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Global Chaos Synchronization of the Forced Van der Pol Chaotic Oscillators via Adaptive Control Method

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Abstract: Chaos theory has a lot of applications in science and engineering. This paper first details the qualitative properties of the forced Van der Pol chaotic oscillator, which has important applications. Since its introduction in the 1920's, the Van der Pol equation has been a prototype model for systems with self-excited limit cycle oscillations. The Van der Pol equation has been studied over wide parameter regimes, from perturbations of harmonic motion to relaxation oscillations. It has been used by scientists to model a variety of physical and biological phenomena. Next, we derive new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators via adaptive control method. MATLAB plots have been shown to illustrate the phase portraits of the forced Van der Pol chaotic oscillator and the adaptive synchronization of the forced Van der Pol chaotic oscillator.
Keywords: Chaos, chaotic systems, Van der Pol oscillator, chaos synchronization, adaptive control, stability.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-42], Pehlivan system [43], Pham system [44], etc.

In control theory, active control method is used when the parameters are available for measurement [45-64]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [65-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-104], biology [105-1121], memristors [113-115], electrical circuits [116], etc.

This paper investigates first the qualitative properties of the forced Van der Pol chaotic oscillator, which was discovered by Van der Pol and Van der Mark ([117], 1927). In [118], it was reported that at certain drive frequencies an irregular noise was heard. This irregular noise was always heard near the natural entrainment frequencies. This was one of the first discovered instances of deterministic chaos. The Van der Pol oscillator has a long history of being used in both the physical and biological sciences. For instance, in biology, Fitzhugh [119] and Nagumo [120] extended the Van der Pol equation in a planar field as a model for action potentials of neurons. A detailed study on forced Van der Pol equation is found in [121].

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system, and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In this paper, we derive new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators [118].

This paper is organized as follows. Section 2 details the dynamics and properties of the forced Van der Pol chaotic oscillator. Section 3 details the global chaos synchronization of the forced Van der Pol chaotic oscillator via adaptive control method. Section 4 details the numerical simulations illustrating the main result derived in this research paper. Section 5 contains the main conclusions of this work.

2. Forced Van der Pol Chaotic Oscillator

The forced Van der Pol chaotic oscillator [118] is described by the second order differential equation

$$\ddot{x} = -x - a(x^2 - 1)\dot{x} + b\cos(\omega t) \tag{1}$$

In this work, we express the forced Van der Pol equation (1) in system form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - a(x_1^2 - 1)x_2 + b\cos(\omega t) \end{cases} \tag{2}$$

In Eq. (2), x_1, x_2 are the states and a, b are constant, positive, parameters.

It is known in the literature [111] that the system (2) is *chaotic*, when the parameter values are taken as

$$a = 5, \quad b = 5, \quad \omega = 2.467 \tag{3}$$

For numerical simulations, we take $x_1(0) = 0.1$ and $x_2(0) = 0.1$.

Figure 1 shows the $x_1 -$ waveform of the Van der Pol system (2), while Figure 2 shows the $x_2 -$ waveform of the Van der Pol system (2). Figure 3 shows the chaotic phase portrait of the Van der Pol system (2).

