

Adaptive Biological Control of Generalized Lotka-Volterra Three-Species Biological System

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Abstract: Recent research has shown the importance of biological control in many biological systems appearing in nature. This paper investigates research in the dynamic and chaotic analysis of the generalized Lotka-Volterra three-species biological system, which was studied by Samardzija and Greller (1988). The generalized Lotka-Volterra biological system consists of two predator and one prey populations. This paper displays the phase portraits of the 3-D generalized Lotka-Volterra system when the system undergoes chaotic behaviour. Next, this paper derives adaptive biological control for globally stabilizing the trajectories of the generalized Lotka-Volterra three-species biological system with unknown parameters. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the three-species generalized Lotka-Volterra biological system.

Keywords: Chaos, chaotic systems, biology, biological control, biological system, Lotka-Volterra system, etc.

Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963[3]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which is algebraically much simpler than the Lorenz system. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-37], Pehlivan system [38], Pham system [39], etc.

An agricultural ecosystem comprises a dynamic web of biological relationships among crop plants or trees, herbivores, predators, preys, disease organisms, etc. Organisms in an ecosystem interact in many ways through competition. These organisms constantly evolve and depend on each other and thereby they create a diverse, complex and dynamic environment.

One of the famous examples of simple biological models is the two-species predator-prey model developed by Lotka and Volterra [40]. Lotka-Volterra system describes the interaction of a two-species predator-prey model and it consists of a system of two nonlinear ordinary differential equations. This is a very popular model and it has many applications of interacting two-species systems. However, this model also has limitations such as it ignores many important factors such as interactions between another species of the same ecosystem, interactions with the environment etc. Thus, three-species models of biological species have more

importance. Arneodo et al. [41] have shown that one can obtain chaotic behaviour for three species in an ecosystem. Three species predator-prey models typically consist of one-prey and two predators, and in this research work, we investigate such a three-species biological generalized Lotka-Volterra system investigated by Samardzija and Greller [42].

This paper discusses the chaotic properties of the three-species generalized Lotka-Volterra biological system [42], and MATLAB plots are shown for the phase portraits of the chaotic system. This paper also derives new results of adaptive biological controller design for the three-species generalized Lotka-Volterra biological system using Lyapunov stability theory [43] and MATLAB plots are shown to illustrate the main results. Active control method is a feedback control strategy which works with the knowledge of system parameters [44-58]. Adaptive control method is a feedback control strategy which is very effective in control theory because it makes use of the estimates of the unknown parameters of the system [59-74]. Chaos theory has many important applications in chemistry [75] and biology [76].

Generalized Lotka-Volterra Three-Species Biological System

Samardzija and Greller (1988, [42]) derived a generalized Lotka-Volterra three-species biological system, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = x_1 - x_1x_2 + cx_1^2 - ax_1^2x_3 \\ \dot{x}_2 = -x_2 + x_1x_2 \\ \dot{x}_3 = -bx_3 + ax_1^2x_3 \end{cases} \quad (1)$$

In (1), x_1 is the prey population, x_2, x_3 are predator populations and a, b, c are positive constants.

In [42], it was shown that the three-species biological system (1) is chaotic when we take $a = 2.9851$, $b = 3$, $c = 2$ (2)

For numerical simulations, we take the initial conditions as $x_1(0) = 1.2$, $x_2(0) = 1.2$ and $x_3(0) = 1.2$.

The 3-D phase portrait of the generalized Lotka-Volterra system(1) is depicted in Figure 1. The 2-D projections of the generalized Lotka-Volterra systems (2) on the coordinate planes are depicted in Figures 2-4.

