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Adaptive Synchronization of Novel 3-D Chemical Chaotic Reactor Systems

Sundarapandian Vaidyanathan*

R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, India

Abstract: Chaos theory has a manifold variety of applications in science and engineering. In this paper, new chemical chaotic reactor equations are derived by modifying the chemical chaotic reactor system obtained by the Huang (2005). This paper gives a summary description of the novel chemical reactor dynamics and the chaos dynamic analysis. Next, new results are obtained for the adaptive synchronization of the novel 3-D chemical chaotic reactor systems. MATLAB plots have been depicted to illustrate the phase portraits of the novel chemical chaotic attractor and the global chaos synchronization of the novel 3-D chemical chaotic reactor systems via adaptive control method.
Keywords: Chaos, chaotic systems, chaos synchronization, chemical reactor, adaptive control, stability.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-40], Pehlivan system [41], Pham system [42], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to device a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [43-62]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [63-76]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [77-83], sliding mode control method [84-95], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [96-97], biology [98-100], memristors [101-103], electrical circuits [104], etc.

This paper investigates first the qualitative properties of a chemical chaotic reactor model discovered by Huang in 2005 [105]. Huang derived the chemical reactor model by considering reactor dynamics with five steps (2 reversible and 3 non-reversible). Then a novel chemical chaotic reactor model is derived. The qualitative properties of the novel chemical chaotic reactor model are described. This paper also derives new results for the global chaos synchronization of the novel 3-D chemical chaotic reactor models via adaptive control method. MATLAB plots are shown to depict the phase portraits and global chaos synchronization of the novel 3-D chemical chaotic reactor designed in this research work.

2. Huang's Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics of Huang and Yang [105] consist of the following five steps given below.

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2X \tag{1a}$$

$$X + Y \xrightarrow{k_2} 2Y \tag{1b}$$

$$A_5 + Y \xrightarrow{k_3} A_2 \tag{1c}$$

$$X + Z \xrightarrow{k_4} A_3 \tag{1d}$$

$$A_4 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftarrows}} 2Z \tag{1e}$$

Equations (1a) and (1e) indicate reversible steps, while equations (1b), (1c) and (1d) indicate non-reversible steps of the Huang chemical reactor [105]. In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z.

Assuming an ideal mixture and a well-stirred reactor, the macroscopic rate equations for the Huang's chemical reactor can be written in non-dimensionalized form as

$$\begin{cases} \dot{x} = a_1 x - k_{-1} x^2 - xy - xz \\ \dot{y} = xy - a_5 y \\ \dot{z} = a_4 z - xz - k_{-5} z^2 \end{cases} \tag{2}$$

In (2), x, y, z are the mole fractions of X, Y and Z. Also, the rate constants k_1, k_3 and k_5 are incorporated in the parameters a_1, a_4 and a_5.

To simplify the notations, we rename the constants and express the chemical reactor system (2) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \tag{3}$$

The system (3) is *chaotic* when the system parameters are chosen as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \tag{4}$$

For numerical simulations, we take the initial conditions

$$x(0) = 1.8, \quad y(0) = 2.5, \quad z(0) = 0.6 \tag{5}$$

The 3-D phase portrait of the chemical chaotic reactor (2) is depicted in Figure 1.

The Lyapunov exponents of the Huang's chemical chaotic attractor (3) are derived in MATLAB as

$$L_1 = 0.4001, \quad L_2 \quad 0, \quad L_3 \quad -11.8762 \tag{6}$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \tag{7}$$

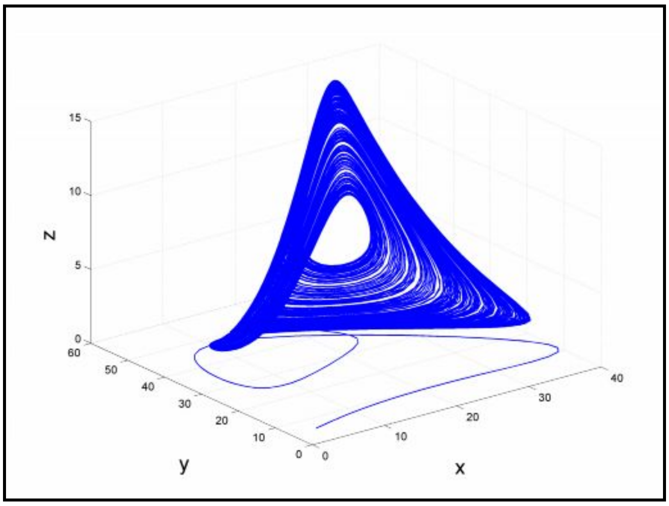

Figure 1. The 3-D phase portrait of the Huang chemical chaotic reactor

3. A Novel Chemical Chaotic Reactor System

In this section, we propose a novel chemical chaotic reactor system by modifying Huang's system (3) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy + rx - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \tag{8}$$

