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Adaptive Synchronization of Chemical Chaotic Reactors

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Abstract: Chaos theory hasa manifold variety of applications in science and engineering. This paper investigates the properties of a chemical chaotic attractor discovered by Haung (2005). This paper gives a summary description of the chemical reactor dynamics and the chaos dynamic analysis.Next, an adaptive synchronizer is designed using control theory for the global chaos synchronization of identical chemical chaotic attractors with unknown parameters. The main results for adaptive synchronization of chemical reactors are established using Lyapunov stability theory. MATLAB plots have been shown to illustrate the phase portraits of the chemical chaotic attractor and the adaptive synchronization of identical chemical chaotic attractors.
Keywords: Chaos, chaotic systems, chemical reactor, adaptive synchronization, stability.

Introduction

Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade Rössler [4]discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system[8], Cai system[9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years such as Sundarapandian systems [11, 12], Vaidyanathan systems [13-20], Pehlivan system [21], Jafari system[22],Pham system [23], etc.

Chaos and control theory havea manifold variety ofapplications in many fields of science and engineering such as oscillators[24], lasers [25-26],biology[27], chemical reactions [28-30], neural networks[31-32],robotics [33-34], electrical circuits [35-36], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to device a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. Various methods have been designed for the synchronization of chaotic systems such as active control [37-45], adaptive control [46-60], sliding mode control [61-68], backstepping control [69-73], etc.

This paper investigates the analysis and adaptive synchronization of the chemical chaotic reactor model discovered by Haung in 2005 [74]. Haung derived the chemical reactor model by considering reactor dynamics with five reversible steps. This paper also derives new results of adaptive synchronizer design for the identical chemical chaotic attractors using Lyapunov stability theory [75] and MATLAB plots are shown to illustrate the main results.

Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics [74] consist of the following five reversible steps given below.

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2X,\ X + Y \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} 2Y,\ A_5 + Y \underset{k_{-3}}{\overset{k_3}{\rightleftharpoons}} A_2,\ X + Z \underset{k_{-4}}{\overset{k_4}{\rightleftharpoons}} A_3,\ A_5 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftharpoons}} 2Z \qquad (1)$$

In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z. The corresponding non-dimensionalized dynamical evolution equations read as

$$\begin{cases} \dot{x} &= (a_1 - k_{-1}x - y - z)x \\ \dot{y} &= (x - a_5)y \\ \dot{z} &= (a_4 - x - k_{-5}z)z \end{cases} \qquad (2)$$

In (2), x, y, z are positive mole functions and a_1, a_4, a_5, k_{-1} and k_{-5} are positive parameters.
To simplify the notations, we rename the constants and express the system (2) as

$$\begin{cases} \dot{x} &= (a - px - y - z)x \\ \dot{y} &= (x - c)y \\ \dot{z} &= (b - x - qz)z \end{cases} \qquad (3)$$

The system (3) is chaotic when the system parameters are chosen as

$$a = 30,\quad b = 16.5,\quad c = 10,\quad p = 0.5,\quad q = 0.5 \qquad (4)$$

For numerical simulations, we take the initial conditions

$$x(0) = 1.8, y(0) = 2.5, \text{ and } z(0) = 0.6. \qquad (5)$$

The 3-D phase portrait of the chemical chaotic reactor is depicted in Fig. 1.

The 2-D projections of the chemical chaotic reactor on the (x, y), (y, z) and (x, z) planes are depicted in Figs. 2-4.

