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Adaptive Synchronization of Generalized Lotka-Volterra Three-Species Biological Systems

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Abstract: Since the recent research has shown the importance of biological control in many biological systems appearing in nature, this research paper investigates research in the dynamic and chaotic analysis of the generalized Lotka-Volterra three-species biological system, which was studied by Samardzija and Greller (1988). The generalized Lotka-Voterra biological system consists of two predator and one prey populations. This paper depicts the phase portraits of the 3-D generalized Lotka-Volttera system when the system undergoes chaotic behaviour. The synchronization of *master* and *slave* chaotic systems deals with synchronizing the respective states of the two systems asymptotically with time. Next, this paper derives adaptive biological control law for globally and exponentially synchronizing the states of the generalized Lotka-Volterra three-species biological systems with unknown parameters. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the three-species generalized Lotka-Volterra biological system and its adaptive synchronization.
Keywords: Chaos, chaotic systems, synchronization, biology, biological system, Lotka-Volterra system, etc.

Introduction

Chaos theory describes the qualitative study of deterministicchaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963[3]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which is algebraically much simpler than the Lorenz system. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-37], Pehlivan system [38], Pham system [39], etc.

One of the famous examples of simple biological models is the two-species predator-prey model developed by Lotka and Volterra [40]. Lotka-Voltera system describes the interaction of a two-species predator-prey model and id consists of a system of two nonlinear ordinary differential equations. This is a very popular model and it has many applications of interacting two-species systems. However, this model also has limitations such as it ignores many important factors such as interactions between another species of the same ecosystem, interactions with the environment etc. Thus, three-species models of biological species have more importance. Arneodo et al. [41] have shown that one can obtain chaotic behaviour for three species in an ecosystem. Three species predator-prey models typically consist of one-prey and two predators, and in this research work, we investigate such a three-species biological generalized Lotka-Volterra system investigated by Samardzija and Greller [42].

An agricultural ecosystem comprises a dynamic web of biological relationships among crop plants or trees, herbivores, predators, preys, disease organisms, etc. Organisms in an ecosystem interact in many ways through competition. These organisms constantly evolve and depend on each other and thereby they create a diverse, complex and dynamic environment.

This paper discusses the chaotic properties of the three-species generalized Lotka-Volterra biological system [42], and MATLAB plots are shown for the phase portraits of the chaotic system. This paper also derives new result using adaptive control method for globally and exponentially synchronizing the respective states of identical three-species generalized Lotka-Volterra biological systems. This main result is established using Lyapunov stability theory [43]. MATLAB plots are shown to illustrate the main results. Active control method is a feedback control strategy which works with the knowledge of system parameters [44-58]. Adaptive control method is a feedback control strategy which makes use of the estimates of the unknown parameters of the system [59-74].Chaos theory has many important applications in chemistry [75] and biology [76].

Generalized Lotka-Volterra Three-Species Biological System

Samardzija and Greller (1988, [42]) derived a generalized Lotka-Volterra three-species biological system, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = x_1 - x_1 x_2 + c x_1^2 - a x_1^2 x_3 \\ \dot{x}_2 = -x_2 + x_1 x_2 \\ \dot{x}_3 = -b x_3 + a x_1^2 x_3 \end{cases} \tag{1}$$

In (1), x_1 is the prey population, x_2, x_3 are predator populations and a, b, c are positive constants.

In [42], it was shown that the three-species biological system (1) is chaotic when we take

$$a = 2.9851, \; b = 3, \; c = 2 \tag{2}$$

For numerical simulations, we take the initial conditions as $x_1(0) = 1.2$, $x_2(0) = 1.2$ and $x_3(0) = 1.2$.

The 3-D phase portrait of the generalized Lotka-Volterra system(1) is depicted in Figure 1. The 2-D projections of the generalized Lotka-Volterra systems (2) on the coordinate planes are depicted in Figures 2-3.

