



Global Chaos Synchronization of Chemical Chaotic Reactors via Novel Sliding Mode Control Method

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. This paper details the qualitative properties of a chemical chaotic attractor discovered by Huang (2005). This paper gives a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, new results are obtained for the global chaos synchronization of identical chemical chaotic reactors using a novel sliding mode control method. MATLAB plots have been shown to illustrate the phase portraits of the chemical chaotic attractor and the global chaos synchronization of identical chemical chaotic attractors.

Keywords: Chaos, chaotic systems, chemical reactor, sliding mode control, stability.

1. Introduction

Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-42], Pehlivan system [43], Pham system [44], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to devise a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [45-64]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [65-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-104], biology [105-112], memristors [113-115], electrical circuits [116], etc.

This paper investigates first the qualitative properties of a chemical chaotic reactor model discovered by Huang in 2005 [117]. Huang derived the chemical reactor model by considering reactor dynamics with five

steps (2 reversible and 3 non-reversible). This paper also derives new results for the global chaos synchronization of chemical chaotic attractors using Lyapunov stability theory using novel sliding mode control method. MATLAB plots are shown to illustrate the phase portraits and global chaos synchronization of the chemical chaotic reactor.

2. Huang's Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics of Huang and Yang [117] consist of the following five steps given below.



Equations (1a) and (1e) indicate reversible steps, while equations (1b), (1c) and (1d) indicate non-reversible steps of the Huang chemical reactor [116]. In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z .

Assuming an ideal mixture and a well-stirred reactor, the macroscopic rate equations for the Huang's chemical reactor can be written in non-dimensionalized form as

$$\begin{cases} \dot{x} = a_1x - k_{-1}x^2 - xy - xz \\ \dot{y} = xy - a_5y \\ \dot{z} = a_4z - xz - k_{-5}z^2 \end{cases} \quad (2)$$

In (2), x, y, z are the mole fractions of X, Y and Z . Also, the rate constants k_1, k_3 and k_5 are incorporated in the parameters a_1, a_4 and a_5 .

To simplify the notations, we rename the constants and express the chemical reactor system (2) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \quad (3)$$

The system (3) is chaotic when the system parameters are chosen as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \quad (4)$$

For numerical simulations, we take the initial conditions

$$x(0) = 1.8, \quad y(0) = 2.5, \quad z(0) = 0.6 \quad (5)$$

The 3-D phase portrait of the chemical chaotic reactor is depicted in Figure 1.

The 2-D projections of the chemical chaotic reactor on the (x, y) , (y, z) and (x, z) planes are depicted in Figures 2-4.

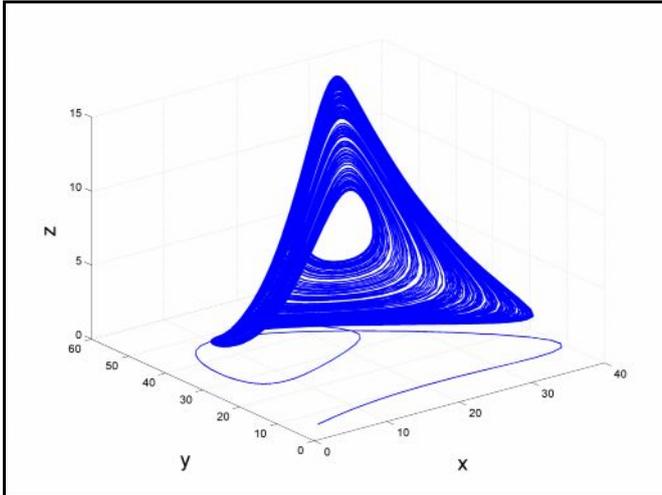


Figure 1. The 3-D phase portrait of the chemical chaotic reactor

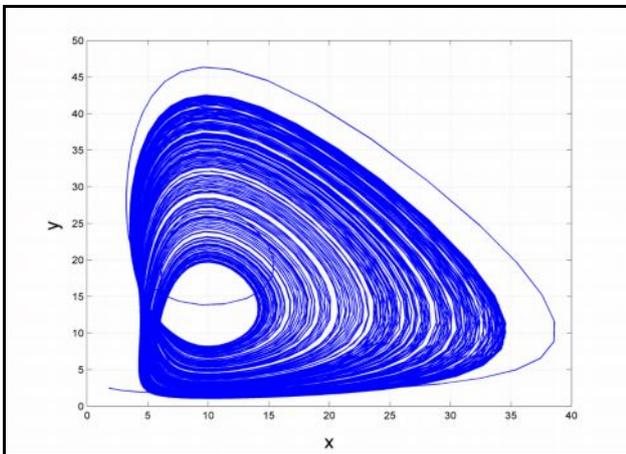


Figure 2. The 2-D projection of the chemical chaotic attractor on the (x,y) plane

