International Journal of ChemTech Research
CODEN (USA): IJCRGG ISSN: 0974-4290
Vol.8, No.8, pp 73-85, 2015

Anti-Synchronization of Chemical Chaotic Reactors via Adaptive Control Method

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. This paper details the qualitative properties of a chemical chaotic attractor discovered by Huang (2005). This paper gives a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, new results are obtained for the anti-synchronization of identical chemical chaotic reactors with unknown system parameters using adaptive control method. MATLAB plots have been shown to illustrate the phase portraits of the chemical chaotic attractor and the anti-synchronization of identical chemical chaotic attractors.
Keywords: Chaos, chaotic systems, anti-synchronization, chemical reactor, sliding mode control, stability.

1. Introduction

Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal of anti-synchronization is to device a feedback mechanism so that the trajectories of the master and slave systems are asymptotically equal in magnitude but opposite in sign. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the anti-synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-107], biology [108-125], memristors [126-128], electrical circuits [129], etc.

This paper investigates first the qualitative properties of a chemical chaotic reactor model discovered by Huang in 2005 [130]. Huang derived the chemical reactor model by considering reactor dynamics with five steps (2 reversible and 3 non-reversible). This paper also derives new results for the anti-synchronization of chemical chaotic attractors with unknown system parameters using Lyapunov stability theory. MATLAB plots are shown to illustrate the phase portraits and anti-synchronization of the chemical chaotic reactor.

2. Huang's Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics of Huang and Yang [130] consist of the following five steps given below.

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2X \tag{1a}$$

$$X + Y \xrightarrow{k_2} 2Y \tag{1b}$$

$$A_5 + Y \xrightarrow{k_3} A_2 \tag{1c}$$

$$X + Z \xrightarrow{k_4} A_3 \tag{1d}$$

$$A_4 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftarrows}} 2Z \tag{1e}$$

Equations (1a) and (1e) indicate reversible steps, while equations (1b), (1c) and (1d) indicate non-reversible steps of the Huang chemical reactor [130]. In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z.

Assuming an ideal mixture and a well-stirred reactor, the macroscopic rate equations for the Huang's chemical reactor can be written in non-dimensionalized form as

$$\begin{cases} \dot{x} = a_1 x - k_{-1} x^2 - xy - xz \\ \dot{y} = xy - a_5 y \\ \dot{z} = a_4 z - xz - k_{-5} z^2 \end{cases} \tag{2}$$

In (2), x, y, z are the mole fractions of X, Y and Z. Also, the rate constants k_1, k_3 and k_5 are incorporated in the parameters a_1, a_4 and a_5.

To simplify the notations, we rename the constants and express the chemical reactor system (2) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \tag{3}$$

The system (3) is chaotic when the system parameters are chosen as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \tag{4}$$

For numerical simulations, we take the initial conditions

$$x(0) = 1.8, \quad y(0) = 2.5, \quad z(0) = 0.6 \tag{5}$$

The 3-D phase portrait of the chemical chaotic reactor is depicted in Figure 1.

The 2-D projections of the chemical chaotic reactor on the (x, y), (y, z) and (x, z) planes are depicted in Figures 2-4.