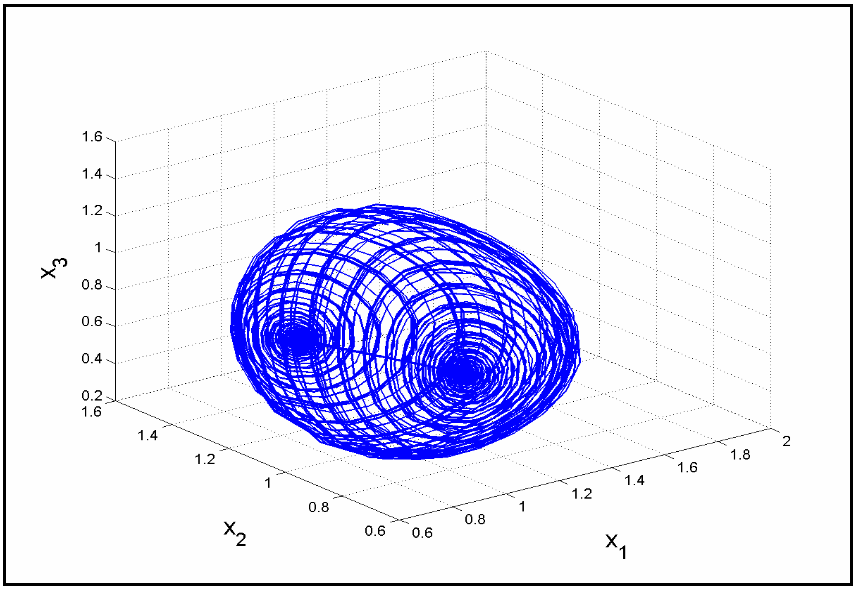

Figure 1. The3-D phase portrait of the generalized Lotka-Volterra chaotic system

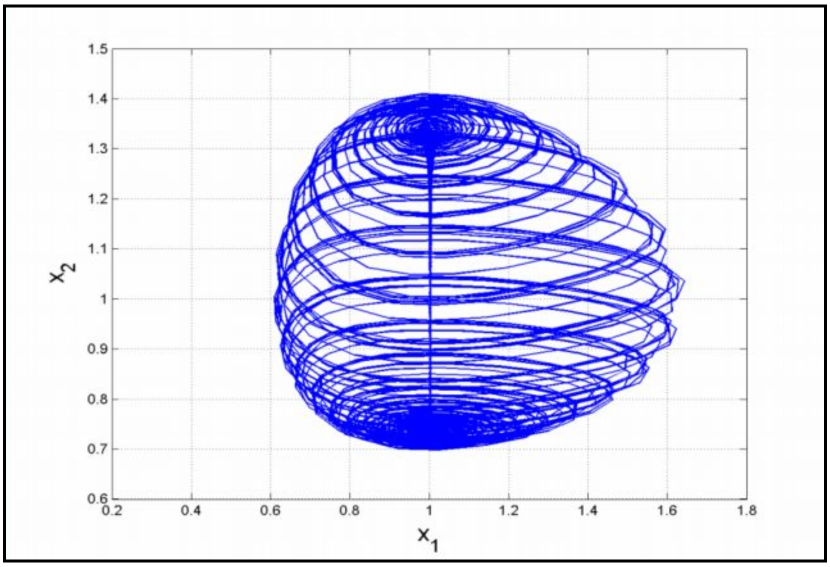

Figure 2. The2-D projection of the generalized Lotka-Volterra system on (x_1, x_2) plane