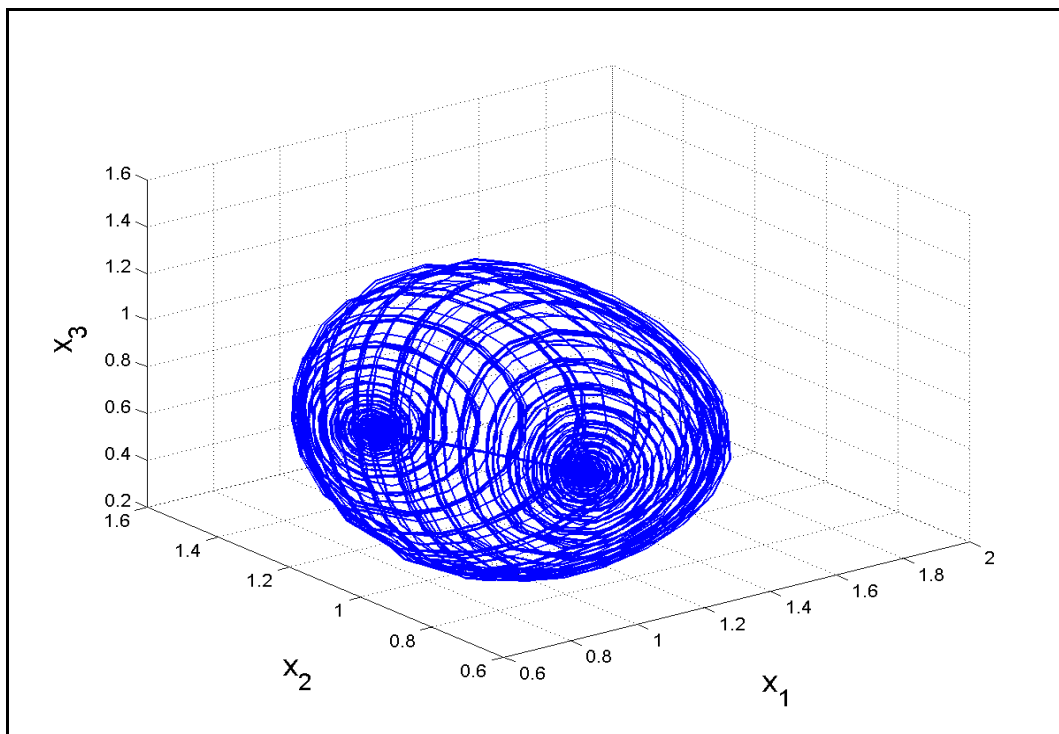


Figure1. The 3-D phase portrait of the generalized Lotka-Volterra chaotic system

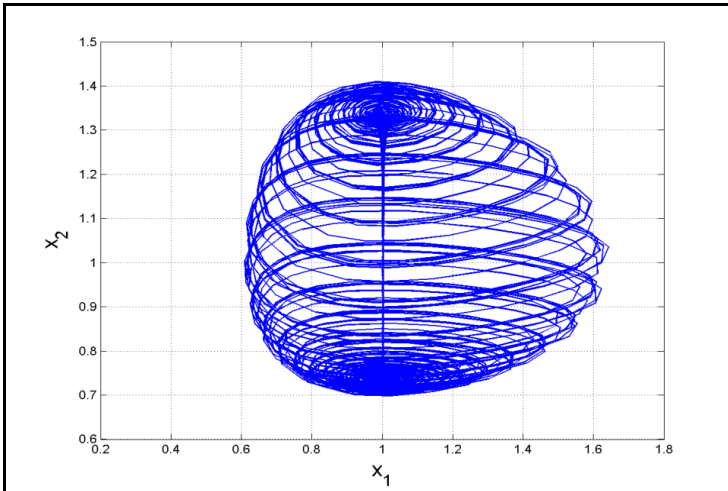


Figure2. The 2-D projection of the generalized Lotka-Volterra system on (x_1, x_2) plane

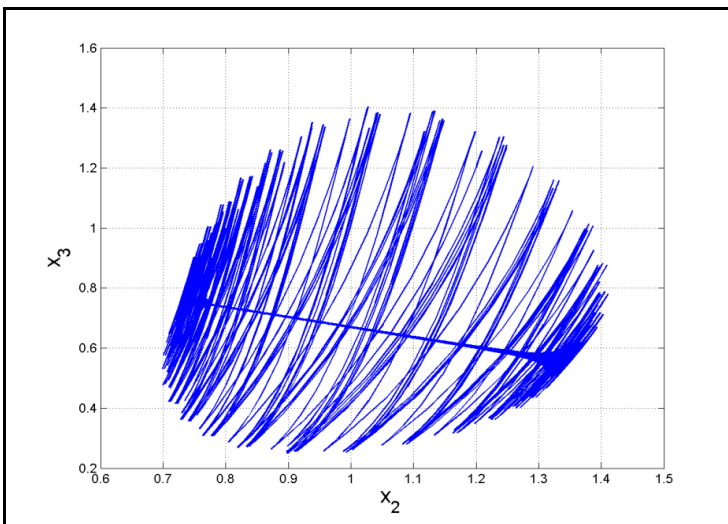


Figure3. The 2-D projection of the generalized Lotka-Volterra system on (x_2, x_3) plane