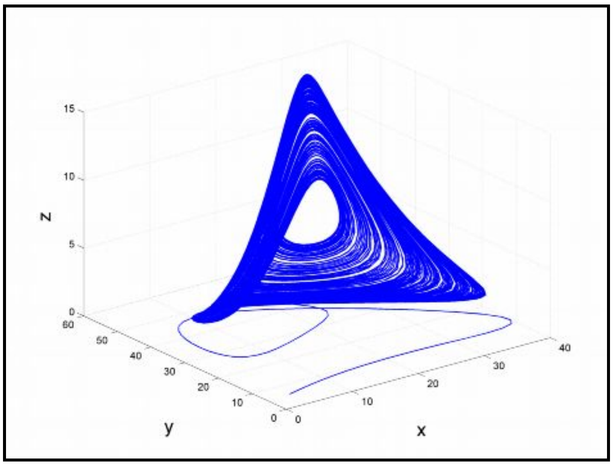

Figure 1. The 3-D phase portrait of the chemical chaotic reactor

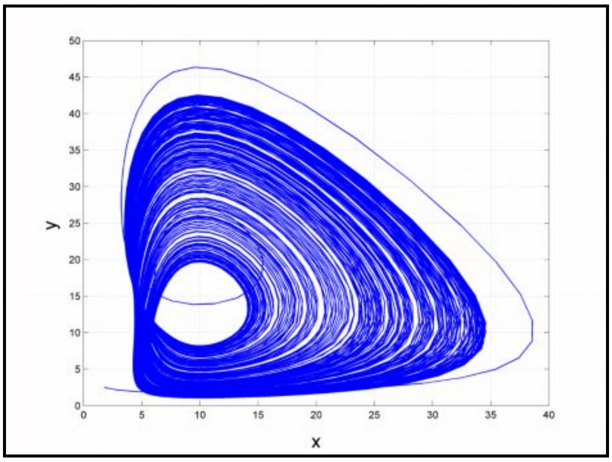

Figure 2. The 2-D projection of the chemical chaotic attractor on the (x, y) plane

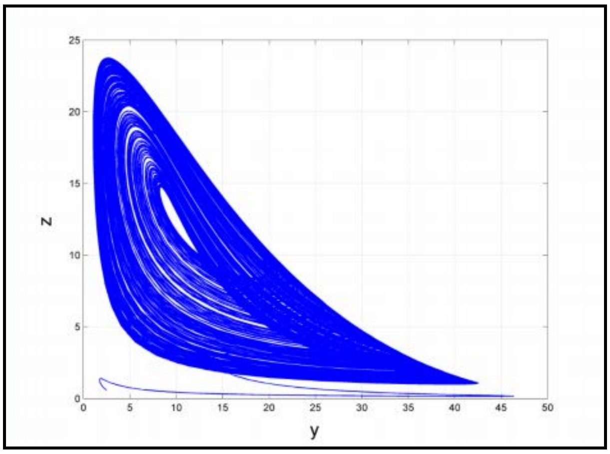

Figure 3. The 2-D projection of the chemical chaotic attractor on the (y, z) plane

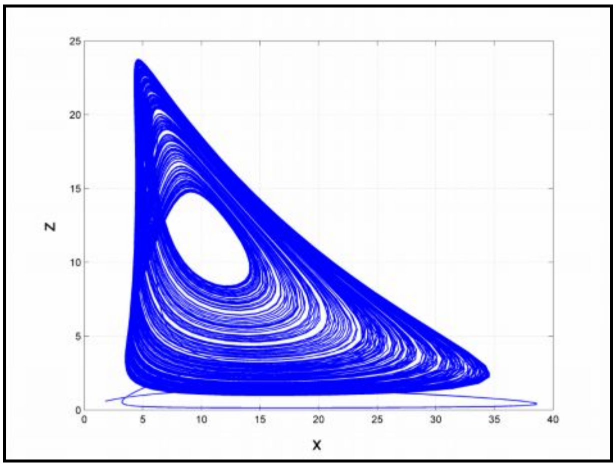

Figure 4. The 2-D projection of the chemical chaotic attractor on the (x, z) plane

3. Computational Analysis of the Chemical Chaotic Attractor

The Lyapunov exponents of the Huang's chemical chaotic attractor (3) are derived in MATLAB as

$$L_1 = 0.4001,\ L_2 = 0,\ L_3 = -11.8762 \tag{6}$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \tag{7}$$

The chemical chaotic attractor has an equilibrium at $(x, y, z) = (0, 0, 0)$.

The eigenvalues of the linearized system matrix of the chemical reactor (3) at the origin are:

$$\lambda_1 = 16.5,\ \lambda_2 = 30,\ \lambda_3 = -10 \tag{8}$$

Thus, the origin is a saddle-point equilibrium, which is unstable.

4. Anti-Synchronization of Chemical Chaotic Attractors via Adaptive Control Method

In this section, we use adaptive control method to design an adaptive feedback synchronizer for achieving global anti-synchronization of the trajectories of identical chemical chaotic reactors with unknown parameters.

Thus, we consider the master system as the chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x}_1 = (a - px_1 - y_1 - z_1)x_1 \\ \dot{y}_1 = (x_1 - c)y_1 \\ \dot{z}_1 = (b - x_1 - qz_1)z_1 \end{cases} \tag{9}$$

In (9), x_1, y_1, z_1 are the states of the master system.