The novel 3-D system (8) is *chaotic* when the parameter values are taken as

$$a = 30, \ \ b = 16.5, \ \ c = 10, \ \ p = 0.5, \ \ q = 0.5, \ \ r = 0.01 \tag{9}$$

For numerical simulations, we take the initial conditions

$$x(0) = 0.1, \ \ y(0) = 0.2, \ \ z(0) = 0.1 \tag{10}$$

The 3-D phase portrait of the novel chemical chaotic reactor (8) is depicted in Figure 2. The 2-D projections of the strange attractor of the novel chemical reactor (8) on the (x, y), (y, z) and (x, z) coordinate planes are depicted in Figures 3-5, respectively.

The Lyapunov exponents of the novel chemical chaotic reactor (8) are obtained as

$$L_1 = 0.4354, \ \ L_2 \ \ 0, \ \ L_3 \ \ -11.9273 \tag{11}$$

Also, the Lyapunov dimension of the novel chemical chaotic reactor (8) is obtained as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0365 \tag{12}$$

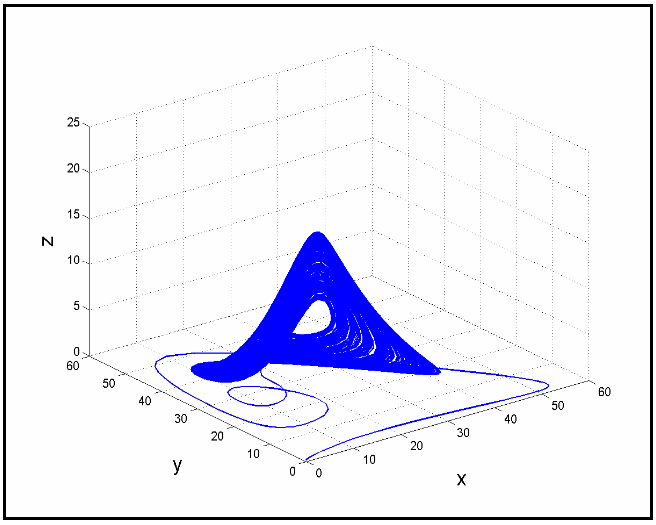

Figure 2. The 3-D phase portrait of the novel chemical chaotic reactor (8)

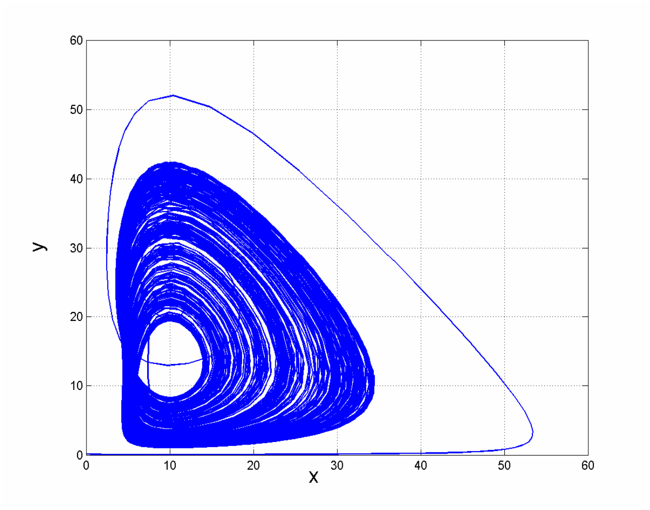

Figure 3. The 2-D projection of the novel chemical chaotic reactor (8) on the (x, y) plane

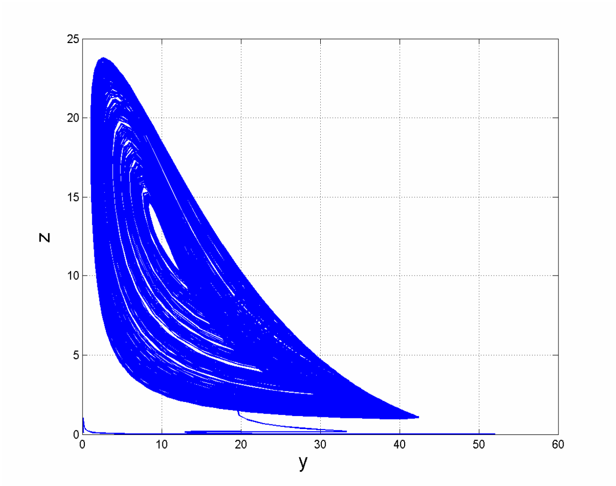

Figure 4. The 2-D projection of the novel chemical chaotic reactor (8) on the (y, z) plane