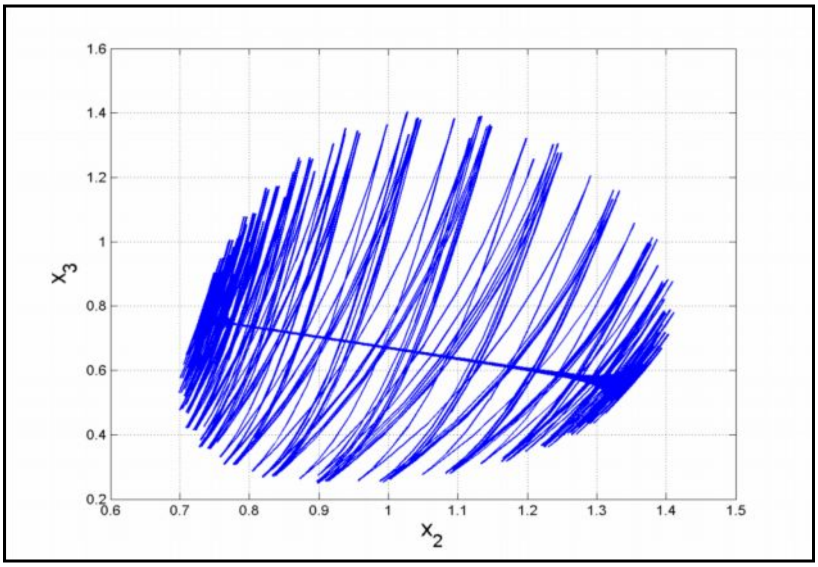

Figure 3. The2-D projection of the generalized Lotka-Volterra system on (x_2, x_3) plane

Adaptive Synchronization of the Generalized Lotka-Volterra Three-Species Biological Systems

The chaotic behaviour of the generalized Lotka-Voterra three-species biological system [42] is an example of the explosive route to chaos and it is attributed to the non-transversal saddle connection type bifurcation. Also, it is observed that the chaotic solution of the generalized Lotka-Volterra biological system (1) portrays a fractal torus in the 3-D phase space. In this section, we use adaptive control method for globally and exponentially synchronizing the states of the generalized Lotka-Volterra three-species biological systems.

As the master system, we consider the generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_1 - x_1 x_2 + c x_1^2 - a x_1^2 x_3 \\ \dot{x}_2 = -x_2 + x_1 x_2 \\ \dot{x}_3 = -b x_3 + a x_1^2 x_3 \end{cases} \tag{3}$$

In (3), x_1, x_2, x_3 are the states and a, b, c are unknown parameters of the system.

As the slave system, we consider the generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{y}_1 = y_1 - y_1 y_2 + c y_1^2 - a y_1^2 y_3 + u_1 \\ \dot{y}_2 = -y_2 + y_1 y_2 + u_2 \\ \dot{y}_3 = -b y_3 + a y_1^2 y_3 + u_3 \end{cases} \tag{4}$$

In (4), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined.

We define the synchronization error between the systems (3) and (4) as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{5}$$

The synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_1 - y_1 y_2 + x_1 x_2 + c\left(y_1^2 - x_1^2\right) - a\left(y_1^2 y_3 - x_1^2 x_3\right) + u_1 \\ \dot{e}_2 = -e_2 + y_1 y_2 - x_1 x_2 + u_2 \\ \dot{e}_3 = -b e_3 + a\left(y_1^2 y_3 - x_1^2 x_3\right) + u_3 \end{cases} \tag{6}$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = -e_1 + y_1 y_2 - x_1 x_2 - \hat{c}(t)\left(y_1^2 - x_1^2\right) + \hat{a}(t)\left(y_1^2 y_3 - x_1^2 x_3\right) - k_1 e_1 \\ u_2 = e_2 - y_1 y_2 + x_1 x_2 - k_2 e_2 \\ u_3 = \hat{b}(t) e_3 - \hat{a}(t)\left(y_1^2 y_3 - x_1^2 x_3\right) - k_3 e_3 \end{cases} \tag{7}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (7) into (6), we get the closed-loop control system given by

$$\begin{cases} \dot{e}_1 = \left[c - \hat{c}(t)\right]\left(y_1^2 - x_1^2\right) - \left[a - \hat{a}(t)\right]\left(y_1^2 y_3 - x_1^2 x_3\right) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -[b - \hat{b}(t)] e_3 + \left[a - \hat{a}(t)\right]\left(y_1^2 y_3 - x_1^2 x_3\right) - k_3 e_3 \end{cases} \tag{8}$$

We define parameter estimation errors as follows:

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \\ e_c = c - \hat{c}(t) \end{cases} \tag{9}$$

Using (9), we can simplify the error dynamics (8) as follows.

$$\begin{cases} \dot{e}_1 = e_c\left(y_1^2 - x_1^2\right) - e_a\left(y_1^2 y_3 - x_1^2 x_3\right) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -e_b e_3 + e_a\left(y_1^2 y_3 - x_1^2 x_3\right) - k_3 e_3 \end{cases} \tag{10}$$

Differentiating the parameter estimation errors (9) with respect to time, we get

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \\ \dot{e}_c = -\dot{\hat{c}}(t) \end{cases} \tag{11}$$

Next, we consider the candidate Lyapunov function given by

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right), \tag{12}$$

which is a positive definite function on R^6.

