x_3 - 0.47x_4) \\ \dot{x}_4 = 1.27x_4(1 - 1.21x_1 - 0.51x_2 - 0.35x_3 - x_4) \end{cases} \tag{3}$$

For numerical simulations, we take the initial conditions as $x_i(0) = 1$ for $i = 1,2,3,4.$

Figure 1 describes the 3-D projection of the four-dimensional Vano competing chaotic model (3) on the $(x_1, x_2, x_3) -$space. Figure 2 describes the 3-D projection of the four-dimensional Vano competing chaotic model (3) on the $(x_1, x_2, x_4) -$space. Figure 3 describes the 3-D projection of the four-dimensional Vano competing chaotic model (3) on the $(x_1, x_3, x_4) -$space. Figure 4 describes the 3-D projection of the four-dimensional Vano competing chaotic model (3) on the $(x_2, x_3, x_4) -$space.

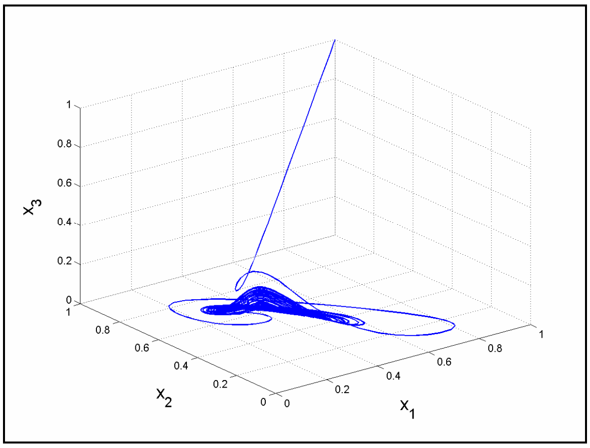

Figure 1. A 3-D projection of the Vano 4-D chaotic system on the $(x_1, x_2, x_3) -$space

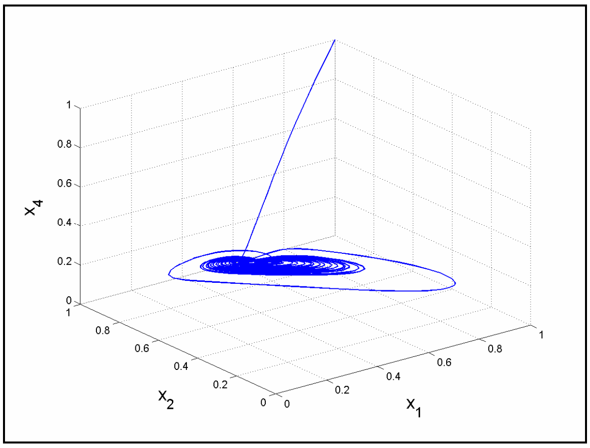

Figure 2. A 3-D projection of the Vano 4-D chaotic system on the $(x_1, x_2, x_4) -$space

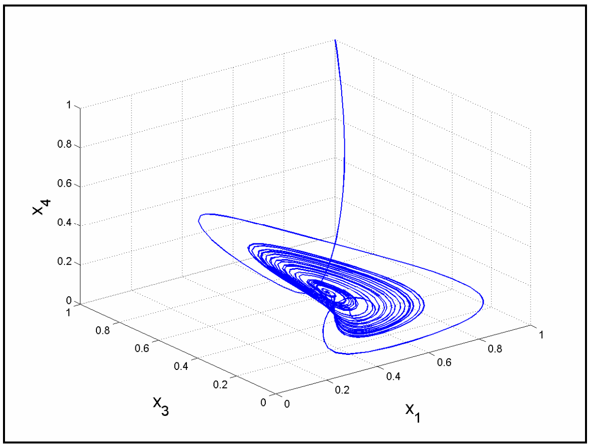

Figure 3. A 3-D projection of the Vano 4-D chaotic system on the $(x_1, x_3, x_4)-$space