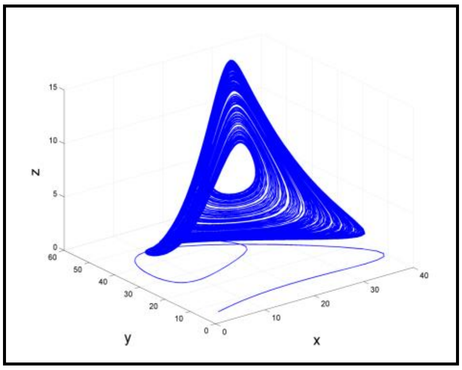

Figure1.The3-D phase portrait of the chemical chaotic reactor

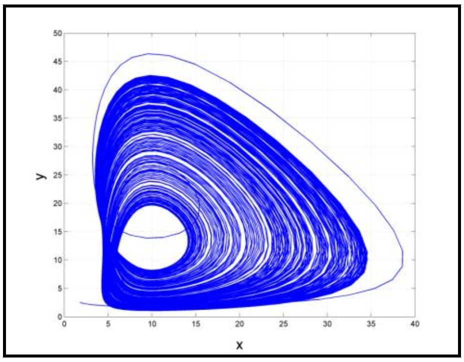

Figure2.The2-D projection of the chemical chaotic attractor on the (x, y) plane

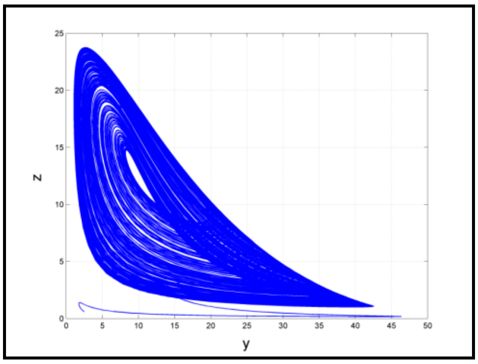

Figure3.The2-D projection of the chemical chaotic attractor on the (y, z) plane

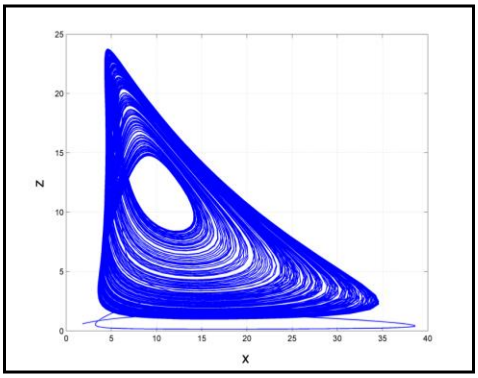

Figure4.The2-D projection of the chemical chaotic attractor on the (x, z) plane

Computational Analysis of the Chemical Chaotic Attractor

The Lyapunov exponents of the chemical chaotic attractor (3) have been obtained in MATLAB as

$$L_1 = 0.4001,\ \ L_2 = 0, L_3 = -11.8762. \tag{6}$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \tag{7}$$

The chemical chaotic attractor has an equilibrium at $(x, y, z) = (0,0,0)$.

The eigenvalues of the linearized system matrix of the attractor (3) at the origin are:

$$\lambda_1 = 16.5,\ \lambda_2 = 30, \lambda_3 = -10. \tag{8}$$

Since there are two positive eigenvalues in the set (7), the origin is an unstable equilibrium of the chemical chaotic attractor (3).

Adaptive Control Design of the Chemical Chaotic Attractor

In this section, we use adaptive control method to design an adaptive feedback synchronizer for globally synchronizing the trajectories of identical chemical chaotic reactors with unknown parameters.

Thus, we consider the master system as the chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x}_1 &= (a - px_1 - y_1 - z_1)x_1 \\ \dot{y}_1 &= (x_1 - c)y_1 \\ \dot{z}_1 &= (b - x_1 - qz_1)z_1 \end{cases} \tag{9}$$

In (9),x_1, y_1, z_1 are the states of the master system.

Also, we consider the slave system as the chemical chaotic attractor given by the dynamics

$$\begin{cases} \dot{x}_2 &= (a - px_2 - y_2 - z_2)x_2 + u_x \\ \dot{y}_2 &= (x_2 - c)y_2 + u_y \\ \dot{z}_2 &= (b - x_2 - qz_2)z_2 + u_z \end{cases} \tag{10}$$

In (10), x_1, y_1, z_1 are the states of the slave system and u_x, u_y, u_z are adaptive controls to be determined using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ of the unknown parameters a, b, c, p, q, respectively.