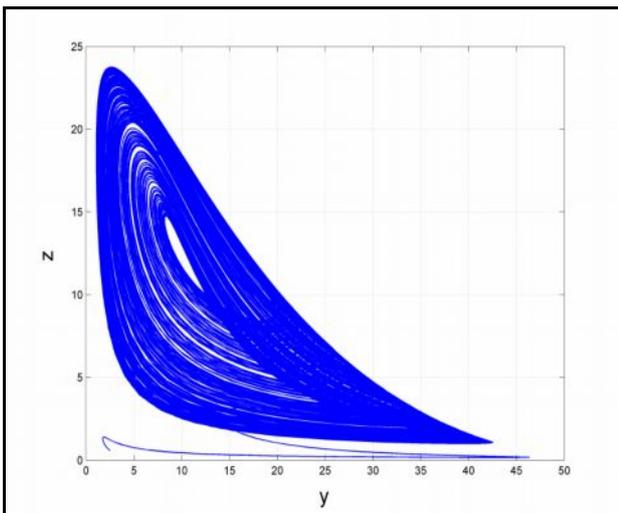


Figure 3. The 2-D projection of the chemical chaotic attractor on the (y,z) plane

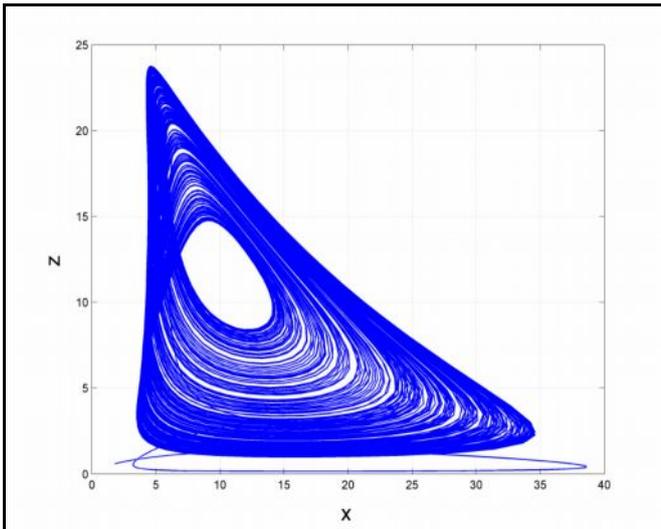


Figure 4. The 2-D projection of the chemical chaotic attractor on the (x, z) plane

3. Computational Analysis of the Chemical Chaotic Attractor

The Lyapunov exponents of the Huang's chemical chaotic attractor (3) are derived in MATLAB as

$$L_1 = 0.4001, L_2 = 0, L_3 = -11.8762 \quad (6)$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \quad (7)$$

The chemical chaotic attractor has an equilibrium at $(x, y, z) = (0, 0, 0)$.

The eigenvalues of the linearized system matrix of the chemical reactor (3) at the origin are:

$$\lambda_1 = -6.5, \lambda_2 = 30, \lambda_3 = -10 \quad (8)$$

Thus, the origin is a saddle-point equilibrium, which is unstable.

4. Global Chaos Synchronization of Chemical Chaotic Attractors via Sliding Mode Control

In this section, we use novel sliding mode control [97] to design a feedback control law for globally synchronizing the trajectories of identical chemical chaotic reactors.

Sliding mode control (SMC) is a nonlinear control technique with the useful properties of accuracy, robustness, easy tuning and implementation. Sliding mode control systems are designed to drive the system states onto a particular surface in the state space, called *sliding surface*. Once the sliding surface is reached, sliding mode control keeps the states on the close neighbourhood of the sliding surface. Hence the sliding mode control is a two part controller design. The first part involves the design of a sliding surface so that the sliding motion satisfies the system design specifications. The second is concerned with the selection of a control law that will make the switching surface attractive to the system state. A major advantage of the sliding mode control method is that the closed loop response becomes totally insensitive to some particular uncertainties. This principle extends to model parameter uncertainties, disturbance and non-linearity that are bounded. From a practical point of view, sliding mode control (SMC) is very useful controlling nonlinear processes which are subject to external disturbances and heavy model uncertainties.

As the master system, we consider the chemical reactor dynamics

$$\begin{cases} \dot{x}_1 = ax_1 - px_1^2 - x_1y_1 - x_1z_1 \\ \dot{y}_1 = x_1y_1 - cy_1 \\ \dot{z}_1 = bz_1 - x_1z_1 - qz_1^2 \end{cases} \quad (9)$$

In (9), x_1, y_1, z_1 are the states of the master system.

