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Adaptive Control of a Chemical Chaotic Reactor

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Abstract: Chaos in nonlinear dynamics occurs widely in physics, chemistry, biology, ecology, secure communications, cryptosystems and many other scientific disciplines. Chaotic systems have many important applications in science and engineering. This paper derives new results for the analysis and adaptive control of a chemical chaotic attractor discovered by Haung (2005). This paper starts with a detailed description of the chemical reactor dynamics and the parameter values for which the chemical reactor exhibits chaotic behaviour. Next, adaptive control law is devised for the global chaos control of the chemical chaotic reactor with unknown parameters. The main results for adaptive control of the chemical chaotic attractor are established using Lyapunov stability theory. Next, the main results are illustrated with numerical simulations using MATLAB.
Keywords: Chaos, chaotic systems, chemical reactor, adaptive control.

Introduction

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. For the motion of a dynamical system to be chaotic, the system variables should contain some nonlinear terms and the system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963[3]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which is algebraically much simpler than the Lorenz system. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years such as Sundarapandian systems [11, 12], Vaidyanathan systems [13-20], Pehlivan system [21], Jafari system[22],Pham system [23], etc.

Chaos theory has very useful applications in many fields of science and engineering such as oscillators[24], lasers [25-26],biology [27], chemical reactions [28-30], neural networks[31-32],robotics [33-34], electrical circuits [35-36], etc.

This paper investigates the analysis and adaptive control of the chemical chaotic reactor model discovered by Haung in 2005 [37]. Haung derived the chemical reactor model by considering reactor dynamics with five reversible steps. For the non-dimensionalized dynamical evolution equations of the Haung's chaotic reactor, the Lyapunov exponents have been obtained as $L_1 = 0.4001, L_2 = 0$ and $L_3 = -11.8762$. The presence of a positive Lyapunov exponent indicates the chaos in the Haung chemical reactor [37].

This paper also derives new results of adaptive controller design for the chemical chaotic attractor using Lyapunov stability theory [38] and MATLAB plots are shown to illustrate the main results. Adaptive control method is a feedback control strategy which is very effective because it uses estimates of the unknown parameters of the system [39-42].

Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics [37] consist of the following five reversible steps given below.

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2X,\ X + Y \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} 2Y,\ A_5 + Y \underset{k_{-3}}{\overset{k_3}{\rightleftharpoons}} A_2,\ X + Z \underset{k_{-4}}{\overset{k_4}{\rightleftharpoons}} A_3,\ A_5 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftharpoons}} 2Z \qquad (1)$$

In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z. The corresponding non-dimensionalized dynamical evolution equations read as

$$\begin{cases} \dot{x} &= (a_1 - k_{-1}x - y - z)x \\ \dot{y} &= (x - a_5)y \\ \dot{z} &= (a_4 - x - k_{-5}z)z \end{cases} \qquad (2)$$

In (2), x, y, z are positive mole functions and a_1, a_4, a_5, k_{-1} and k_{-5} are positive parameters.

To simplify the notations, we rename the constants and express the system (2) as

$$\begin{cases} \dot{x} &= (a - px - y - z)x \\ \dot{y} &= (x - c)y \\ \dot{z} &= (b - x - qz)z \end{cases} \qquad (3)$$

The system (3) is chaotic when the system parameters are chosen as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \qquad (4)$$

For numerical simulations, we take the initial conditions $x(0) = 0.8, y(0) = 0.5,$ and $z(0) = 0.8.$

The 3-D phase portrait of the chemical chaotic reactor is depicted in Fig. 1.