The global chaos synchronization error is defined by

$$\begin{cases} e_x &= x_2 - x_1 \\ e_y &= y_2 - y_1 \\ e_z &= z_2 - z_1 \end{cases} \tag{11}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x &= a e_x - p(x_2^2 - x_1^2) - x_2 y_2 - x_2 z_2 + x_1 y_1 + x_1 z_1 + u_x \\ \dot{e}_y &= -c e_y + x_2 y_2 - x_1 y_1 + u_y \\ \dot{e}_z &= b e_z - q(z_2^2 - z_1^2) - x_2 z_2 + x_1 z_1 + u_z \end{cases} \tag{12}$$

We consider the adaptive control law defined by

$$\begin{cases} u_x &= -\hat{a}(t) e_x + \hat{p}(t)(x_2^2 - x_1^2) + x_2 y_2 + x_2 z_2 - x_1 y_1 - x_1 z_1 - k_x e_x \\ u_y &= \hat{c}(t) e_y - x_2 y_2 + x_1 y_1 - k_y e_y \\ u_z &= -\hat{b}(t) e_z + \hat{q}(t)(z_2^2 - z_1^2) + x_2 z_2 - x_1 z_1 - k_z e_z \end{cases} \tag{13}$$

In (13), $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ are estimates of the unknown parameters a, b, c, p, q, respectively, and k_x, k_y, k_z are positive gain constants.

Substituting (13) into (12), we obtain the closed-loop error dynamical system

$$\begin{cases} \dot{e}_x &= [a - \hat{a}(t)] e_x - [p - \hat{p}(t)](x_2^2 - x_1^2) - k_x e_x \\ \dot{e}_y &= -[c - \hat{c}(t)] e_y - k_y e_y \\ \dot{e}_z &= [b - \hat{b}(t)] e_z - [q - \hat{q}(t)](z_2^2 - z_1^2) - k_z e_z \end{cases} \tag{14}$$

Now, we define the parameter estimation errors as

$$\begin{cases} e_a &= a - \hat{a}(t) \\ e_b &= b - \hat{b}(t) \\ e_c &= c - \hat{c}(t) \\ e_p &= p - \hat{p}(t) \\ e_q &= q - \hat{q}(t) \end{cases} \tag{15}$$

Using (15), we can simplify the error dynamics (14) as

$$\begin{cases} \dot{e}_x &= e_a e_x - e_p(x_2^2 - x_1^2) - k_x e_x \\ \dot{e}_y &= -e_c e_y - k_y e_y \\ \dot{e}_z &= e_b e_z - e_q(z_2^2 - z_1^2) - k_z e_z \end{cases} \tag{16}$$

Differentiating (16) with respect to t, we get

$$\begin{cases} \dot{e}_a &= -\dot{\hat{a}}(t) \\ \dot{e}_b &= -\dot{\hat{b}}(t) \\ \dot{e}_c &= -\dot{\hat{c}}(t) \\ \dot{e}_p &= -\dot{\hat{p}}(t) \\ \dot{e}_q &= -\dot{\hat{q}}(t) \end{cases} \tag{17}$$

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{18}$$

Clearly, V is a positive definite function on $\mathbf{R}^8$.

Differentiating V along the trajectories of (16) and (17), we obtain

$$\begin{aligned} \dot{V} = &-k_x e_x^2 - k_y e_y^2 - k_z e_z^2 + e_a\left[e_x^2 - \dot{\hat{a}}(t)\right] + e_b\left[e_z^2 - \dot{\hat{b}}(t)\right] + e_c\left[-e_y^2 - \dot{\hat{c}}(t)\right] \\ &+ e_p\left[-e_x(x_2^2 - x_1^2) - \dot{\hat{p}}(t)\right] + e_q\left[-e_z(z_2^2 - z_1^2) - \dot{\hat{q}}(t)\right] \end{aligned} \tag{19}$$

In view of (19), we take the parameter update law as follows.

$$\begin{cases} \dot{\hat{a}}(t) &= e_x^2 \\ \dot{\hat{b}}(t) &= e_z^2 \\ \dot{\hat{c}}(t) &= -e_y^2 \\ \dot{\hat{p}}(t) &= -e_x\left(x_2^2 - x_1^2\right) \\ \dot{\hat{q}}(t) &= -e_z\left(z_2^2 - z_1^2\right) \end{cases} \tag{20}$$

Theorem 1. The identical chemical chaotic attractors (9) and (10) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (13) and the parameter update law (20), where k_x, k_y, k_z are positive gain constants.