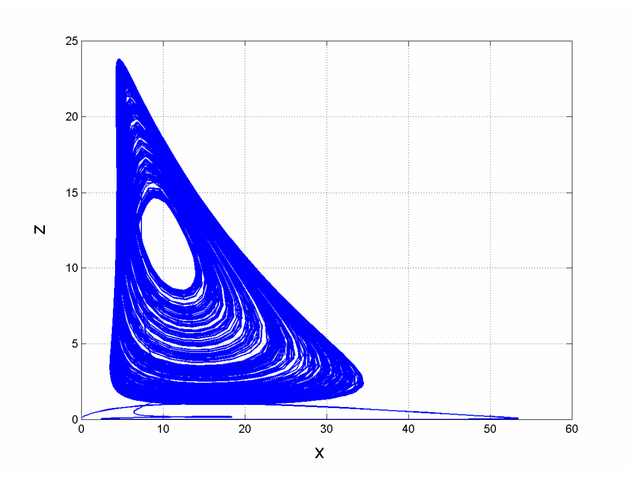

Figure 5. The 2-D projection of the novel chemical chaotic reactor (8) on the (x, z) plane

4. Global Chaos Synchronization of the Novel Chemical Chaotic Reactors via Adaptive Control

In this section, we use adaptive control method to achieve global chaos synchronization of the identical states of the novel 3-D chemical chaotic reactors with unknown system parameters. We use Lyapunov stability theory to prove the main adaptive control result derived in this section.

As the master system, we consider the novel chemical reactor dynamics given by

$$\begin{cases} \dot{x}_1 = ax_1 - px_1^2 - x_1y_1 - x_1z_1 \\ \dot{y}_1 = x_1y_1 + rx_1 - cy_1 \\ \dot{z}_1 = bz_1 - x_1z_1 - qz_1^2 \end{cases} \tag{13}$$

In (13), x_1, y_1, z_1 are the states of the system. We suppose that the system parameters a, b, c, p, q, r are unknown and use estimates of the parameters to derive the adaptive controls u_x, u_y, u_z.

As the slave system, we consider the controlled novel chemical reactor dynamics given by

$$\begin{cases} \dot{x}_2 = ax_2 - px_2^2 - x_2y_2 - x_2z_2 + u_x \\ \dot{y}_2 = x_2y_2 + rx_2 - cy_2 + u_y \\ \dot{z}_2 = bz_2 - x_2z_2 - qz_2^2 + u_z \end{cases} \tag{14}$$

The complete synchronization errors are defined by

$$\begin{cases} e_x = x_2 - x_1 \\ e_y = y_2 - y_1 \\ e_z = z_2 - z_1 \end{cases} \tag{15}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x = ae_x - p(x_2^2 - x_1^2) - x_2y_2 + x_1y_1 - x_2z_2 + x_1z_1 + u_x \\ \dot{e}_y = re_x - ce_y + x_2y_2 - x_1y_1 + u_y \\ \dot{e}_z = be_z - x_2z_2 + x_1z_1 - q(z_2^2 - z_1^2) + u_z \end{cases} \tag{16}$$

We consider the adaptive controller defined by

$$\begin{cases} u_x = -\hat{a}(t)e_x + \hat{p}(t)(x_2^2 - x_1^2) + x_2y_2 - x_1y_1 + x_2z_2 - x_1z_1 - k_xe_x \\ u_y = -\hat{r}(t)e_x + \hat{c}(t)e_y - x_2y_2 + x_1y_1 - k_ye_y \\ u_z = -\hat{b}(t)e_z + x_2z_2 - x_1z_1 + \hat{q}(t)(z_2^2 - z_1^2) - k_ze_z \end{cases} \tag{17}$$

where k_x, k_y, k_z are positive gain constants.