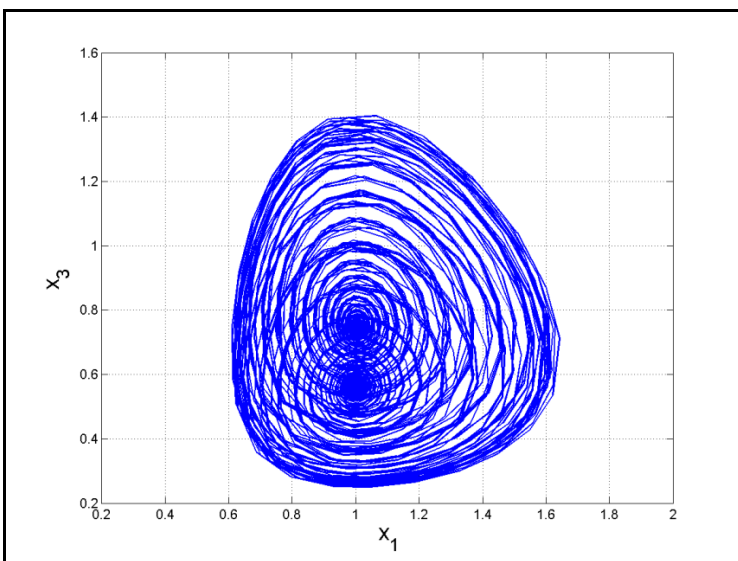


Figure4. The 2-D projection of the generalized Lotka-Volterra system on (x_1, x_3) plane

Adaptive Control of the Generalized Lotka-Volterra Three-Species Biological System

The chaotic behaviour of the generalized Lotka-Volterra three-species biological system [42] is an example of the explosive route to chaos and it is attributed to the non-transversal saddle connection type bifurcation. Also, it is observed that the chaotic solution of the generalized Lotka-Volterra biological system (1) portrays a fractal torus in the 3-D phase space.

In this section, we consider the controlled generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_1 - x_1x_2 + cx_1^2 - ax_1^2x_3 + u_1 \\ \dot{x}_2 = -x_2 + x_1x_2 + u_2 \\ \dot{x}_3 = -bx_3 + ax_1^2x_3 + u_3 \end{cases} \quad (3)$$

In (3), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are the adaptive biological controls to be found using estimates of the unknown parameters a, b, c of the system (3).

Our control goal is to drive the states of the generalized 3-species Lotka-Volterra system (3) to desired values of the states, say, α, β, γ , respectively.

Thus, we define the tracking errors as

$$\begin{cases} e_1 = x_1 - \alpha \\ e_2 = x_2 - \beta \\ e_3 = x_3 - \gamma \end{cases} \quad (4)$$

The tracking error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = (e_1 + \alpha) - (e_1 + \alpha)(e_2 + \beta) + c(e_1 + \alpha)^2 - a(e_1 + \alpha)^2(e_3 + \gamma) + u_1 \\ \dot{e}_2 = -(e_2 + \beta) + (e_1 + \alpha)(e_2 + \beta) + u_2 \\ \dot{e}_3 = -b(e_3 + \gamma) + a(e_1 + \alpha)^2(e_3 + \gamma) + u_3 \end{cases} \quad (5)$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = -(e_1 + \alpha) + (e_1 + \alpha)(e_2 + \beta) - \hat{c}(t)(e_1 + \alpha)^2 + \hat{a}(t)(e_1 + \alpha)^2(e_3 + \gamma) - k_1e_1 \\ u_2 = (e_2 + \beta) - (e_1 + \alpha)(e_2 + \beta) - k_2e_2 \\ u_3 = \hat{b}(t)(e_3 + \gamma) - \hat{a}(t)(e_1 + \alpha)^2(e_3 + \gamma) - k_3e_3 \end{cases} \quad (6)$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (6) into (5), we get the closed-loop control system given by

