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Chaos in Neurons and Adaptive Control of Birkhoff-Shaw Strange Chaotic Attractor

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Abstract: Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations. In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally. In this research work, we discuss the properties of the Birkhoff-Shaw chaotic attractor, which is a forced oscillator and this strange chaotic attractor exhibits the structure of beaks and wings, typically observed in chaotic neuronal models. We also derive new results for the adaptive control of the Birkhoff-Shaw chaotic attractor (1981).All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the Birkhoff-Shaw chaotic attractor.
Keywords: Chaos, chaotic systems,biology, neurons, Hodgkin-Huxley equations, Birkhoff-Shaw attractor, etc.

Introduction

Chaos theory describes the qualitative study of deterministicchaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963[3]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which is algebraically much simpler than the Lorenz system. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-37], Pehlivan system [38], Pham system [39], etc.

Chaos is an important applied area in nonlinear dynamical systems and it is applicable to many real-world systems including the biological systems. Nerve membranes are known to exhibit their own nonlinear dynamics which generate and propagate action potentials. Such nonlinear dynamics in nerve membranes can produce chaos in neurons and related bifurcations. In 1952, A.L. Hodgkin and A.F. Huxley proposed a nonlinear dynamical system as a mathematical model of nerve membranes based on their electrophysiological experiments with squid giant atoms. Their mathematical model is referred to as

Hodgkin-Huxley equations in the literature [40]. Chaos in nerve membranes have been studied in the chaos literature both theoretically and experimentally.

In this research work, we discuss the properties of the Birkhoff-Shaw chaotic attractor (1981, [41]), which is a forced oscillator and this strange chaotic attractor exhibits the structure of beaks and wings, typically observed in chaotic neuronal models. We also derive new results for the adaptive control of the Birkhoff-Shaw chaotic attractor. All the main results are proved using Lyapunov stability theory [42].

In control theory, active control method is used when the parameters are available for measurement [43-60]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [61-74]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [75-81], sliding mode control method [82-93], etc.

Recently, chaos theory is found to have applications in many areas such as chemistry [94], biology [95], memristors [96-98], electrical circuits [99-100], etc.

Birkhoff-Shaw Chaotic Attractor

Shaw(1981, [41]) derived a strange chaotic attractor, called as the *Birkhoff-Shaw chaotic attractor*, which is described by the 2-D system of differential equations

$$\begin{cases} \dot{x}_1 = ax_2 + x_1 - bx_1x_2^2 \\ \dot{x}_2 = -x_1 + 0.25\sin(1.57t) \end{cases} \tag{1}$$

where x_1, x_2 are the states and a, b are positive constants.

The Birkhoff-Shaw system (1) is chaotic [41] when the parameter values are taken as

$$a = 0.7, \quad b = 10 \tag{2}$$

For numerical simulations, we take the initial conditions as $x_1(0) = -0.2$ and $x_2(0) = 0.2$.

The 2-D phase portrait of the Birkhoff-Shaw strange chaotic attractor is depicted in Figure 1. This type of chaotic behaviour is a common feature for the chaotic behaviour observed in neurons.

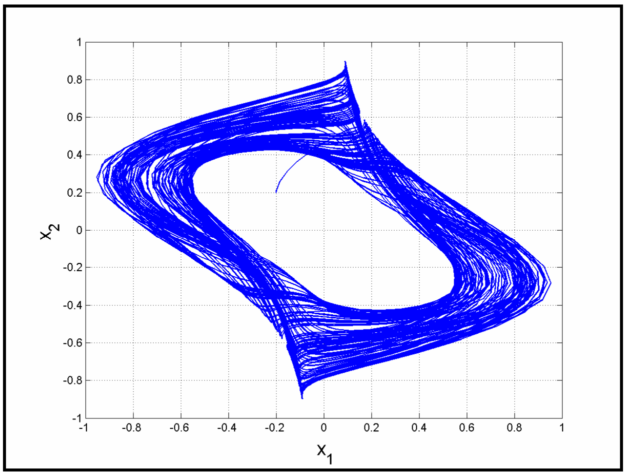

Figure1.The2-D phase portrait of the Birkhoff-Shaw Chaotic Attractor

Adaptive Control of the Birkhoff-Shaw Chaotic Attractor

The chaotic behaviour of the Birkhoff-Shaw chaotic behaviour [41] is similar to the chaotic behaviour observed in neurons. In this section, we consider the controlled Birkhoff-Shaw chaotic attractor given by the 2-D dynamics

$$\begin{cases} \dot{x}_1 = ax_2 + x_1 - bx_1x_2^2 + u_1 \\ \dot{x}_2 = -x_1 + 0.25\sin(1.57t) + u_2 \end{cases} \tag{3}$$

In (3), x_1, x_2 are the states and u_1, u_2 are the adaptive controls to be found using estimates of the unknown parameters a, b of the Birkhoff-Shaw system.

