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Anti-Synchronization of Rikitake Two-Disk Dynamo Chaotic Systems via Adaptive Control Method

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. The frequent and irregular reversals of the earth's magnetic field has motivated a number of research studies involving electrical currents within the earth's molten core. One of the first such nonlinear models that exhibited the frequent and irregular reversals of the earth's magnetic field was Rikitake's two-disk dynamo system (1958). Rikitake two-disk dynamo system is a chaotic system that predated the pioneering work of Lorenz (1963). In this paper, we describe the dynamic equations and qualitative properties of the Rikitae two-disk dynamo chaotic system. We also derive new results for the global anti-synchronization of the Rikitake two-disk dynamo chaotic systems. MATLAB plots have been depicted to illustrate the phase portraits of the Rikitake two-disk dynamo chaotic attractor and the global anti-synchronization of the Rikitake two-disk dynamo chaotic systems via adaptive control method.
Keywords: Chaos, chaotic systems, anti-synchronization, earth's magnetic field, electrical currents, Rikitake dynamo system, two-disk model, nonlinear model, adaptive control, stability.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal of anti-synchronization is to device a feedback mechanism so that the state trajectories of the master and slave systems are equal in magnitude and opposite in sign asymptotically. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the anti-synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [46-

65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

The frequent and irregular reversals of the earth's magnetic field has motivated a number of research studies involving electrical currents within the earth's molten core. One of the first such nonlinear models that exhibited the frequent and irregular reversals of the earth's magnetic field was Rikitake two-disk dynamo system [143]. Rikitake two-disk dynamo system (1958) is a classical chaotic system that predated the pioneering work of Lorenz (1963).

First, this research paper details the dynamic equations of the Rikitake two-disk dynamo system [143] and discusses its qualitative properties.

This paper also derives new results for the global anti-synchronization of the Rikitake two-disk dynamo system via adaptive control method. MATLAB simulation plots are shown to depict the phase portraits and global anti-synchronization of the Rikitake two-disk dynamo systems.

2. Rikitake Two-Disk Dynamo Chaotic System

Rikitake two-disk dynamo chaotic system [143] is governed by the system model

$$\begin{cases} \dot{x}_1 = -ax_1 + x_2 x_3 \\ \dot{x}_2 = -ax_2 + x_1 (x_3 - b) \\ \dot{x}_3 = 1 - x_1 x_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b are constant positive parameters. The parameter a represents the resistive dissipation and the parameter b represents the difference in the angular velocities of the two disks.

We note that the Rikitake two-disk dynamo chaotic system (1) has the same number of terms as the Lorenz chaotic system, but with one additional nonlinearity.

The Rikitake two-disk dynamo system (1) is *chaotic* when the system parameters are chosen as

$$a = 1, \quad b = 1 \tag{2}$$

For numerical simulations, we take the initial conditions

$$x_1(0) = 1.0, \quad x_2(0) = 0, \quad x_3(0) = 0.8 \tag{3}$$

Figure 1 shows the 3-D phase portrait of the Rikitake two-disk dynamo system (1). Figures 2-4 show the 2-D projections of the Rikitake two-disk dynamo system on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, respectively.

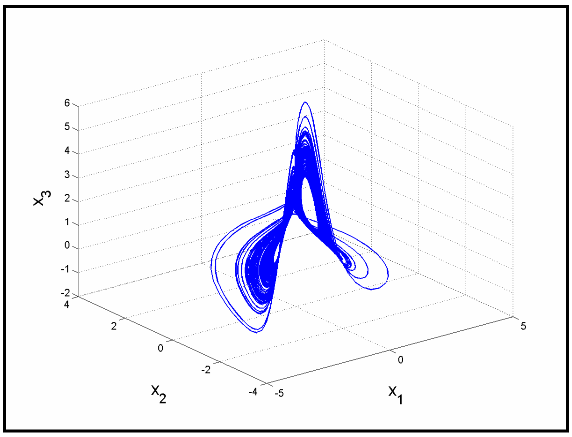

Figure 1. The 3-D phase portrait of the Rikitake two-disk dynamo system

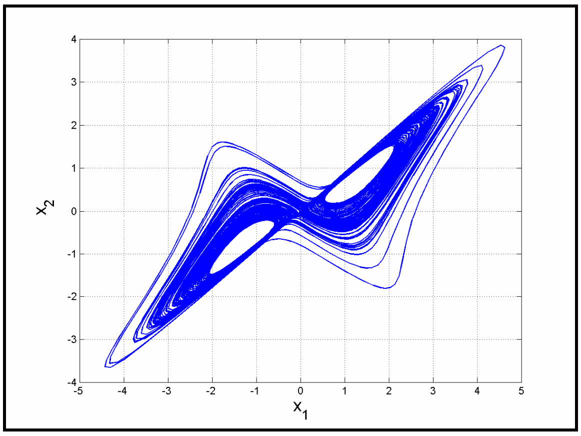

Figure 2. The 2-D projection of the Rikitake two-disk dynamo system on the (x_1, x_2) plane