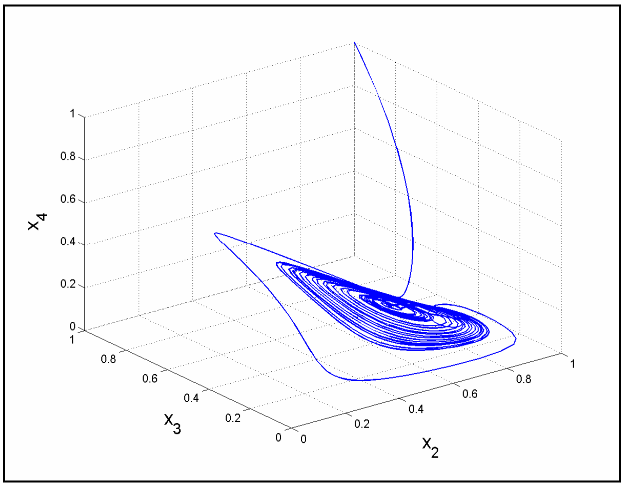

Figure 4. A 3-D projection of the Vano 4-D chaotic System on the $(x_2, x_3, x_4)-$space

3. Anti-Synchronization of the Vano 4-D Competitive Biological Chaotic Systems

In this section, we apply active control method to derive new results for the global chaos anti-synchronization of the identical Vano 4-D competitive chaotic systems. The main result is established via Lyapunov stability theory [133].

As the master system, we consider the Vano 4-D competitive chaotic system given by

$$\begin{cases} \dot{x}_1 = x_1\left(1 - x_1 - 1.09x_2 - 1.52x_3\right) \\ \dot{x}_2 = 0.72x_2\left(1 - x_2 - 0.44x_3 - 1.36x_4\right) \\ \dot{x}_3 = 1.53x_3\left(1 - 2.33x_1 - x_3 - 0.47x_4\right) \\ \dot{x}_4 = 1.27x_4\left(1 - 1.21x_1 - 0.51x_2 - 0.35x_3 - x_4\right) \end{cases} \tag{4}$$

where x_1, x_2, x_3, x_4 are the four states of the system (4).

As the slave system, we consider the Vano 4-D competitive chaotic system given by

$$\begin{cases} \dot{y}_1 = y_1\left(1 - y_1 - 1.09y_2 - 1.52y_3\right) + u_1 \\ \dot{y}_2 = 0.72y_2\left(1 - y_2 - 0.44y_3 - 1.36y_4\right) + u_2 \\ \dot{y}_3 = 1.53y_3\left(1 - 2.33y_1 - y_3 - 0.47y_4\right) + u_3 \\ \dot{y}_4 = 1.27y_4\left(1 - 1.21y_1 - 0.51y_2 - 0.35y_3 - y_4\right) + u_4 \end{cases} \tag{5}$$

where y_1, y_2, y_3, y_4 are the four states of the system (5) and u_1, u_2, u_3, u_4 are active controls to be determined.
We define the anti-synchronization errors between the Vano 4-D chaotic systems (4) and (5) as follows:

$$\begin{cases} e_1(t) = y_1(t) + x_1(t) \\ e_2(t) = y_2(t) + x_2(t) \\ e_3(t) = y_3(t) + x_3(t) \\ e_4(t) = y_4(t) + x_4(t) \end{cases} \tag{6}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_1 - y_1^2 - x_1^2 - 1.09\left(y_1 y_2 + x_1 x_2\right) - 1.52(y_1 y_3 + x_1 x_3) + u_1 \\ \dot{e}_2 = 0.72 e_2 - 0.72\left(y_2^2 + x_2^2\right) - 0.3168\left(y_2 y_3 + x_2 x_3\right) - 0.9792\left(y_2 y_4 + x_2 x_4\right) + u_2 \\ \dot{e}_3 = 1.53 e_3 - 3.5649\left(y_1 y_3 + x_1 x_3\right) - 1.53\left(y_3^2 + x_3^2\right) - 0.7191\left(y_3 y_4 + x_3 x_4\right) + u_3 \\ \dot{e}_4 = 1.27 e_4 - 1.5367\left(y_1 y_4 + x_1 x_4\right) - 0.6477\left(y_2 y_4 + x_2 x_4\right) - 0.4445\left(y_3 y_4 + x_3 x_4\right) \\ \qquad - 1.27\left(y_4^2 + x_4^2\right) + u_4 \end{cases} \tag{7}$$