$$\begin{cases} \dot{e}_1 = [c - \hat{c}(t)](e_1 + \alpha)^2 - [a - \hat{a}(t)](e_1 + \alpha)^2(e_3 + \gamma) - k_1e_1 \\ \dot{e}_2 = -k_2e_2 \\ \dot{e}_3 = -[b - \hat{b}(t)](e_3 + \gamma) + [a - \hat{a}(t)](e_1 + \alpha)^2(e_3 + \gamma) - k_3e_3 \end{cases} \quad (7)$$

We define parameter estimation errors as follows:

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \\ e_c = c - \hat{c}(t) \end{cases} \quad (8)$$

Using (8), we can simplify the error dynamics (7) as follows.

$$\begin{cases} \dot{e}_1 = e_c(e_1 + \alpha)^2 - e_a(e_1 + \alpha)^2(e_3 + \gamma) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -e_b(e_3 + \gamma) + e_a(e_1 + \alpha)^2(e_3 + \gamma) - k_3 e_3 \end{cases} \quad (9)$$

Differentiating the parameter estimation errors (8) with respect to time, we get

$$\begin{cases} \dot{e}_a = -\hat{a}(t) \\ \dot{e}_b = -\hat{b}(t) \\ \dot{e}_c = -\hat{c}(t) \end{cases} \quad (10)$$

Next, we consider the candidate Lyapunov function given by

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2), \quad (11)$$

which is a positive definite function on R^6 .

Differentiating V along the trajectories of (9) and (10), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a \left[(e_3 - e_1)(e_1 + \alpha)^2(e_3 + \gamma) - \hat{a} \right] + e_b \left[-e_3(e_3 + \gamma) - \hat{b} \right] + e_c \left[e_1(e_1 + \alpha)^2(e_3 + \gamma) - \hat{c} \right] \quad (12)$$

In view of (12), we take the parameter estimates as follows:

$$\begin{cases} \hat{a} = (e_3 - e_1)(e_1 + \alpha)^2(e_3 + \gamma) \\ \hat{b} = -e_3(e_3 + \gamma) \\ \hat{c} = e_1(e_1 + \alpha)^2 \end{cases} \quad (13)$$

Theorem 1. *The generalized Lotka-Volterra three-species biological system (3) is exponentially regulated to the steady-state values α, β, γ by the adaptive biological control law (6) and the parameter update law (13), where k_1, k_2, k_3 are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by Eq. (11) is a positive definite function on R^6 .

Substituting the parameter update law (13) into (12), the time-derivative of V is obtained as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \quad (14)$$

which is a negative semi-definite function on R^6 .

Thus, by Lyapunov stability theory, we conclude that the tracking error $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for all initial conditions $e(0) \in R^3$. This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (3) and (13).

We take the gain constants as $k_i = 8$ for $i = 1, 2, 3$. We take $\alpha = 2$, $\beta = 3$ and $\gamma = 4$.

The parameter values are taken as in the chaotic case, viz. $a = 2.9851$, $b = 3$ and $c = 2$.

We take the initial conditions of the system(3) as $x_1(0) = 5.4$, $x_2(0) = 8.1$ and $x_3(0) = 6.9$.

Also, we take $\hat{a}(0) = 12.5$, $\hat{b}(0) = 15.3$ and $\hat{c}(0) = 20.7$.

Figures 5-7 show the time-history of the exponential convergence of the states x_1, x_2, x_3 to desired target values α, β, γ respectively.