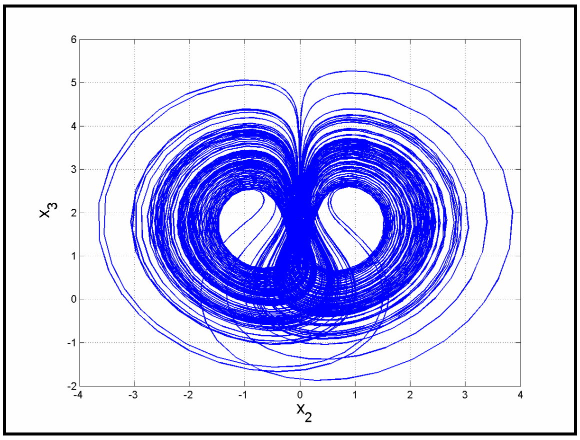

Figure 3. The 2-D projection of the Rikitake two-disk dynamo system on the (x_2, x_3) plane

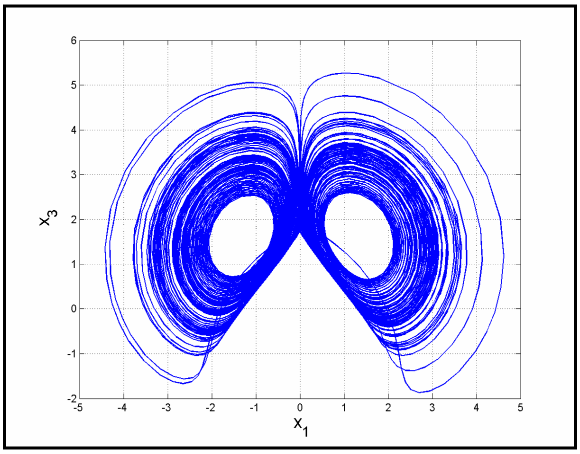

Figure 4. The 2-D projection of the Rikitake two-disk dynamo system on the (x_1, x_3) plane

The Lyapunov exponents of the Rikitake two-disk dynamo system (1) are numerically found as

$$L_1 = 0.12749, \quad L_2 \quad 0, \quad L_3 \quad -2.12704 \tag{4}$$

From the LE spectrum (4), it is immediate that the Rikitake two-disk dynamo system (1) is a chaotic system and the Maximal Lyapunov Exponent (MLE) of the Rikitake dynamo system (1) is $L_1 = 0.12749$.

Since the sum of the Lyapunov exponents in (4) is negative, it follows that the Rikitake two-disk dynamo system (1) is dissipative.

Also, the Lyapunov dimension of the Rikitake two-disk dynamo system (1) is derived as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0599 \tag{5}$$

3. Global Anti-Synchronization of the Rikitake Two-Disk Dynamo Chaotic Systems via Adaptive Control

In this section, we use adaptive control method to achieve global anti-synchronization of the identical states of the Rikitake two-disk dynamo chaotic systems with unknown parameters. We use Lyapunov stability theory [144] to prove the main adaptive control result derived in this section.

As the master system, we consider the Rikitake two-disk dynamo dynamics given by

$$\begin{cases} \dot{x}_1 = -ax_1 + x_2 x_3 \\ \dot{x}_2 = -ax_2 + x_1(x_3 - b) \\ \dot{x}_3 = 1 - x_1 x_2 \end{cases} \tag{6}$$

In ((6), x_1, x_2, x_3 are the states of the Rikitake two-disk dynamo system and a, b are unknown parameters.
As the slave system, we consider the controlled Rikitake two-disk dynamo dynamics given by

$$\begin{cases} \dot{y}_1 = -ay_1 + y_2 y_3 + u_1 \\ \dot{y}_2 = -ay_2 + y_1(y_3 - b) + u_2 \\ \dot{y}_3 = 1 - y_1 y_2 + u_3 \end{cases} \tag{7}$$