Differentiating V along the trajectories of (10) and (11), we obtain

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[\left(y_1^2y_3 - x_1^2x_3\right)(e_3 - e_1) - \dot{\hat{a}}\right] + e_b\left[-e_3^2 - \dot{\hat{b}}\right] + e_c\left[e_1\left(y_1^2 - x_1^2\right) - \dot{\hat{c}}\right] \quad (13)$$

In view of (13), we take the parameter estimates as follows:

$$\begin{cases} \dot{\hat{a}} = \left(y_1^2y_3 - x_1^2x_3\right)(e_3 - e_1) \\ \dot{\hat{b}} = -e_3^2 \\ \dot{\hat{c}} = e_1\left(y_1^2 - x_1^2\right) \end{cases} \quad (14)$$

Theorem 1. *The generalized Lotka-Volterra three-species biological systems (3) and (4) are globally and exponentially synchronized for all initial states* $x(0), y(0) \in R^3$ *by the adaptive biological control law (7) and the parameter update law (14), where* k_1, k_2, k_3 *are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by Eq. (12) is a positive definite function on R^6.

Substituting the parameter update law (14) into (13), the time-derivative of V is obtained as

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2, \quad (15)$$

which is a negative semi-definite function on R^6.

Thus, by Lyapunov stability theory [43], we conclude that the synchronization error $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (3), (4) and the parameter update law (14).

We take the gain constants as

$$k_1 = 6, \quad k_2 = 6, \quad k_3 = 6 \quad (16)$$

The parameter values of the systems (3) and (4) are taken as in the chaotic case, i.e.

$$a = 2.9851, \quad b = 3, \quad c = 2 \quad (17)$$

We take the initial conditions of the master system (3) as

$$x_1(0) = 5.2, \quad x_2(0) = 12.7, \quad x_3(0) = 3.8 \quad (18)$$

We take the initial conditions of the slave system (4) as

$$y_1(0) = 14.5, \quad y_2(0) = 3.4, \quad y_3(0) = 10.1 \quad (19)$$

Also, we take the initial conditions of the parameter estimates as

$$\hat{a}(0) = 5.1, \quad \hat{b}(0) = 7.4, \quad \hat{c}(0) = 20.8 \quad (20)$$

Figures 4-6 show the synchronization of the states of the generalized Lotka-Volterra 3-species biological systems (3) and (4). Figure 7 shows the time-history of the synchronization errors e_1, e_2, e_3.