Substituting (17) into (16), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_x = [a - \hat{a}(t)]e_x - [p - \hat{p}(t)](x_2^2 - x_1^2) - k_xe_x \\ \dot{e}_y = [r - \hat{r}(t)]e_x - [c - \hat{c}(t)]e_y - k_ye_y \\ \dot{e}_z = [b - \hat{b}(t)]e_z - [q - \hat{q}(t)](z_2^2 - z_1^2) - k_ze_z \end{cases} \tag{18}$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \\ e_r(t) = r - \hat{r}(t) \end{cases} \tag{19}$$

Using (19), the closed-loop system (18) can be simplified as

$$\begin{cases} \dot{e}_x = e_a e_x - e_p (x_2^2 - x_1^2) - k_x e_x \\ \dot{e}_y = e_r e_x - e_c e_y - k_y e_y \\ \dot{e}_z = e_b e_z - e_q (z_2^2 - z_1^2) - k_z e_z \end{cases} \tag{20}$$

Differentiating (19) with respect to time, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \\ \dot{e}_r(t) = -\dot{\hat{r}}(t) \end{cases} \tag{21}$$

Next, we consider the candidate Lyapunov function defined by

$$V(e_x, e_y, e_z, e_a, e_b, e_c, e_p, e_q, e_r) = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2 + e_r^2\right) \tag{22}$$

Differentiating (22) along the trajectories of (20) and (21), we get the following dynamics

$$\begin{aligned} \dot{V} = &-k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a\left[e_x^2 - \dot{\hat{a}}\right] + e_b\left[e_z^2 - \dot{\hat{b}}\right] + e_c\left[-e_y^2 - \dot{\hat{c}}\right] \\ &+ e_p\left[-e_x\left(x_2^2 - x_1^2\right) - \dot{\hat{p}}\right] + e_q\left[-e_z\left(z_2^2 - z_1^2\right) - \dot{\hat{q}}\right] + e_r\left[e_x e_y - \dot{\hat{r}}\right] \end{aligned} \tag{23}$$

In view of (22), we take the following parameter update law:

$$\begin{cases} \dot{\hat{a}}(t) = e_x^2 \\ \dot{\hat{b}}(t) = e_z^2 \\ \dot{\hat{c}}(t) = -e_y^2 \\ \dot{\hat{p}}(t) = -e_x\left(x_2^2 - x_1^2\right) \\ \dot{\hat{q}}(t) = -e_z(z_2^2 - z_1^2) \\ \dot{e}_r(t) = e_x e_y \end{cases} \tag{24}$$

Next, we state and prove the main result of this section.

Theorem 1. The adaptive control law (17) and the parameter update law (24) achieve global chaos synchronization of the identical novel 3-D chemical chaotic reactors defined by (13) and (14), where k_x, k_y, k_z are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [106].

The quadratic Lyapunov function V defined by (22) is positive definite on R^9.

Substituting the parameter update law (24) into (23), we get the time derivative of V as

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2, \tag{25}$$

which is negative semi-definite on R^9.

Thus, by Barbalat's lemma in Lyapunov stability theory [106], it follows that the closed-loop error dynamics (20) is globally exponentially stable.

This completes the proof. ■

5. Numerical Simulations

We use the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ to solve the system of differential equations (14) and (24), when the adaptive control law (17) is implemented.

We take the parameter values of the novel chemical chaotic reactors as in the chaotic case, viz.

$$a = 30,\quad b = 16.5,\quad c = 10,\quad p = 0.5,\quad q = 0.5,\quad r = 0.01 \tag{26}$$

We take the gain constants as $k_x = 6$, $k_y = 6$ and $k_z = 6$.

We take the initial values of the chemical chaotic reactor (13) as

$x_1(0) = 3.2,\quad y_1(0) = 7.4,\quad z_1(0) = 12.5$

We take the initial values of the chemical chaotic reactor (14) as

$x_2(0) = 9.7,\quad y_2(0) = 2.1,\quad z_2(0) = 4.8$

We take the initial values of the parameter estimates as

$\hat{a}(0) = 2.1,\quad \hat{b}(0) = 8.5,\quad \hat{c}(0) = 4.3,\quad \hat{p}(0) = 6.2,\quad \hat{q}(0) = 1.8,\quad \hat{r}(0) = 5.9$

Figures 6-8 show the complete of the novel chemical chaotic reactor systems (13) and (14).
Figure 9 shows the time-history of the synchronization errors e_x, e_y, e_z.