Also, we consider the slave system as the chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x}_2 = ax_2 - px_2^2 - x_2y_2 - x_2z_2 + u_x \\ \dot{y}_2 = x_2y_2 - cy_2 + u_y \\ \dot{z}_2 = bz_2 - x_2z_2 - qz_2^2 + u_z \end{cases} \quad (10)$$

In (10), x_2, y_2, z_2 are the states of the slave system and u_x, u_y, u_z are sliding controls to be determined.

The global chaos synchronization error is defined by

$$\begin{cases} e_x = x_2 - x_1 \\ e_y = y_2 - y_1 \\ e_z = z_2 - z_1 \end{cases} \quad (11)$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x = ae_x - p(x_2^2 - x_1^2) - x_2y_2 + x_1y_1 - x_2z_2 + x_1z_1 + u_x \\ \dot{e}_y = -ce_y + x_2y_2 - x_1y_1 + u_y \\ \dot{e}_z = be_z - x_2z_2 + x_1z_1 - q(z_2^2 - z_1^2) + u_z \end{cases} \quad (12)$$

In this paper, we use Vaidyanathan's theorem [97] to devise a novel sliding mode controller to drive the synchronization error to zero asymptotically.

We take the parameter values as in the chaotic case

First, we write the error system (12) in matrix form as

$$\dot{e} = Ae + \psi(x, y) + u \quad (13)$$

where

$$e = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}, A = \begin{bmatrix} a & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & b \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 16.5 \end{bmatrix}, u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \quad (14)$$

and

$$\psi(x, y) = \begin{bmatrix} -p(x_2^2 - x_1^2) - x_2y_2 + x_1y_1 - x_2z_2 + x_1z_1 \\ x_2y_2 - x_1y_1 \\ -x_2z_2 + x_1z_1 - q(z_2^2 - z_1^2) \end{bmatrix} \quad (15)$$

We find $B \in R^3$ so that (A, B) is completely controllable. A simple choice of B is

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \quad (16)$$

Thus, we set the nonlinear feedback control u as

$$u = -\psi(x, y) + Bv \quad (17)$$

where v is the sliding control, which is determined as follows.

The sliding variable is selected as

$$s = Ce \quad [2.1 \quad 0 \quad -2.0]e = 2.1e_1 - 2e_3 \quad (18)$$

With the choice of $C = [2.1 \quad 0 \quad -2.0]$, the eigenvalues of the matrix $E = [I - B(CB)^{-1}C]A$ are given by

$$\text{eig}(E) = \{-10, -253.5, 0\}. \quad (19)$$

This shows that the dynamics along the sliding manifold is globally asymptotically stable.

Next, we take the sliding constants as $k = 6$ and $q = 0.2$. Then the novel sliding control v is obtained by the Vaidyanathan's theorem [94] as

$$v(t) = -(CB)^{-1} [C(kI + A)e + qs^2 \text{sgn}(s)] \quad (20)$$

A simplification gives

$$v(t) = -756e_x + 450e_z - 2s^2 \text{sgn}(s) \quad (21)$$

As an application of Vaidyanathan's theorem [97], we obtain the following result.

Theorem 1. The identical chemical chaotic attractors (9) and (10) with unknown system parameters are globally and asymptotically synchronized for all initial conditions by the sliding control law (17), where v is defined by (21), B is defined by (16) and $\psi(x, y)$ is defined by (15). ■

5. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (9) and (10), when the sliding control law (17) is applied.

We take the sliding constants as $k = 6$ and $q = 0.2$.

We take the initial conditions of the chemical reactor (9) as

$$x_1(0) = 6.3, \quad y_1(0) = 8.1, \quad z_1(0) = 2.7 \quad (22)$$

We take the initial conditions of the chemical reactor (10) as

$$x_2(0) = 3.5, \quad y_2(0) = 4.9, \quad z_2(0) = 9.2 \quad (23)$$

The parameter values of the chemical reactor are taken as in the chaotic case, viz.

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \quad (24)$$

Figures 5-7 show the complete chaos synchronization of the chemical chaotic reactors (9) and (10).

Figure 8 shows the time-history of the chaos synchronization errors e_x, e_y, e_z .