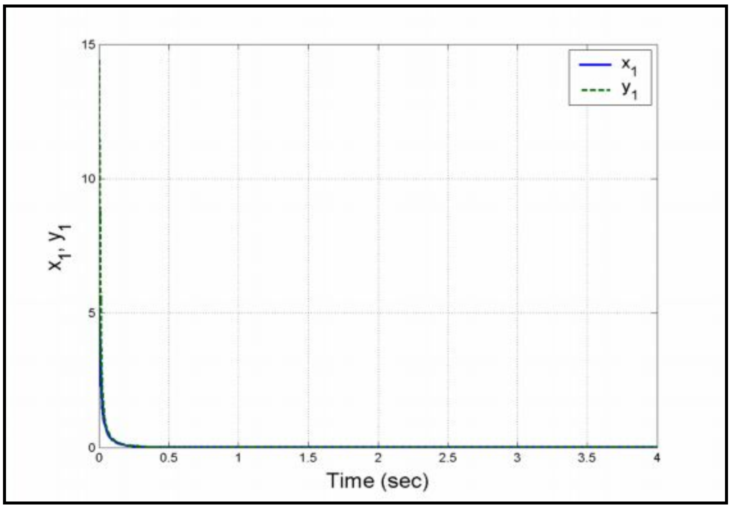

Figure 4. Synchronization of the states x_1 **and** y_1

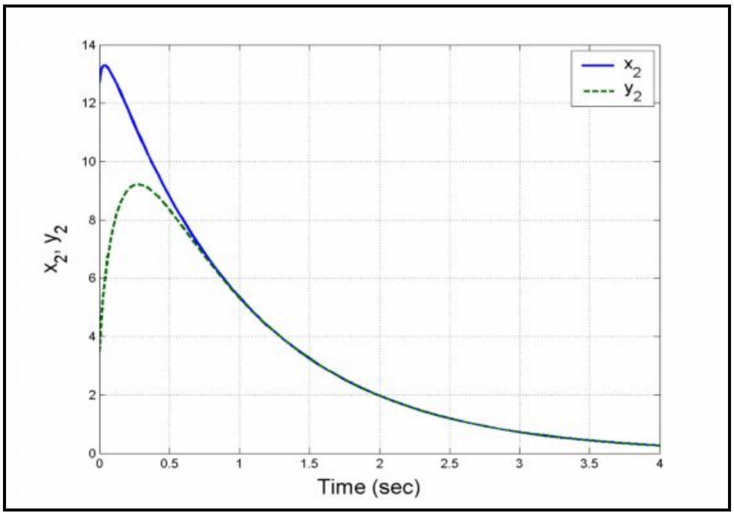

Figure 5. Synchronization of the states x_2 **and** y_2

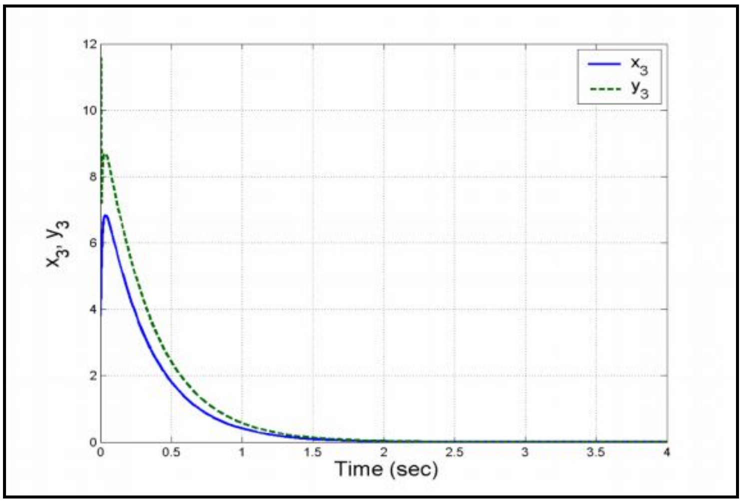

Figure 6. Synchronization of the states x_3 **and** y_3

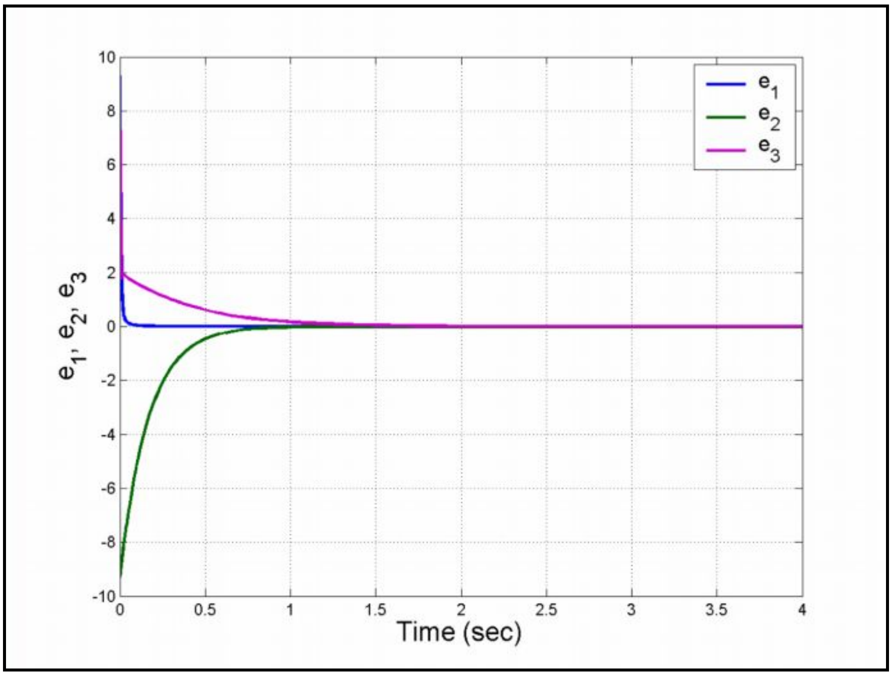

Figure 7. Time-history of the synchronization errors e_1, e_2, e_3

Conclusions

In this paper, new results have been derived for the analysis and adaptivesynchronization of the three-species generalized Lokta-Volterra biological systems discovered by Samardzija and Greller (1988). After a description and dynamic analysis of the chaotic 3-D three species Samardzija-Greller model, we have designed an adaptive biological feedback controller for the global exponential and complete synchronization of the states of the three-species generalized Lotka-Volterra biological systems.The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

References

1. Azar, A. T., and Vaidyanathan, S., Chaos Modeling and Control Systems Design, Studies in Computational Intelligence, Vol. 581, Springer, New York, USA, 2015.
2. Azar, A. T., and Vaidyanathan, S., Computational Intelligence Applications in Modeling and Control, Studies in Computational Intelligence, Vol. 575, Springer, New York, USA, 2015.