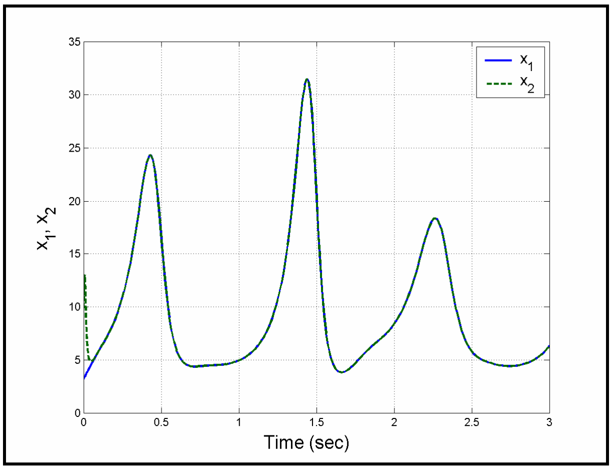

Figure 6. Complete synchronization of the states $x_1(t)$ and $x_2(t)$

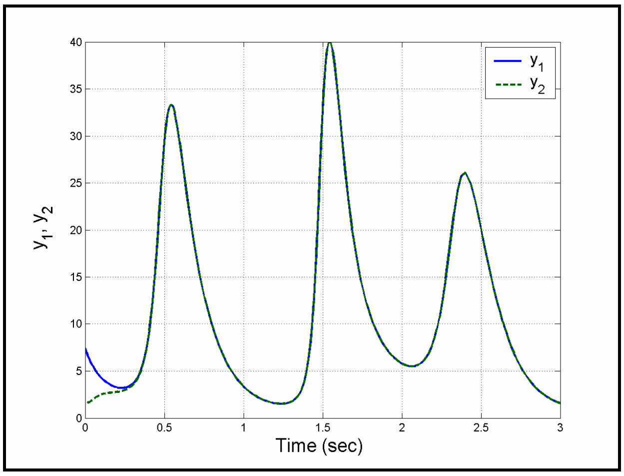

Figure 7. Complete synchronization of the states $y_1(t)$ and $y_2(t)$

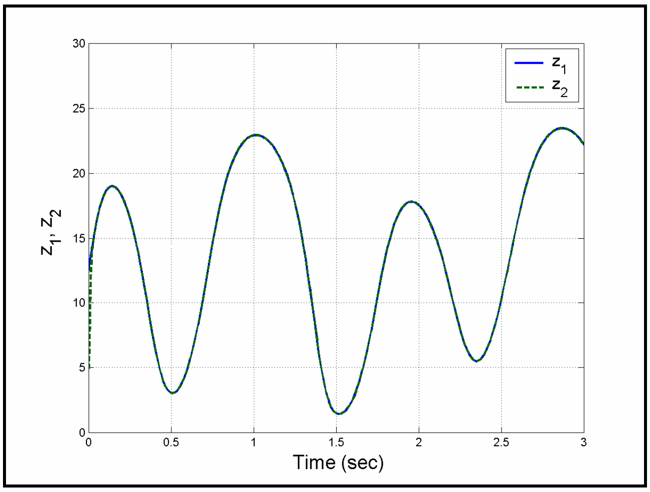

Figure 8. Complete synchronization of the states $z_1(t)$ and $z_2(t)$

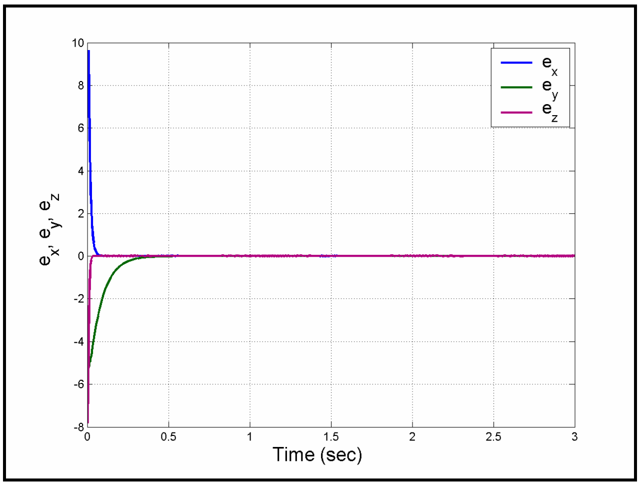

Figure 9. Time-history of the synchronization errors $e_x(t), e_y(t), e_z(t)$

6. Conclusions

In this paper, new chemical chaotic reactor equations are derived by modifying the chemical chaotic reactor system obtained by the Huang (2005). We gave a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, new results were obtained for the adaptive synchronization of the novel chemical chaotic reactor system with unknown system parameters. MATLAB plots were shown to illustrate the phase portraits of the novel chemical chaotic attractor and the global chaos synchronization of the novel chemical chaotic reactor systems via adaptive control method.

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