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Adaptive Backstepping Control of Enzymes-Substrates System with Ferroelectric Behaviour in Brain Waves

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, INDIA

Abstract: Recent research has shown that many physical, chemical and biological nonlinear dynamical systems can exhibit an apparently unpredictable and chaotic manner. This realization has broad implications and applications in many fields of science such asphysics, chemistry, biology, ecology, secure communications, cryptosystems, etc. This paper investigates research in the dynamic and chaotic analysis of enzymes-substrate reactions system with ferroelectric behaviour in brain waves which was studied by Enjieu Kadji, Chabi Orou, Yamapi and Woafo (2007). The enzymes-substrates system is a 2-D non-autonomous system with a cosinusoidal forcing term. This paper displays the phase portraits of the 2-D enzymes-substrates system when the system undergoes chaotic behaviour. Next, this paper derives adaptive backstepping control for globally stabilizing the trajectories of the enzyme-substrates system with unknown parameters. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the enzyme-substrates system.
Keywords: Chaos, chaotic systems, enzymes-substrate reactions, biology, backstepping control, etc.

Introduction

Chaos theory describes the qualitative study of deterministicchaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963[3]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which is algebraically much simpler than the Lorenz system. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-28], Pehlivan system [29], Pham system [30], etc.

Coherent oscillations in biological systems are studied by Frohlich [31] and the following suggestions were made which are taken as a physical basis for theoretical investigation of enzymatic substrate reaction with ferroelectric behaviour in brain waves model [32].

1. When metabolic energy is available, long-wavelength electric vibrations are very strongly and coherently excited in active biological system.
2. Biological systems have metastable states with a very high electric polarization.

These long range interactions may lead to a selective transport of enzymes, and hence specific chemical reactions may become possible. From Frohlich ideas [31], it may be supposed that in large regions of the system of proteins, substrates, ions and structured water are activated by the chemical energy available from substrate enzyme reactions. After a deep study, Enjieu Kadji, Chabi Orou, Yamapi and Woafo (2007) derived enzymes-substrates reactions system with ferroelectric behaviour in brain waves [33]. Specifically, chaotic behaviour was noted for the 2-D enzyme-substrate reactions system, which is further investigated in this research work.

This paper discusses the chaotic properties of the enzyme-substrates reactions system, and MATLAB plots are shown for the phase portraits of the chaotic system. This paper also derives new results of adaptive backstepping controller design for the enzymes-substrate system using Lyapunov stability theory [34] and MATLAB plots are shown to illustrate the main results. Adaptive control method is a feedback control strategy which is very effective because it uses estimates of the unknown parameters of the system [35-40].The backsteppingcontrol method is a recursive procedure that links the choice of a Lyapunov functionwith the design of a controller and guarantees global asymptotic stability of the strict feedbacksystems [41-45].

Enzymes-Substrates Reaction System

Enjieu Kadji, Chabi Orou, Yamapi and Woafo derived enzyme-substrate reactions system with ferroelectric behaviour in brain waves [33], which is given by the differential equation

$$\ddot{x} - \mu(1 - x^2 + ax^4 - bx^6)\dot{x} + x = E\cos(\Omega t) \quad (1)$$

In (1), a, b are positive parameters, μ is the parameter of nonlinearity, while E and Ω are the amplitude and the frequency of the external cosinusoidal excitation, respectively.

The enzymes-substrates reaction system (1) can be compactly put in system form as

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= \mu(1 - x^2 + ax^4 - bx^6)y - x + E\cos(\Omega t) \end{cases} \quad (2)$$

For the external excitation, we take the constants as

$$E = 8.27, \ \ \Omega = 3.465 \quad (3)$$

The biological system (2) is chaotic when the system parameters are chosen as

$$a = 2.55, \ \ b = 1.70, \ \ \mu = 2.001 \quad (4)$$

For numerical simulations, we take the initial conditions $x(0) = 0.1$and $y(0) = 0.1$.