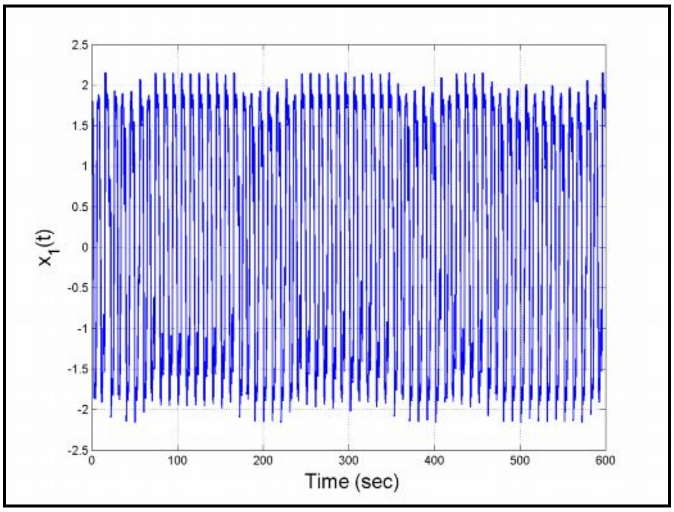

Figure 1. x_1 **− waveform of the Van der Pol system**

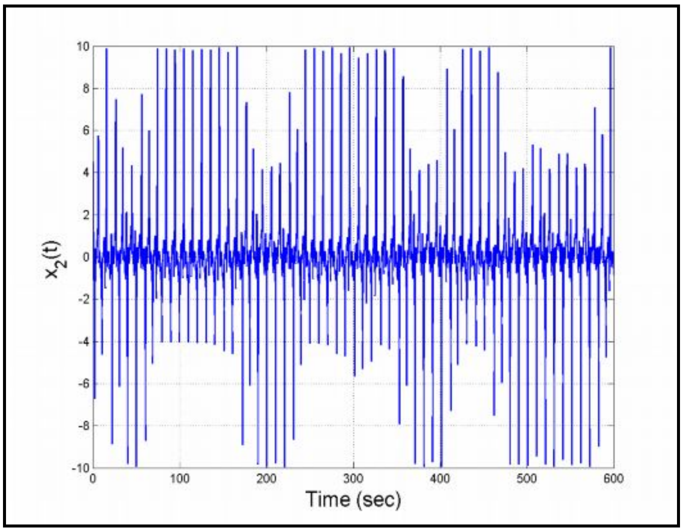

Figure 2. x_2 **– waveform of the Van der Pol system**

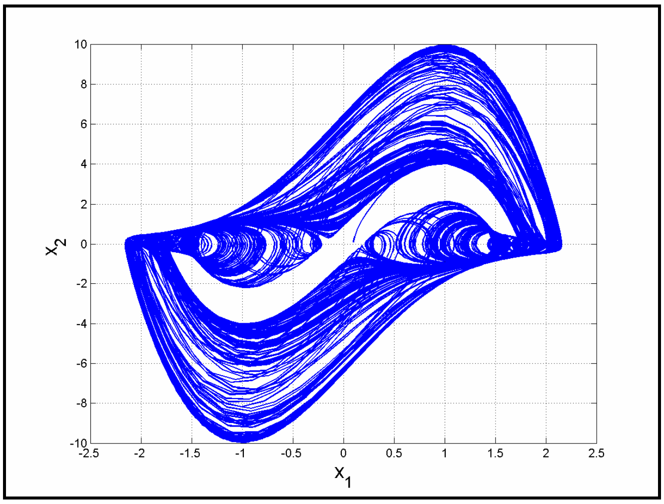

Figure 3. Chaotic phase portrait of the Van der Pol system

3. Adaptive Synchronization of the Forced Van der Pol Chaotic Oscillators

In this section, we derive new results for the global chaos synchronization of the forced Van der Pol chaotic oscillators via adaptive control method. The main result is established using Lyapunov stability theory [122].

As the master system, we consider the forced Van der Pol chaotic oscillator given by the 2-D dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - a(x_1^2 - 1)x_2 + b\cos(\omega t) \end{cases} \tag{4}$$

where a is an unknown system parameter, but the external periodic force $f(t) = b\cos(\omega t)$ is known.
As the slave system, we consider the forced Van der Pol chaotic oscillator given by the 2-D dynamics

$$\begin{cases} \dot{y}_1 = y_2 + u_1 \\ \dot{y}_2 = -y_1 - a(y_1^2 - 1)y_2 + b\cos(\omega t) + u_2 \end{cases} \tag{5}$$

In (5), u_1, u_2 are adaptive controls to be determined using an estimate $\hat{a}(t)$ of the unknown parameter a.
We define the synchronization error between the forced Van der Pol systems (4) and (5) as follows:

$$\begin{cases} e_1(t) = y_1(t) - x_1(t) \\ e_2(t) = y_2(t) - x_2(t) \end{cases} \tag{6}$$

Then the synchronization error dynamics is obtained as follows:

$$\begin{cases} \dot{e}_1 = e_2 + u_1 \\ \dot{e}_2 = -e_1 - a\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] + u_2 \end{cases} \tag{7}$$

We consider the adaptive control defined by

$$\begin{cases} u_1 = -e_2 - k_1 e_1 \\ u_2 = e_1 + \hat{a}(t)\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] - k_2 e_2 \end{cases} \tag{8}$$

where k_1 and k_2 are positive gain constants.