3. Lorenz, E. N., Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, 1963, 20, 130-141.
4. Rössler, O. E., An equation for continuous chaos, Physics Letters A, 1976, 57, 397-398.
5. Arneodo, A., Coullet, P., and Tresser, C., Possible new strange attractors with spiral structure, Communications in Mathematical Physics, 1981, 79, 573-579.
6. Sprott, J. C., Some simple chaotic flows, Physical Review E, 1994, 50, 647-650.
7. Chen, G., and Ueta, T., Yet another chaotic attractor, International Journal of Bifurcation and Chaos, 1999, 9, 1465-1466.
8. Lü, J., and Chen, G., A new chaotic attractor coined, International Journal of Bifurcation and Chaos, 2002, 12, 659-661.
9. Cai, G., and Tan, Z., Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain Systems, 2007, 1, 235-240.
10. Tigan, G., and Opris, D., Analysis of a 3D chaotic system, Chaos, Solitons and Fractals, 2008, 36, 1315-1319.
11. Sundarapandian, V., and Pehlivan, I., Analysis, control, synchronization and circuit design of a novel chaotic system, Mathematical and Computer Modelling, 2012, 55, 1904-1915.
12. Sundarapandian, V., Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, 2013, 6, 45-52.
13. Vaidyanathan, S., A new six-term 3-D chaotic system with an exponential nonlinearity, Far East