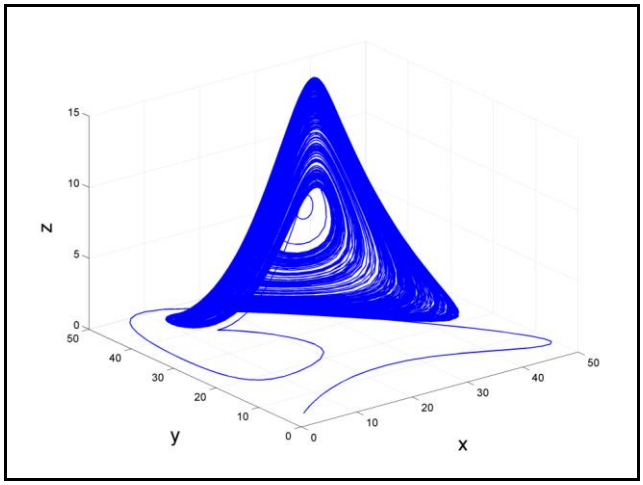

Figure 1.The3-D phase portrait of the chemical chaotic reactor

Computational Analysis of the Chemical Chaotic Attractor

The Lyapunov exponents of the chemical chaotic attractor (3) have been obtained in MATLAB as

$$L_1 = 0.4001,\ L_2 = 0, L_3 = -11.8762. \qquad (5)$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \qquad (6)$$

The chemical chaotic attractor has an equilibrium at $(x, y, z) = (0,0,0)$.

The eigenvalues of the linearized system matrix of the attractor (3) at the origin are:

$$\lambda_1 = 16.5,\ \lambda_2 = 30, \lambda_3 = -10. \tag{7}$$

Since there are two positive eigenvalues in the set (7), the origin is an unstable equilibrium of the chemical chaotic attractor (3).

Adaptive Control Design of the Chemical Chaotic Attractor

In this section, we use adaptive control method to design an adaptive feedback control law for globally stabilizing the chemical chaotic reactor with unknown parameters.

Thus, we consider the controlled chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x} &= (a - px - y - z)x + u_x \\ \dot{y} &= (x - c)y + u_y \\ \dot{z} &= (b - x - qz)z + u_z \end{cases} \tag{8}$$

In (8), x, y, z are the states and u_x, u_y, u_z are adaptive controls to be determined using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ of the unknown parameters a, b, c, p, q, respectively.

We consider the adaptive control law defined by

$$\begin{cases} u_x &= -(\hat{a}(t) - \hat{p}(t)x - y - z)x - k_x x \\ u_y &= -(x - \hat{c}(t))y - k_y y \\ u_z &= -\left(\hat{b}(t) - x - \hat{q}(t)z\right)z - k_z z \end{cases} \tag{9}$$

In (9), $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ are estimates of the unknown parameters a, b, c, p, q, respectively, and k_x, k_y, k_z are positive gain constants.

Substituting (9) into (8), we obtain the closed-loop dynamical system

$$\begin{cases} \dot{x} &= [a - \hat{a}(t)]x - [p - \hat{p}(t)]x^2 - k_x x \\ \dot{y} &= -[c - \hat{c}(t)]y - k_y y \\ \dot{z} &= \left[b - \hat{b}(t)\right]z - [q - \hat{q}(t)]z^2 - k_z z \end{cases} \tag{10}$$

Now, we define the parameter estimation errors as

$$\begin{cases} e_a &= a - \hat{a}(t) \\ e_b &= b - \hat{b}(t) \\ e_c &= c - \hat{c}(t) \\ e_p &= p - \hat{p}(t) \\ e_q &= q - \hat{q}(t) \end{cases} \tag{11}$$

Using (11), we can simplify the closed-loop system (10) as

$$\begin{cases} \dot{x} &= e_a x - e_p x^2 - k_x x \\ \dot{y} &= -e_c y - k_y y \\ \dot{z} &= e_b z - e_q z^2 - k_z z \end{cases} \tag{12}$$

Differentiating (11) with respect to t, we get

$$\begin{cases} \dot{e}_a &= -\dot{\hat{a}}(t) \\ \dot{e}_b &= -\dot{\hat{b}}(t) \\ \dot{e}_c &= -\dot{\hat{c}}(t) \\ \dot{e}_p &= -\dot{\hat{p}}(t) \\ \dot{e}_q &= -\dot{\hat{q}}(t) \end{cases} \tag{13}$$

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(x^2 + y^2 + z^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{14}$$

Clearly, V is a positive definite function on R^8.