The 2-D phase portrait of the enzymes-substrates biological reaction system is depicted in Fig. 1.

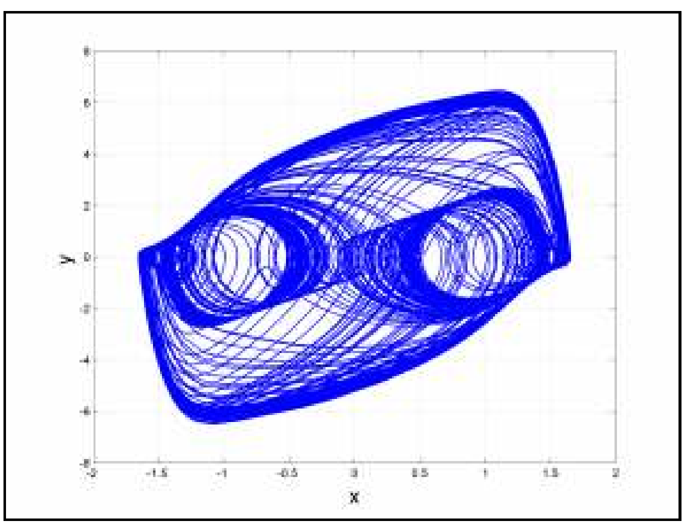

Figure1.The2-D phase portrait of the enzymes-substrates biological reaction system

Adaptive Backstepping Control of the Enzymes-Substrates Reaction System

In this section, we design an adaptive backstepping feedback control law for globally stabilizing the enzymes-substrates reaction system with unknown parameters$a$, and b. It is supposed that the constants E and Ω

associated with the external excitation $f(t) - E\cos(\Omega t)$ are maintained at the constant values given in equation (3). It is also supposed that the nonlinear parameter μ is maintained at the constant value given in equation (4).

Thus, we consider the controlled enzymes-substrates reaction system given by the dynamics

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= \mu\left(1 - x^2 + ax^4 - bx^6\right)y - x + E\cos(\Omega t) + u \end{cases} \tag{5}$$

In (5), $x, y,$ are the states and u is the adaptive backstepping control to be found using estimates $\hat{a}(t), \hat{b}(t)$ of the unknown parameters $a, b,$ respectively.

Now, we define the parameter estimation errors as

$$\begin{cases} e_a &= a - \hat{a}(t) \\ e_b &= b - \hat{b}(t) \end{cases} \tag{6}$$

Differentiating (6) with respect to t, we get

$$\begin{cases} \dot{e}_a &= -\dot{\hat{a}} \\ \dot{e}_b &= -\dot{\hat{b}} \end{cases} \tag{7}$$

Next, we shall state and prove the main result of this section.

Theorem 1. The enzymes-substrates reaction system (5) with unknown system parameters a and b is globally and exponentially stabilized for all initial conditions by the adaptive control law

$$u = -x - 2y - \mu\,(1 - x^2)y - \hat{a}(t)\mu x^4 y + \hat{b}(t)\mu x^6 y - E\cos(\Omega t) - kz_2 \tag{8}$$

where $k > 0$ is a gain constant,

$$z_2 = x + y, \tag{9}$$

and the update law for the parameter updates $\hat{a}(t), \hat{b}(t)$ is given by

$$\begin{cases} \dot{\hat{a}} &= \mu z_2 x^4 y \\ \dot{\hat{b}} &= -\mu z_2 x^6 y \end{cases} \tag{10}$$

Proof. We prove this result by applying backstepping control and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2, \tag{11}$$

where

$$z_1 = x \tag{12}$$

Differentiating V_1 along the dynamics (5), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = -z_1^2 + z_1(x + y) \tag{13}$$

Now, we define

$$z_2 = x + y \tag{14}$$

Substituting (14) into (13), we obtain

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{15}$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2, e_a, e_b) = V_1(z_1) + \frac{1}{2} z_2^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2, \tag{16}$$

which is positive definite on $\mathbf{R}^4$.