In (7), y_1, y_2, y_3 are the states of the controlled Rikitake two-disk dynamo system.
The anti-synchronization errors are defined by

$$\begin{cases} e_1 = y_1 + x_1 \\ e_2 = y_2 + x_2 \\ e_3 = y_3 + x_3 \end{cases} \tag{8}$$

The anti-synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = -ae_1 + y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 = -be_1 - ae_2 + y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 = 2 - y_1 y_2 - x_1 x_2 + u_3 \end{cases} \tag{9}$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = \hat{a}(t)e_1 - y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 = \hat{b}(t)e_1 + \hat{a}(t)e_2 - y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 = -2 + y_1 y_2 + x_1 x_2 - k_3 e_3 \end{cases} \tag{10}$$

where k_1, k_2, k_3 are positive gain constants.
Substituting (10) into (9), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = -[a - \hat{a}(t)]e_1 - k_1 e_1 \\ \dot{e}_2 = -[b - \hat{b}(t)]e_1 - [a - \hat{a}(t)]e_2 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{11}$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{12}$$

Using (12), the closed-loop system (11) can be simplified as

$$\begin{cases} \dot{e}_1 = -e_a e_1 - k_1 e_1 \\ \dot{e}_2 = -e_b e_1 - e_a e_2 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{13}$$

Differentiating (12) with respect to time, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{14}$$

Next, we consider the candidate Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \tag{15}$$

Differentiating (15) along the trajectories of (13) and (14), we get the following dynamics

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[-e_1^2 - e_2^2 - \dot{\hat{a}}\right] + e_b\left[-e_1 e_2 - \dot{\hat{b}}\right] \tag{16}$$

In view of (15), we take the following parameter update law:

$$\begin{cases} \dot{\hat{a}}(t) = -e_1^2 - e_2^2 \\ \dot{\hat{b}}(t) = -e_1 e_2 \end{cases} \tag{17}$$

Next, we state and prove the main result of this section.

Theorem 1. The adaptive control law (10) and the parameter update law (17) achieve global and exponential anti-synchronization of the identical 3-D Rikitake two-disk dynamo chaotic systems defined by (6) and (7), where k_1, k_2, k_3 are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [144].

The quadratic Lyapunov function V defined by (15) is positive definite on R^5.

Substituting the parameter update law (17) into (16), we get the time derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \tag{18}$$

which is negative semi-definite on R^5.

Thus, by Barbalat's lemma in Lyapunov stability theory [144], it follows that the closed-loop error dynamics (13) is globally exponentially stable.

Hence, it is immediate that the identical 3-D Rikitake two-disk dynamo chaotic systems (6) and (7) are globally and exponentially anti-synchronized.

This completes the proof. ■

4. Numerical Simulations

We use the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ to solve the system of differential equations (6) and (7), when the adaptive control law (10) is implemented.

We take the parameter values of the Rikitake two-disk dynamo chaotic systems as in the chaotic case, viz.

$$a = 1, \quad b = 1 \tag{19}$$

We take the gain constants as

$$k_1 = 8, \quad k_2 = 8, \quad k_3 = 8 \tag{20}$$

We take the initial values of the Rikitake dynamo system (6) as

$$x_1(0) = 21.2, \quad x_2(0) = 5.6, \quad x_3(0) = 12.3 \tag{21}$$

We take the initial values of the Rikitake dynamo system (7) as

$$y_1(0) = 9.7, \quad y_2(0) = 2.5, \quad y_3(0) = 4.2 \tag{22}$$

We take the initial values of the parameter estimates as

$$\hat{a}(0) = 7.4, \quad \hat{b}(0) = 5.3 \tag{23}$$

Figures 5-7 show the anti-synchronization of the Rikitake dynamo chaotic systems (6) and (7).

Figure 8 shows the time-history of the anti-synchronization errors e_1, e_2, e_3.