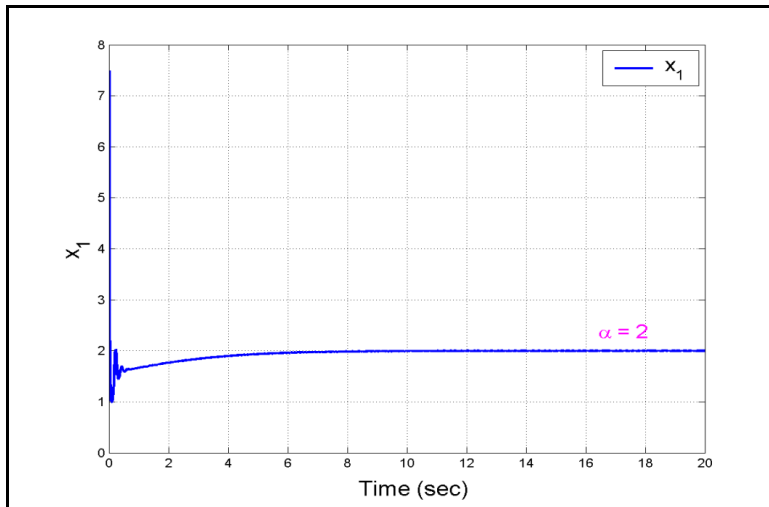


Figure5.Regulation of the controlled state x_1 to $\alpha = 2$

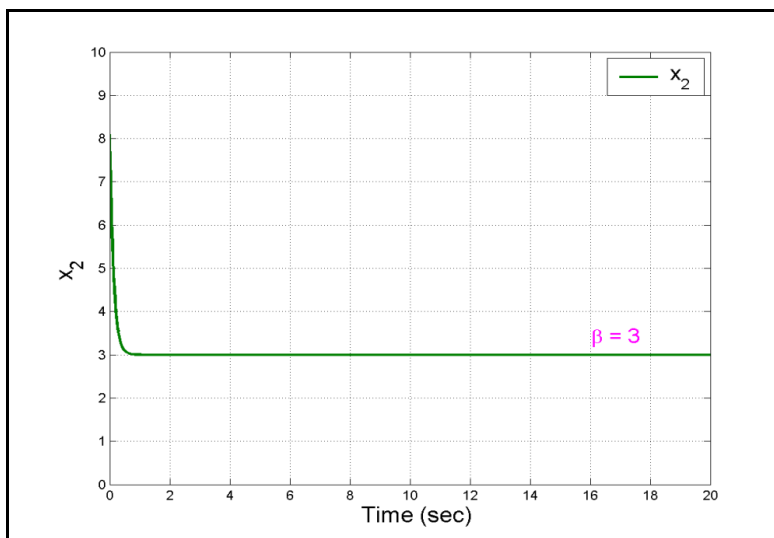


Figure6. Regulation of the controlled state x_2 to $\beta = 3$

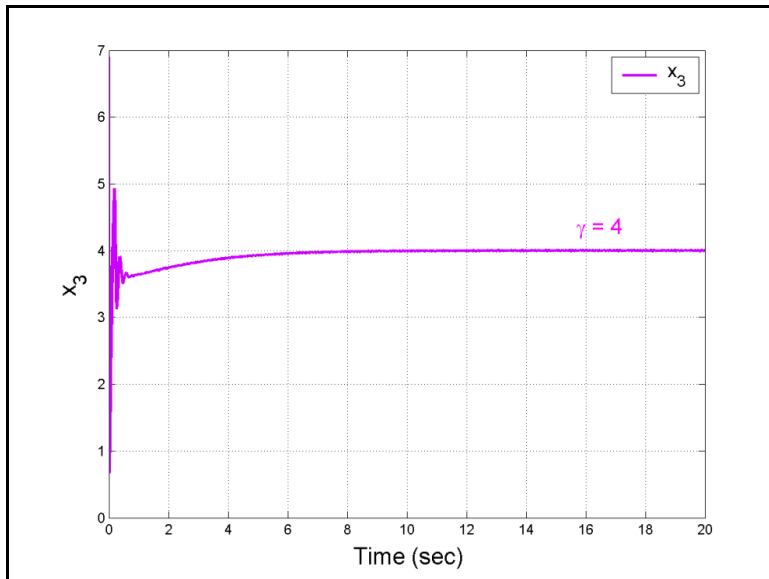


Figure7. Regulation of the controlled state x_3 to $\gamma = 4$ Conclusions

In this paper, new results have been derived for the analysis and adaptive biological control of the three-species generalized Lotka-Volterra biological system discovered by Samardzija and Greller (1988). After a description and dynamic analysis of the chaotic 3-D three species Samardzija-Greller model, we have designed an adaptive biological feedback controller for the global exponential regulation of the states of the three-species generalized Lotka-Volterra biological system. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

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