We consider the active nonlinear control defined by

$$\begin{cases} u_1 = -e_1 + y_1^2 + x_1^2 + 1.09\left(y_1 y_2 + x_1 x_2\right) + 1.52(y_1 y_3 + x_1 x_3) - k_1 e_1 \\ u_2 = -0.72 e_2 + 0.72\left(y_2^2 + x_2^2\right) + 0.3168\left(y_2 y_3 + x_2 x_3\right) + 0.9792\left(y_2 y_4 + x_2 x_4\right) - k_2 e_2 \\ u_3 = -1.53 e_3 + 3.5649\left(y_1 y_3 + x_1 x_3\right) + 1.53\left(y_3^2 + x_3^2\right) + 0.7191\left(y_3 y_4 + x_3 x_4\right) - k_3 e_3 \\ u_4 = -1.27 e_4 + 1.5367\left(y_1 y_4 + x_1 x_4\right) + 0.6477\left(y_2 y_4 + x_2 x_4\right) + 0.4445\left(y_3 y_4 + x_3 x_4\right) \\ \qquad + 1.27\left(y_4^2 + x_4^2\right) - k_4 e_4 \end{cases} \tag{8}$$

where k_1, k_2, k_3, k_4 are positive gain constants.
Substituting (8) into (7), we get the closed-loop control system as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \\ \dot{e}_4 = -k_4 e_4 \end{cases} \tag{9}$$

Next, we state and prove the main result of this section.
Theorem 1. The identical Vano 4-D competitive biological chaotic systems (4) and (5) are globally and exponentially anti-synchronized by the active control law (8), where k_1, k_2, k_3, k_4 are positive gain constants.
Proof. This result is a consequence of the Lyapunov stability theory [121].
We consider the quadratic Lyapunov function V defined by

$$V(e_1, e_2, e_3, e_4) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{10}$$

which is positive definite on R^4.
Differentiating V along the trajectories of the error dynamics (9), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2, \tag{11}$$

which is negative definite on R^4.

Thus, using Lyapunov stability theory [133], we conclude that the error dynamics (9) is globally exponentially stable. This completes the proof. ■

4. Numerical Simulations

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with step-size

$h = 10^{-8}$ to solve the Vano 4-D competitive biological chaotic systems (4) and (5), when the active nonlinear feedback control law (8) is implemented. We take the positive gain constants as $k_i = 6$ for $i = 1, 2, 3, 4$.

We take the initial conditions of the master system (4) as

$$x_1(0) = 4.7, \quad x_2(0) = 16.2, \quad x_3(0) = 17.4, \quad x_4(0) = 3.8 \tag{12}$$

We take the initial conditions of the slave system (5) as

$$y_1(0) = 5.3, \quad y_2(0) = 12.9, \quad y_3(0) = 21.8, \quad y_4(0) = 17.6 \tag{13}$$

Figures 5-8 show the anti-synchronization of the Vano 4-D competitive biological chaotic systems (4) and (5).