We consider the adaptive controller defined by

$$\begin{cases} u_1 = -\hat{a}(t)x_2 - x_1 + \hat{b}(t)x_1x_2^2 - k_1x_1 \\ u_2 = x_1 - 0.25\sin(1.57t) - k_2x_2 \end{cases} \tag{4}$$

where k_1, k_2 are positive gain constants.

Substituting (4) into (3, we get the closed-loop control system given by

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)]x_2 - [b - \hat{b}(t)]x_1x_2^2 - k_1x_1 \\ \dot{x}_2 = -k_2x_2 \end{cases} \tag{5}$$

We define the parameter estimation errors as follows:

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \end{cases} \tag{6}$$

Using (6), we can simplify the closed-loopplant dynamics (5) as follows.

$$\begin{cases} \dot{x}_1 = e_a x_2 - e_b x_1 x_2^2 - k_1x_1 \\ \dot{x}_2 = -k_2x_2 \end{cases} \tag{7}$$

Differentiating the parameter estimation errors (6) with respect to time, we get

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \end{cases} \tag{8}$$

Next, we consider the candidate Lyapunov function given by

$$V(x_1, x_2, e_a, e_b) = \frac{1}{2}\left(x_1^2 + x_2^2 + e_a^2 + e_b^2\right), \tag{9}$$

which is a positive definite function on R^4.

Differentiating V along the trajectories of (7) and (8), we obtain

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 + e_a\left[x_1x_2 - \dot{\hat{a}}\right] + e_b\left[-x_1^2x_2^2 - \dot{\hat{b}}\right] \tag{10}$$

In view of (10), we take the parameter estimates as follows:

$$\begin{cases} \dot{\hat{a}} = x_1x_2 \\ \dot{\hat{b}} = -x_1^2x_2^2 \end{cases} \tag{11}$$

Next, we state and prove the main result of this section.

Theorem 1. *The Birkhoff-Shaw chaotic attractor (3) is globally and exponentially stabilized by the adaptive control law (4) and the parameter update law (11), where k_1, k_2 are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by Eq. (9) is a positive definite function on R^4.

Substituting the parameter update law (11) into (10), the time-derivative of V is obtained as

$$\dot{V} = -k_1x_1^2 - k_2x_2^2, \tag{12}$$

which is a negative semi-definite function on R^4.

Thus, by Lyapunov stability theory [42], we conclude that the controlled state vector $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^2$.

This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (3) and (11). We take the gain constants as $k_1 = 10$ and $k_2 = 10$.

The parameter values of the Birkhoff-Shaw chaotic attractor (3) are taken as in the chaotic case (2), i.e.

$a = 0.7, \quad b = 10$

We take the initial conditions of theBirkhoff-Shaw chaotic attractor(3) as $x_1(0) = 1.8$ and $x_2(0) = -2.3$.

Also, we take the initial conditions of the parameter estimates as $\hat{a}(0) = 4$ and $\hat{b}(0) = 5$.

Figure 2showsthe time-history of the exponential convergence of the states x_1, x_2 to zero.

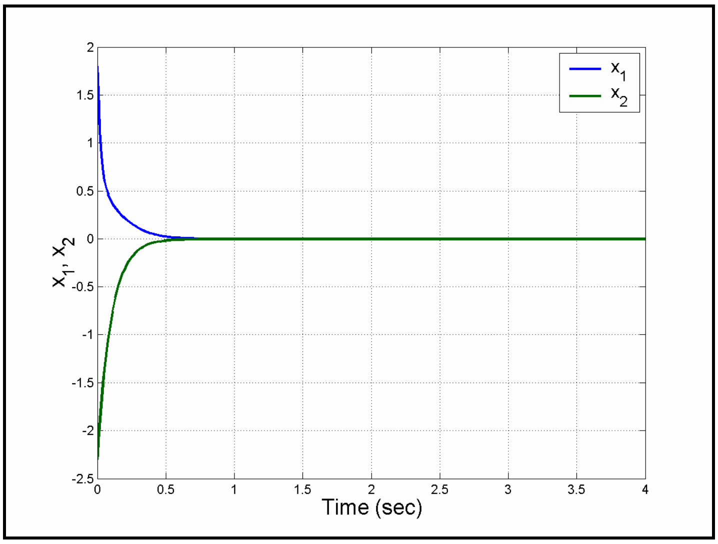

Figure2.Time-history of the controlled states x_1, x_2 of the Birkhoff-Shaw system

Conclusions

In this paper, chaos in neurons was discussion and new results have been derived for the analysis and adaptivecontrol of theBirkhoff-Shaw chaotic attractor (1981) with unknown system parameters. Main results were proved using Lyapunov stability theory. MATLAB simulations have been show to demonstrate and validate all the results derived in this paper for the Birkhoff-Shaw chaotic attractor.

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