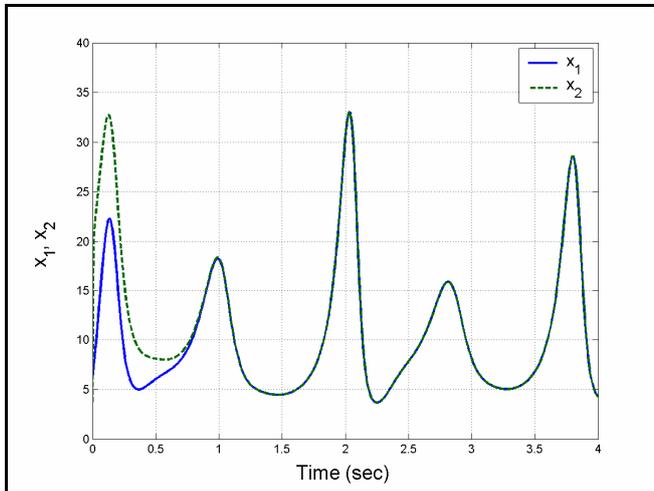


Figure 5. Complete synchronization of the states $x_1(t)$ and $x_2(t)$

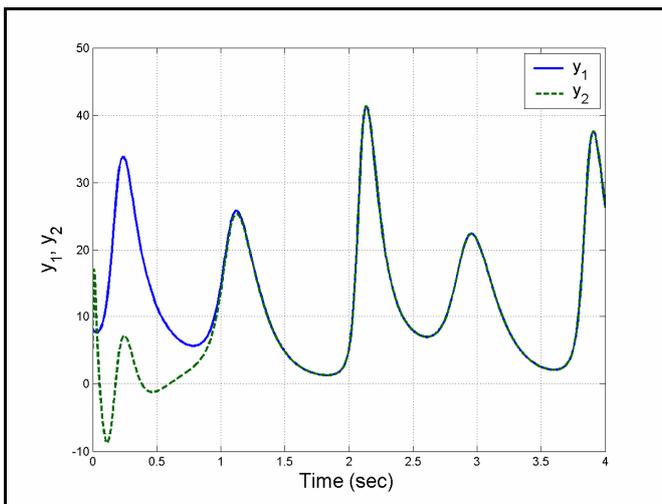


Figure 6. Complete synchronization of the states $y_1(t)$ and $y_2(t)$

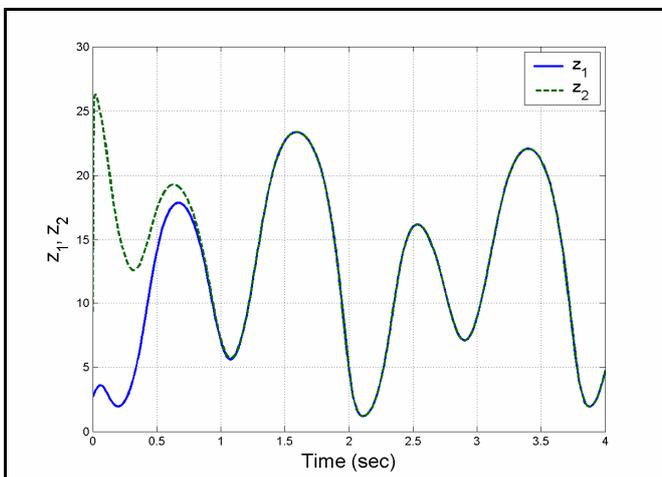


Figure 7. Complete synchronization of the states $z_1(t)$ and $z_2(t)$

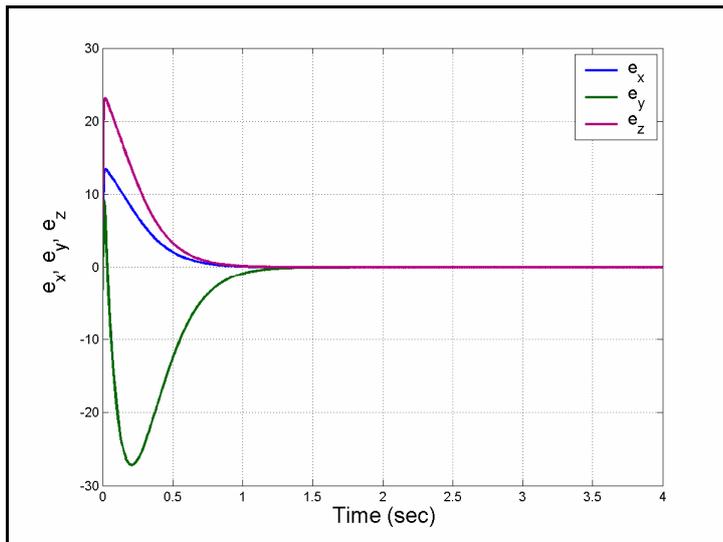


Figure 8. Time-history of the chaos synchronization errors $e_x(t), e_y(t), e_z(t)$

6. Conclusions

In this paper, new results have been derived for the dynamic analysis and global chaos synchronization of a chemical chaotic attractor discovered by Huang and Yang (2005) via novel sliding mode control method. First, the paper discussed the qualitative properties, Lyapunov exponents, stability of equilibrium point at the origin and phase portraits of the chemical chaotic attractor discovered by Huang. Then this paper derived new results for the global synchronization of the states of the identical chemical chaotic reactors via novel sliding mode controller (SMC). We have given MATLAB simulations to illustrate all the main results presented in this work.

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