Also, we consider the slave system as the chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x}_2 = (a - px_2 - y_2 - z_2)x_2 + u_x \\ \dot{y}_2 = (x_2 - c)y_2 + u_y \\ \dot{z}_2 = (b - x_2 - qz_2)z_2 + u_z \end{cases} \tag{10}$$

In (10), x_2, y_2, z_2 are the states of the slave system and u_x, u_y, u_z are adaptive controls to be determined using estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{p}(t),\ \hat{q}(t),$ of the unknown parameters $a, b, c, p, q,$ respectively.

The anti-synchronization error between the chemical chaotic attractors is defined by

$$\begin{cases} e_x = x_2 + x_1 \\ e_y = y_2 + y_1 \\ e_z = z_2 + z_1 \end{cases} \tag{11}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x = ae_x - p(x_2^2 + x_1^2) - x_2 y_2 - x_2 z_2 - x_1 y_1 - x_1 z_1 + u_x \\ \dot{e}_y = -ce_y + x_2 y_2 + x_1 y_1 + u_y \\ \dot{e}_z = be_z - q(z_2^2 + z_1^2) - x_2 z_2 - x_1 z_1 + u_z \end{cases} \quad (12)$$

We consider the adaptive control law defined by

$$\begin{cases} u_x = -\hat{a}(t)e_x + \hat{p}(t)(x_2^2 + x_1^2) + x_2 y_2 + x_2 z_2 + x_1 y_1 + x_1 z_1 - k_x e_x \\ u_y = \hat{c}(t)e_y - x_2 y_2 - x_1 y_1 - k_y e_y \\ u_z = -\hat{b}(t)e_z + \hat{q}(t)(z_2^2 + z_1^2) + x_2 z_2 + x_1 z_1 - k_z e_z \end{cases} \quad (13)$$

In (13), $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{p}(t)$, $\hat{q}(t)$ are estimates of the unknown parameters a, b, c, p, q respectively, and k_x, k_y, k_z are positive gain constants.

Substituting (13) into (12), we obtain the closed-loop error dynamical system

$$\begin{cases} \dot{e}_x = [a - \hat{a}(t)]e_x - [p - \hat{p}(t)](x_2^2 + x_1^2) - k_x e_x \\ \dot{e}_y = -[c - \hat{c}(t)]e_y - k_y e_y \\ \dot{e}_z = [b - \hat{b}(t)]e_z - [q - \hat{q}(t)](z_2^2 + z_1^2) - k_z e_z \end{cases} \quad (14)$$

Now, we define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \quad (15)$$

Using (15), we can simplify the error dynamics (14) as

$$\begin{cases} \dot{e}_x = e_a e_x - e_p (x_2^2 + x_1^2) - k_x e_x \\ \dot{e}_y = -e_c e_y - k_y e_y \\ \dot{e}_z = e_b e_z - e_q (z_2^2 + z_1^2) - k_z e_z \end{cases} \quad (16)$$

Differentiating (16) with respect to t, we get

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \\ \dot{e}_c = -\dot{\hat{c}}(t) \\ \dot{e}_p = -\dot{\hat{p}}(t) \\ \dot{e}_q = -\dot{\hat{q}}(t) \end{cases} \quad (17)$$

We consider the quadratic Lyapunov function defined by

$$\dot{V} = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \quad (18)$$

Clearly, V is a positive definite function on R^8.

Differentiating V along the trajectories of (16) and (17), we obtain

$$\begin{aligned} \dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a\left[e_x^2 - \dot{\hat{a}}(t)\right] + e_b\left[e_z^2 - \dot{\hat{b}}(t)\right] + e_c\left[-e_y^2 - \dot{\hat{c}}(t)\right] \\ + e_p\left[-e_x(x_2^2 + x_1^2) - \dot{\hat{p}}(t)\right] + e_q\left[-e_z(z_2^2 + z_1^2) - \dot{\hat{q}}(t)\right] \end{aligned} \quad (19)$$

In view of (19), we take the parameter update law as follows.

$$\begin{cases} \dot{\hat{a}}(t) &= e_x^2 \\ \dot{\hat{b}}(t) &= e_z^2 \\ \dot{\hat{c}}(t) &= -e_y^2 \\ \dot{\hat{p}}(t) &= -e_x\left(x_2^2 + x_1^2\right) \\ \dot{\hat{q}}(t) &= -e_z\left(z_2^2 + z_1^2\right) \end{cases} \tag{20}$$

Theorem 1. The identical chemical chaotic attractors (9) and (10) with unknown system parameters are globally and exponentially anti-synchronized for all initial conditions by the adaptive control law (13) and the parameter update law (20), where k_x, k_y, k_z are positive gain constants.