Substituting (8) into (7), we get the closed-loop error system as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -[a - \hat{a}(t)]\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] - k_2 e_2 \end{cases} \tag{9}$$

We define the parameter estimation error as

$$e_a(t) = a - \hat{a}(t) \tag{10}$$

Using (10), the closed-loop system (9) can be simplified as follows:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -e_a\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] - k_2 e_2 \end{cases} \tag{11}$$

Differentiating (10) with respect to t, we get

$$\dot{e}_a(t) = -\dot{\hat{a}}(t) \tag{12}$$

We consider the Lyapunov function defined by

$$V(e_1, e_2, e_a) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_a^2 \right) \tag{13}$$

which is positive definite on R^3.

Differentiating V along the trajectories of (10) and (11), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 + e_a\left[-e_2\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] - \dot{\hat{a}} \right] \tag{14}$$

In view of (14), we take the parameter update law as

$$\dot{\hat{a}} = -e_2\left[y_1^2 y_2 - x_1^2 x_2 - e_2 \right] \tag{15}$$

Next, we state and prove the main result of this section.

Theorem 1. The forced Van der Pol chaotic oscillator systems (4) and (5) are globally and exponentially synchronized by the adaptive control law (8) and the parameter update law (15), where k_1, k_2 are positive gain constants.

Proof. This result is a consequence of Lyapunov stability theory [122].

The quadratic Lyapunov function V defined by (13) is positive definite on R^3.

Substituting (15) into (14), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 \tag{15}$$

which is negative semi-definite on R^3.

Thus, using Barbalat's lemma [122], we conclude that the error dynamics (11) is globally exponentially stable.

Hence, we conclude that the forced Van der Pol chaotic oscillator systems (4) and (5) are globally and exponentially synchronized by the adaptive control law (8) and the parameter update law (15).

This completes the proof. ■

4. Numerical Simulations

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with step-size $h = 10^{-6}$ to solve the forced Van der Pol chaotic oscillator systems (4) and (5), when the adaptive control law (8) and the parameter update law (15) are implemented.

We take the parameter values as in the chaotic case, *i.e.*

$$a = 5,\ b = 5 \text{ and } \omega = 2.467. \tag{16}$$

We take the positive gain constants as

$$k_1 = 5 \text{ and } k_2 = 5. \tag{17}$$

We take the initial conditions of the forced Van der Pol chaotic oscillator system (4) as

$$x_1(0) = 7.4 \quad \text{and} \quad x_2(0) = 3.5. \tag{18}$$

We take the initial conditions of the forced Van der Pol chaotic oscillator system (5) as

$$y_1(0) = 2.1 \quad \text{and} \quad y_2(0) = 11.9. \tag{19}$$

We take the initial condition of the parameter estimate as

$$\hat{a}(0) = 11.8. \tag{20}$$

Figures 4 and 5 show the complete synchronization of the forced Van der Pol chaotic oscillator systems (4) and (5). Figure 6 shows the time-history of the synchronization errors $e_1(t)$ and $e_2(t)$.