Figure 9 shows the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

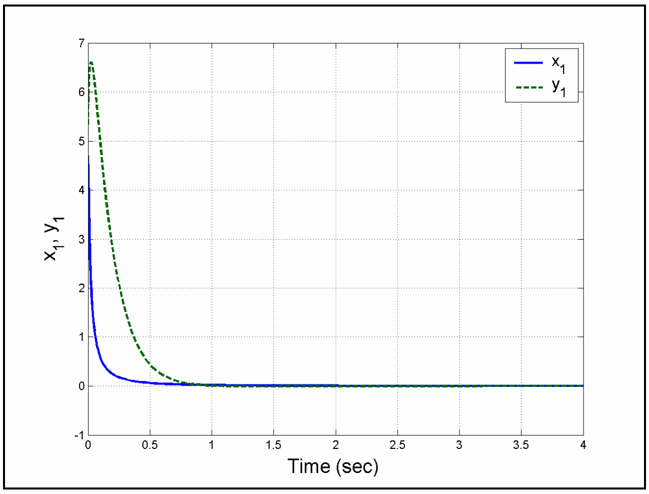

Figure 5. Anti-synchronization of the states x_1 and y_1

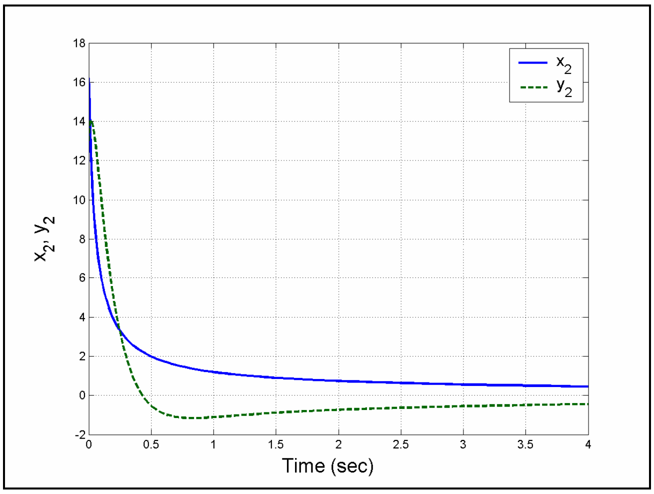

Figure 6. Anti-synchronization of the states x_2 and y_2

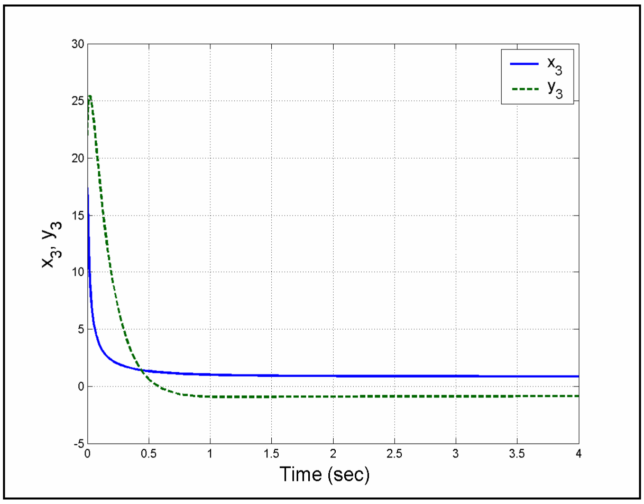

Figure 7. Anti-synchronization of the states x_3 and y_3

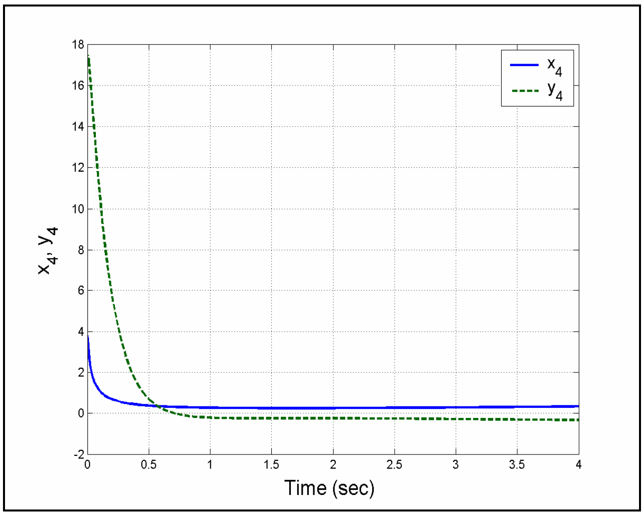

Figure 8. Anti-synchronization of the states x_4 and y_4

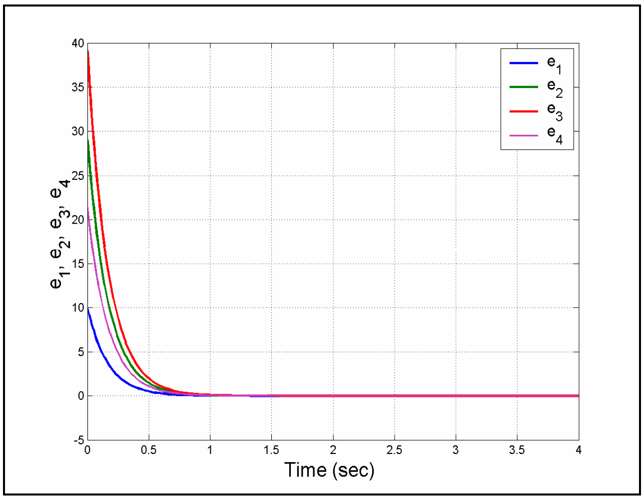

Figure 9. Time-history of the anti-synchronization errors e_1, e_2, e_3, e_4

5. Conclusions

In 2005, Sprott, Wildenberg and Azizi found a high-dimensional Lotka-Volterra multi-species competitive model that exhibits spatiotemporal chaos. In 2006, Vano, Wildenberg, Anderson, Noel and Sprott discovered chaos in basic Lotka-Volterra models of four competing species. In this paper, we applied active control method to derive new results for the anti-synchronization of Lotka-Volterra biological systems with four competing species discovered by Vano, Wildenberg, Anderson, Noel and Sprott. MATLAB plots have been shown to demonstrate all the main results derived in this work.

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