Proof. We prove this result by Lyapunov stability theory [131].

We consider the quadratic Lyapunov function V defined in (18), which is positive definite on R^8.

Substituting the parameter update law (20) into (19), we obtain

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2 \tag{21}$$

By (21), it follows that $\dot{V}$ is a negative semi-definite function on R^8.

By Barbalat's lemma in Lyapunov stability theory [131], it follows that the errors e_x, e_y, e_z exponentially converge to zero as $t \to \infty$ for all initial conditions.

This completes the proof. ■

5. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (9) and (10), when the adaptive control law (13) and parameter update law (20) are applied. We take the gain constants as $k_x = 5, k_y = 5$ and $k_z = 5$.

We take the initial conditions of the chemical reactor (9) as

$$x_1(0) = 3.5, \quad y_1(0) = 5.1, \quad z_1(0) = 4.7 \tag{22}$$

We take the initial conditions of the chemical reactor (10) as

$$x_2(0) = 1.4, \quad y_2(0) = 16.3, \quad z_2(0) = 10.8 \tag{23}$$

The parameter values of the chemical reactor are taken as in the chaotic case, viz.

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \tag{24}$$

We take the initial conditions for the parameter updates as

$$\hat{a}(0) = 6.4, \quad \hat{b}(0) = 12, \quad \hat{c}(0) = 4.3, \quad \hat{p}(0) = 7.6, \quad \hat{q}(0) = 3.4 \tag{35}$$

Figures 5-7 show the anti-synchronization of the chemical chaotic reactors (9) and (10).

Figure 8 shows the time-history of the anti-synchronization errors e_x, e_y, e_z.