Journal of Mathematical Sciences, 2013, 79, 135-143.
14. Vaidyanathan, S., Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters, Journal of Engineering Science and Technology Review, 2013, 6, 53-65.
15. Vaidyanathan, S., A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East Journal of Mathematical Sciences, 2014, 84, 219-226.
16. Vaidyanathan, S., Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2014, 22, 41-53.
17. Vaidyanathan, S., and Madhavan, K., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system, International Journal of Control Theory and Applications, 2013, 6, 121-137.
18. Vaidyanathan, S., Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities, European Physical Journal: Special Topics, 2014, 223, 1519-1529.
19. Vaidyanathan, S., Volos, C., Pham, V. T., Madhavan, K., and Idowu, B. A., Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities, Archives of Control Sciences, 2014, 24, 257-285.
20. Vaidyanathan, S., Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control, International Journal of Modelling, Identification and Control, 2014, 22, 207-217.
21. Vaidyanathan, S., and Azar, A.T., Analysis and control of a 4-D novel hyperchaotic system, Studies in Computational Intelligence, 2015, 581, 3-17.
22. Vaidyanathan, S., Volos, C., Pham, V.T., and Madhavan, K., Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation, Archives of Control Sciences, 2015, 25, 135-158.
23. Vaidyanathan, S., Volos, C., and Pham, V.T., Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation, Archives of Control Sciences, 2014, 24, 409-446.
24. Vaidyanathan, S., A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control, International Journal of Control Theory and Applications, 2013, 6, 97-109.
25. Vaidyanathan, S., Analysis, properties and control of an eight-term 3-D chaotic system with an exponential nonlinearity, International Journal of Modelling, Identification and Control, 2015, 23, 164-172.
26. Vaidyanathan, S., Azar, A.T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23, 267-277.
27. Vaidyanathan, S., Qualitative analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with a quartic nonlinearity, International Journal of Control Theory and Applications, 2014, 7, 1-20.
28. Vaidyanathan, S., Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities, International Journal of Control Theory and Applications, 2014, 7, 35-47.
29. Vaidyanathan, S., and Pakiriswamy, S., A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control, Journal of Engineering Science and Technology Review, 2015, 8, 52-60.
30. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, adaptive control and adaptive synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 181-191.
31. Vaidyanathan, S., Rajagopal, K., Volos, C.K., Kyprianidis, I.M., and Stouboulos, I.N., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW, Journal of Engineering Science and Technology Review, 2015, 8, 130-141.
32. Pham, V.T., Volos, C.K., Vaidyanathan, S., Le, T.P., and Vu, V.Y., A memristor-based hyperchaotic system with hidden attractors: Dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8, 205-214.
33. Pham, V.T., Volos, C.K., and Vaidyanathan, S., Multi-scroll chaotic oscillator based on a first-order delay differential equation, Studies in Computational Intelligence, 2015, 581, 59-72.
34. Vaidyanathan, S., Volos, C.K., Kyprianidis, I.M., Stouboulos, I.N., and Pham, V.T., Analysis, adaptive

control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 24-36.
35. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium, Journal of Engineering Science and Technology Review, 2015, 8, 232-244.
36. Sampath, S., Vaidyanathan, S., Volos, C.K., and Pham, V.T., An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 1-6.
37. Vaidyanathan, S., A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters, Journal of Engineering Science and Technology Review, 2015, 8, 106-115.
38. Pehlivan, I., Moroz, I. M., and Vaidyanathan, S., Analysis, synchronization and circuit design of a novel butterfly attractor, Journal of Sound and Vibration, 2014, 333, 5077-5096.
39. Pham, V. T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristic neural network, Optoelectronics and Advanced Materials – Rapid Communications, 2014, 8, 1157-1163.
40. Freedman, H.I., Deterministic Mathematical Models in Population Ecology, Marcel Dekker, New York, USA, 1980.
41. Arneodo, A., Coullet, P., and Tresser, C., Occurrence of strange attractors in three-dimensional Volterra equation, Physics Letters A, 1980, 259-263.
42. Samardzija, N., and Greller, L.D., Explosive route to chaos through a fractal torus in a generalized Lotka-Volterra model, Bulletin of Mathematical Biology, 1988, 50, 465-491.
43. Khalil, H.K., Nonlinear Systems, Prentice Hall, New Jersey, USA, 2001.
44. Sarasu, P., and Sundarapandian, V., Active controller design for generalized projective synchronization of four-scroll chaotic systems, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 26-33.
45. Vaidyanathan, S., and Rajagopal, K., Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 55-61.
46. Sundarapandian, V., and Sivaperumal, S., Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, 2011, 6, 224-231.
47. Sarasu, P., and Sundarapandian, V., The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control, International Journal of Soft Computing, 2011, 6, 216-233.
48. Vaidyanathan, S., and Sampath, S., Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, 2012, 9, 274-279.
49. Sundarapandian, V., and Karthikeyan, R., Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control, Journal of Engineering and Applied Sciences, 2012, 7, 254-264.
50. Vaidyanathan, S., Analysis and synchronization of the hyperchaotic Yujun systems via sliding mode control, Advances in Intelligent Systems and Computing, 2012, 176, 329-337.
51. Vaidyanathan, S., Global chaos control of hyperchaotic Liu system via sliding control method, International Journal of Control Theory and Applications, 2012, 5, 117-123.
52. Vaidyanathan, S., Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, 2012, 5, 15-20.
53. Karthikeyan, R., and Sundarapandian, V., Hybrid chaos synchronization of four-scroll systems via active control, Journal of Electrical Engineering, 2014, 65, 97-103.
54. Vaidyanathan, S., Global chaos synchronisation of identical Li-Wu chaotic systems via sliding mode control, International Journal of Modelling, Identification and Control, 2014, 22, 170-177.
55. Pham, V.T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristive neural network, Optoelectronics and Advanced Materials, Rapid Communications, 2014, 8, 1157-1163.
56. Vaidyanathan, S., and Azar, A.T., Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems, Studies in Computational Intelligence, 2015, 576, 527-547.
57. Vaidyanathan, S., and Azar, A.T., Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems, Studies in Computational