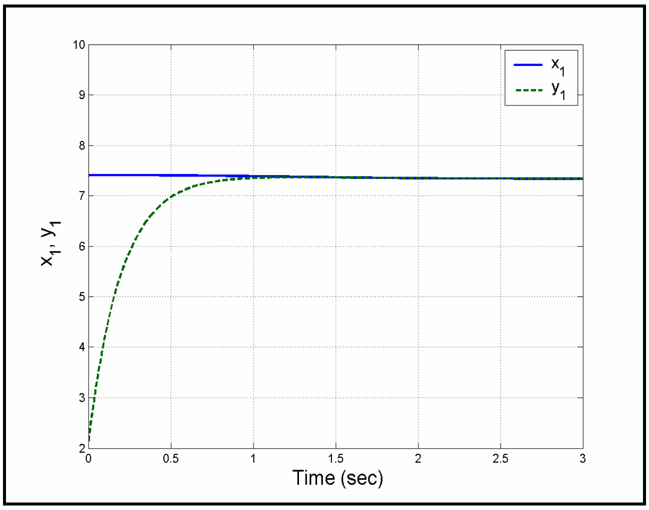

Figure 4. Complete synchronization of the states x_1 and y_1

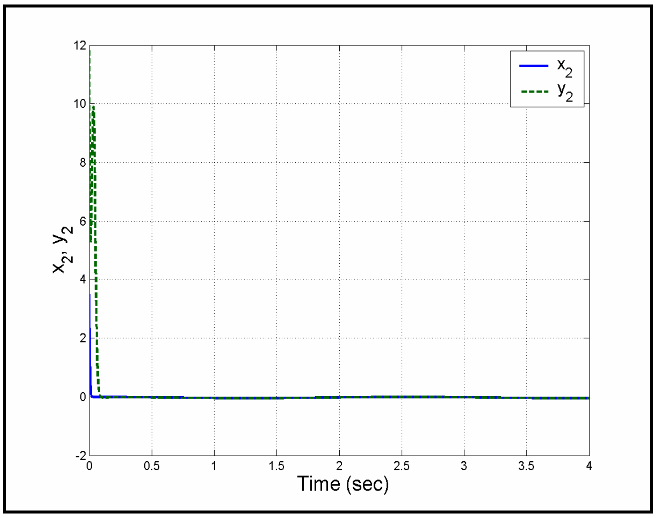

Figure 5. Complete synchronization of the states x_2 and y_2

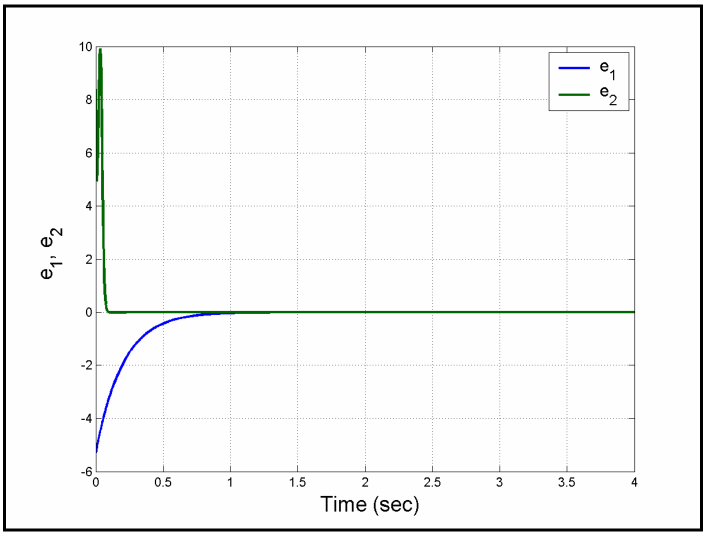

Figure 6. Time-history of the synchronization errors $e_1(t), e_2(t)$

5. Conclusions

In this paper, we first discussed the qualitative properties of the forced Van der Pol chaotic oscillator, which has important applications. Since its introduction in the 1920's, the Van der Pol equation has been a prototype model for systems with self-excited limit cycle oscillations. The Van der Pol equation has been studied over wide parameter regimes, from perturbations of harmonic motion to relaxation oscillations. It has been used by scientists to model a variety of physical and biological phenomena. Next, we derived new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators via adaptive control method. MATLAB plots have been shown to illustrate the phase portraits of the forced Van der Pol chaotic oscillator and the adaptive synchronization of the forced Van der Pol chaotic oscillator.

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