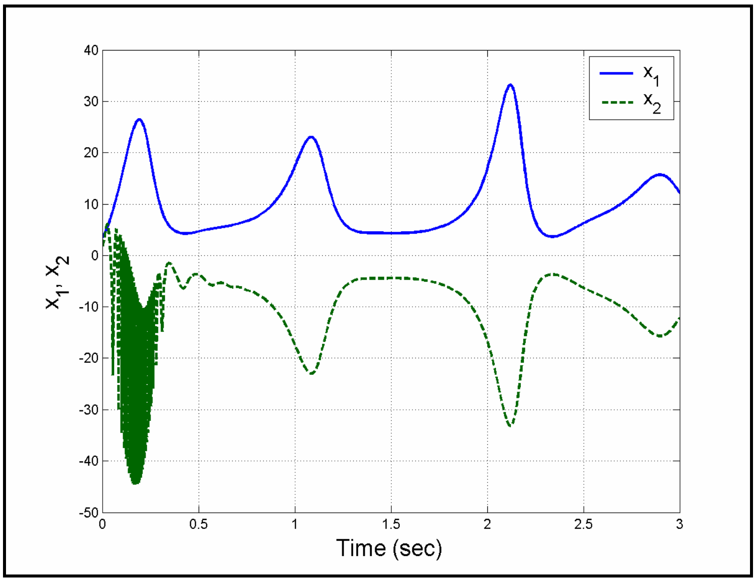

Figure 5. Anti-synchronization of the states $x_1(t)$ and $x_2(t)$

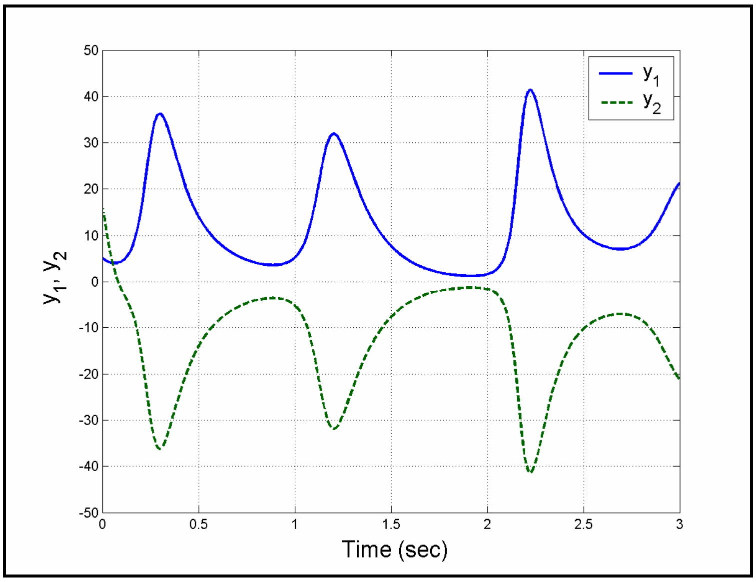

Figure 6. Anti-synchronization of the states $y_1(t)$ and $y_2(t)$

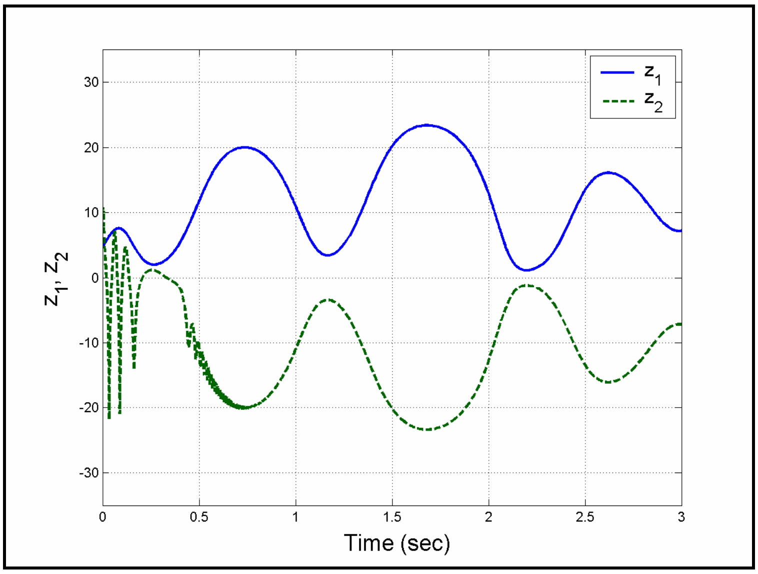

Figure 7. Anti-synchronization of the states $z_1(t)$ and $z_2(t)$

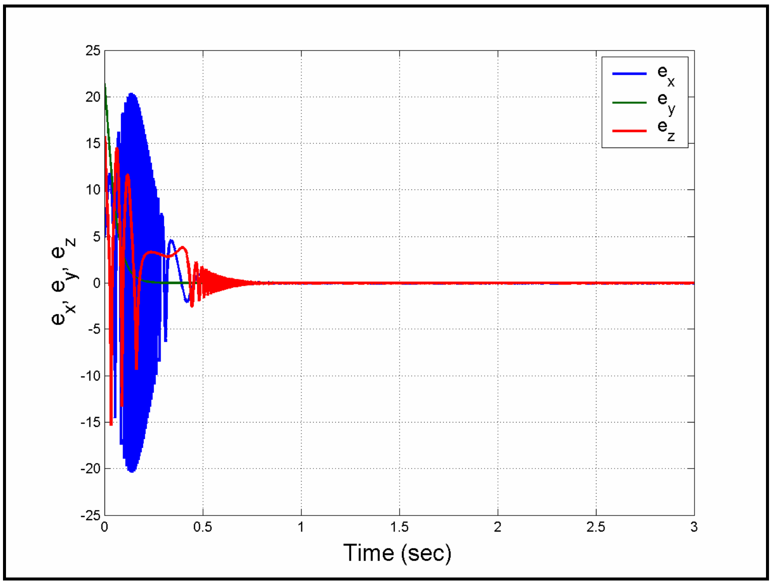

Figure 8. Time-history of the chaos synchronization errors $e_x(t), e_y(t), e_z(t)$

6. Conclusions

In this paper, new results have been derived for the dynamic analysis and anti-synchronization of a chemical chaotic attractor discovered by Huang and Yang (2005) via adaptive control method. First, the paper discussed the qualitative properties, Lyapunov exponents, stability of equilibrium point at the origin and phase portraits of the chemical chaotic attractor discovered by Huang. Then this paper derived new results for the anti-synchronization of the states of the identical chemical chaotic reactors via adaptive control method. We have given MATLAB simulations to illustrate all the main results presented in this work.

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