Differentiating V along the trajectories of (10) and (13), we obtain

$$\dot{V} = -k_x x^2 - k_y y^2 - k_z z^2 + e_a\left[x^2 - \dot{\hat{a}}(t)\right] + e_b\left[z^2 - \dot{\hat{b}}(t)\right] + e_c\left[-y^2 - \dot{\hat{c}}(t)\right] + e_p\left[-x^3 - \dot{\hat{p}}(t)\right] + e_q\left[-z^3 - \dot{\hat{q}}(t)\right] \quad (15)$$

In view of (15), we take the parameter update law as follows.

$$\begin{cases} \dot{\hat{a}}(t) = x^2 \\ \dot{\hat{b}}(t) = z^2 \\ \dot{\hat{c}}(t) = -y^2 \\ \dot{\hat{p}}(t) = -x^3 \\ \dot{\hat{q}}(t) = -z^3 \end{cases} \quad (16)$$

Theorem 1. The chemical chaotic attractor (8) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (9) and the parameter update law (16), where k_x, k_y, k_z are positive gain constants.

Proof. We prove this result by Lyapunov stability theory. We consider the quadratic Lyapunov function V defined in (14), which is positive definite on $\boldsymbol{R^8}$.

Substituting the parameter update law (16) into (15), we obtain

$$\dot{V} = -k_x x^2 - k_y y^2 - k_z z^2 \quad (17)$$

By (17), it follows that $\dot{V}$ is a negative semi-definite function on $\boldsymbol{R^8}$.

By Barbalat's lemma in Lyapunov stability theory, it follows that the states $x(t), y(t), z(t)$ exponentially converge to zero as $t \to \infty$ for all initial conditions.This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (8) and (16), when the adaptive control law (9) is applied.

We take the gain constants as $k_x = 20, k_y = 20, k_z = 20.$ We take the initial conditions of the chemical reactor (8) as $x(0) = 3.2, y(0) = 1.3, z(0) = 2.7.$ The parameter values are taken as in (4) for the chaotic case.

Also, we take $\hat{a}(0) = 2.1, \quad \hat{b}(0) = 0.3, \quad \hat{c}(0) = 3.1, \quad \hat{p}(0) = 1.6, \quad \hat{q}(0) = 2.3.$

Fig. 2 shows the time-history of the exponential convergence of the controlled states $x(t), y(t), z(t).$

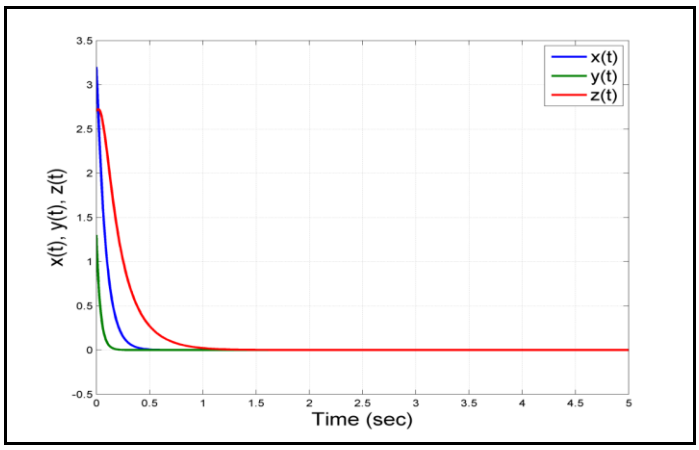

Figure2.Time-history of the controlled states of the chemical chaotic reactor

Conclusions

In this paper, new results have been derived for the analysis and adaptive control of a chemical chaotic attractor discovered by Haung (2005). After analyzing the qualitative properties of the chemical chaotic attractor discovered by Haung, we have designed an adaptive controller for the global exponential stabilization of the states of the chemical chaotic reactor. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

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