Intelligence, 2015, 576, 549-569.
58. Vaidyanathan, S., Sampath, S., and Azar, A.T., Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system, International Journal of Modelling, Identification and Control, 2015, 23, 92-100.
59. Sundarapandian, V., Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lectures on Electrical Engineering, 2013, 131, 319-327.
60. Vaidyanathan, S., Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, 2013, 177, 1-10.
61. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of two-scroll systems via adaptive control, International Journal of Soft Computing, 2012, 7, 146-156.
62. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of three-scroll chaotic systems via adaptive control, European Journal of Scientific Research, 2012, 72, 504-522.
63. Sundarapandian, V., and Karthikeyan, R., Adaptive anti-synchronization of uncertain Tigan and Li systems, Journal of Engineering and Applied Sciences, 2012, 7, 45-52.
64. Suresh, R., and Sundarapandian, V., Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback, Far East Journal of Mathematical Sciences, 2013, 73, 73-95.
65. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback, Malaysian Journal of Mathematical Sciences, 2013, 7, 219-246.
66. Vaidyanathan, S., Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, 2013, 177, 1-10.
67. Sundarapandian, V., Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lecture Notes in Electrical Engineering, 2013, 131, 319-327.
68. Rasappan, S., and Vaidyanathan, S., Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design, Kyungpook Mathematical Journal, 2014, 54, 293-320.
69. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback, Arabian Journal for Science and Engineering, 2014, 39, 3351-3364.
70. Vaidyanathan, S., Idowu, B.A., and Azar, A.T., Backstepping controller design for the global chaos synchronization of Sprott's jerk systems, Studies in Computational Intelligence, 2015, 581, 39-58.
71. Vaidyanathan, S., and Pakiriswamy, S., Adaptive controller design for the generalized projective synchronization of circulant chaotic systems with unknown parameters, International Journal of Control Theory and Applications, 2014, 7, 55-74.
72. Vaidyanathan, S., Volos, C.K., Rajagopal, K., Kyprianidis, I.M. and Stouboulos, I.N., Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, Journal of Engineering Science and Technology Review, 2015, 8, 74-82.
73. Volos, C.K., Kyrpianidis, I.M., Stouboulos, I.N. Tlelo-Cuautle, E., and Vaidyanathan, S., Memristor: A new concept in synchronization of coupled neuromorphic circuits, Journal of Engineering Science and Technology Review, 2015, 8, 157-173.
74. Volos, C.K., Pham, V.T., Vaidyanathan, S., Kyprianidis, I.M., and Stouboulos, I.N., Synchronization phenomena in coupled Colpitts circuits, Journal of Engineering Science and Technology Review, 2015, 8, 142-151.
75. Vaidyanathan, S., Adaptive synchronization of chemical chaotic reactors, International Journal of ChemTech Research, 2015, 8, 612-621.
76. Vaidyanathan, S., Adaptive backstepping control of enzymes-substrates system with ferroelectric behaviour in brain waves, International Journal of PharmTech Research, 2015, 8, 256-261.

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