Differentiating (16) along the dynamics (5) and (7), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} \tag{17}$$

where

$$S = z_1 + z_2 + \dot{z}_2 = x + 2y + \mu(1 - x^2)y + a\mu x^4 y - b\mu x^6 y + E\cos(\Omega t) + u \tag{18}$$

Substituting the feedback control law (8) into (18), we obtain

$$S = [a - \hat{a}(t)]\mu x^4 y - [b - \hat{b}(t)]\mu x^6 y - kz_2 \tag{19}$$

Using (6), the equation (19) can be simplified as

$$S = e_a \mu x^4 y - e_b \mu x^6 y - kz_2 \tag{20}$$

Substituting the value of S from (20) into (17), we get

$$\dot{V}_2 = -z_1^2 - (1+k)z_2^2 + e_a\left[\mu z_2 x^4 y - \dot{\hat{a}}\right] + e_b\left[-\mu z_2 x^6 y - \dot{\hat{b}}\right] \quad (21)$$

Substituting the parameter update law (10) into (21), we get

$$\dot{V}_2 = -z_1^2 - (1+k)z_2^2 \quad (22)$$

Which is a negative semi-definite function on $\boldsymbol{R}^4$.

By Barbalat's lemma in Lyapunov stability theory [34], it follows that the states $x(t), y(t)$ exponentially converge to zero as $t \to \infty$ for all initial conditions. This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$for solving the systems of differential equations given by (5) and (10), when the backstepping control law (8) is applied.

We take the gain constant as $k = 8$. We take the initial conditions of the enzymes-substrates reaction system (5) as $x(0) = 2.4, y(0) = 0.5$. The parameter values are taken as in (3) and (4) for the chaotic case.

Also, we take $\hat{a}(0) = 5.8$ and $\hat{b}(0) = 4.3$.

Fig. 2 shows the time-history of the exponential convergence of the controlled states $x(t), y(t)$.

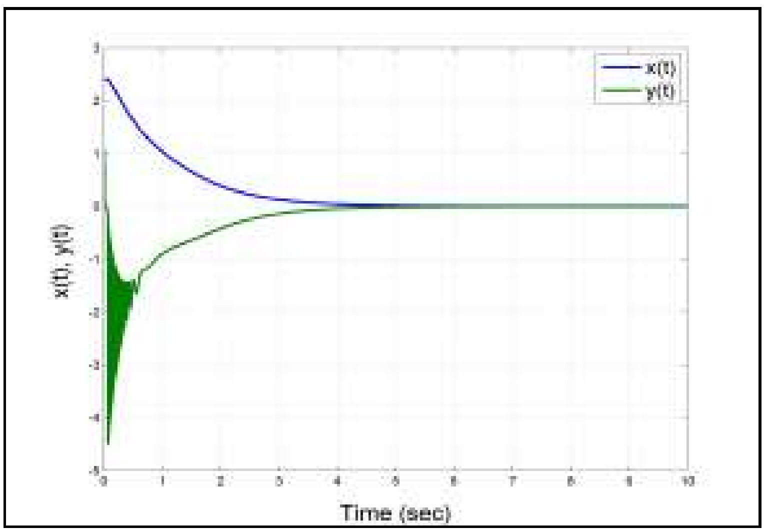

Figure2. Time-history of the controlled states of the enzymes-substrates reaction system

Conclusions

In this paper, new results have been derived for the analysis and adaptive control of enzymes-substrates reaction with ferroelectric behaviour in brain waves discovered by Enjieu Kadji, Chabi Orou, Yamapi and Woafo (2007). After a description and dynamic analysis of the chaotic 2-D non-autonomous attractor describing the enzymes-substrates reaction systems, we have designed an adaptive backstepping feedback controller for the global exponential stabilization of the states of the enzymes-substrates reaction system. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

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