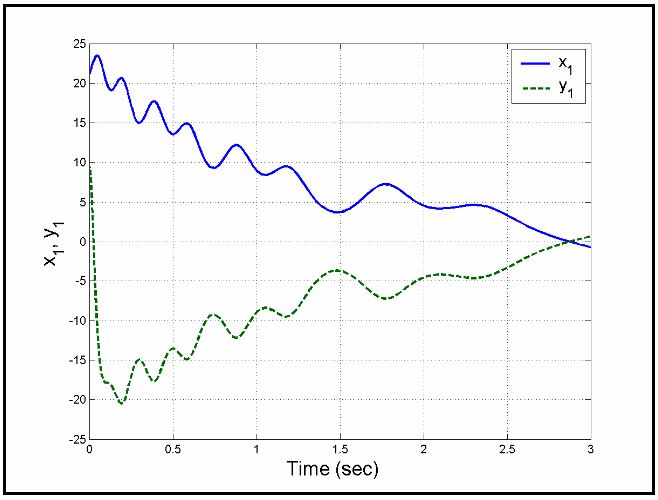

Figure 5. Anti-synchronization of the states $x_1(t)$ and $y_1(t)$

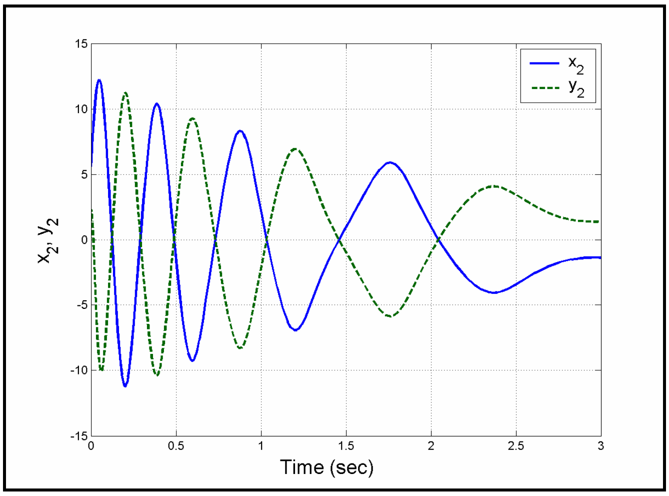

Figure 6. Anti-synchronization of the states $x_2(t)$ and $y_2(t)$

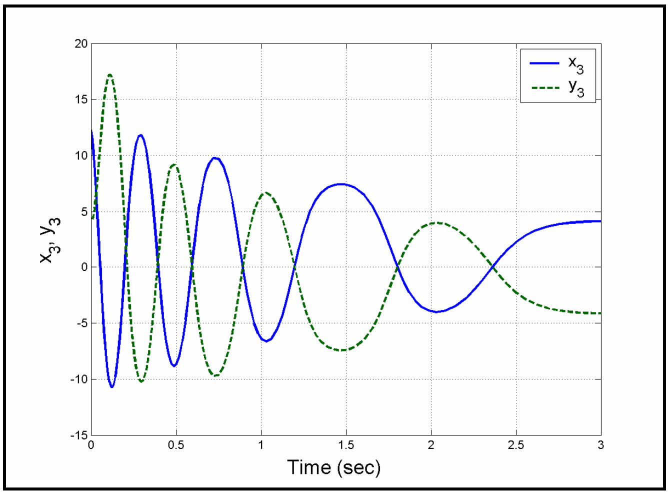

Figure 7. Anti-synchronization of the states $x_3(t)$ and $y_3(t)$

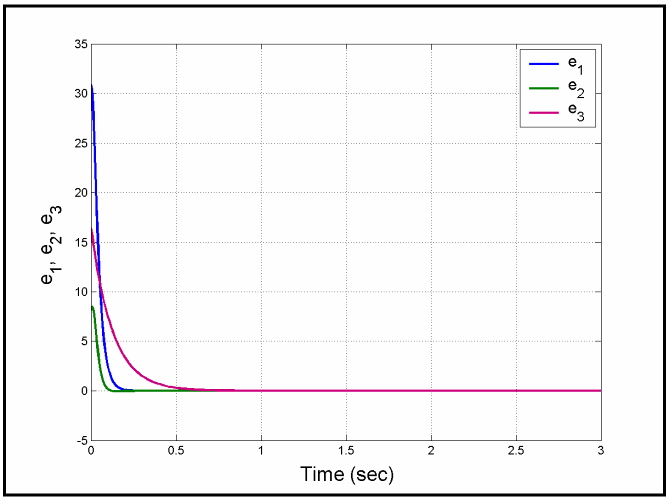

Figure 8. Time-history of the anti-synchronization errors $e_1(t), e_2(t), e_3(t)$

5. Conclusions

In this paper, we described the dynamic equations and qualitative properties of the Rikitae two-disk dynamo chaotic system. We also derived new results for the global anti-synchronization of the Rikitake two-disk dynamo chaotic systems. MATLAB plots were depicted to illustrate the phase portraits of the Rikitake two-disk dynamo chaotic attractor and the global anti-synchronization of the Rikitake two-disk dynamo chaotic systems via adaptive control method.

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