Proof. We prove this result by Lyapunov stability theory.
We consider the quadratic Lyapunov function V defined in (18), which is positive definite on $\boldsymbol{R}^8$.
Substituting the parameter update law (20) into (19), we obtain

$$\dot{V} = -k_x x^2 - k_y y^2 - k_z z^2 \tag{21}$$

By (21), it follows that $\dot{V}$ is a negative semi-definite function on $\boldsymbol{R}^8$.
By Barbalat's lemma in Lyapunov stability theory [75], it follows that the states $x(t), y(t), z(t)$ exponentially converge to zero as $t \to \infty$ for all initial conditions.

Hence, identical chemical chaotic attractors (9) and (10) are globally and exponentially synchronized for all initial conditions by the adaptive control law (13) and the parameter update law (20).

This completes the proof. ■

Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (9) and (10), when the adaptive control law (13) is applied.

We take the gain constants as $k_x = 6, k_y = 6, k_z = 6.$

We take the initial conditions of the chemical reactor (9) as
$x_1(0) = 2.3, y_1(0) = 1.7, z_1(0) = 0.8.$

We take the initial conditions of the chemical reactor (10) as
$x_2(0) = 0.7, y_2(0) = 2.9, z_2(0) = 2.5.$

The parameter values of the chemical reactor are taken as in the chaotic case, viz.
$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5$

Also, we take $\hat{a}(0) = 3.1, \quad \hat{b}(0) = 2.3, \quad \hat{c}(0) = 5.1, \quad \hat{p}(0) = 3.6, \quad \hat{q}(0) = 2.8.$

Figs. 5-7 show the complete chaos synchronization of the chemical chaotic reactors (9) and (10).
Fig. 8 shows the time-history of the chaos synchronization errors e_x, e_y, e_z.

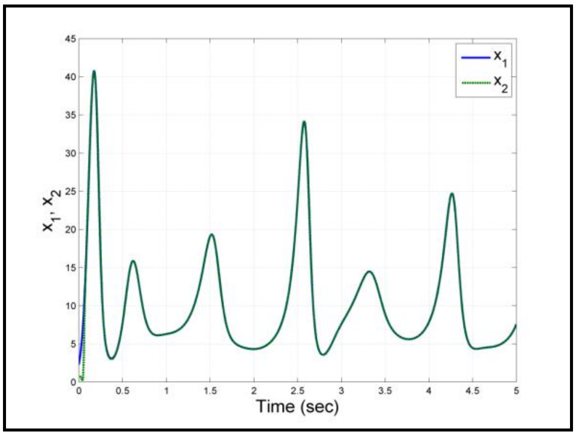

Figure5.Complete synchronization of the states $x_1(t), x_2(t)$

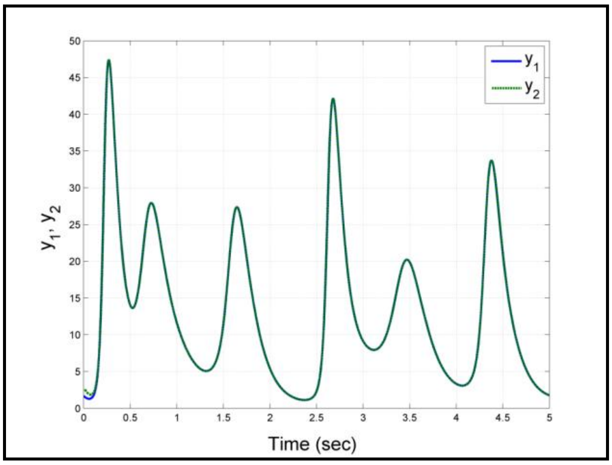

Figure6. Complete synchronization of the states $y_1(t), y_2(t)$

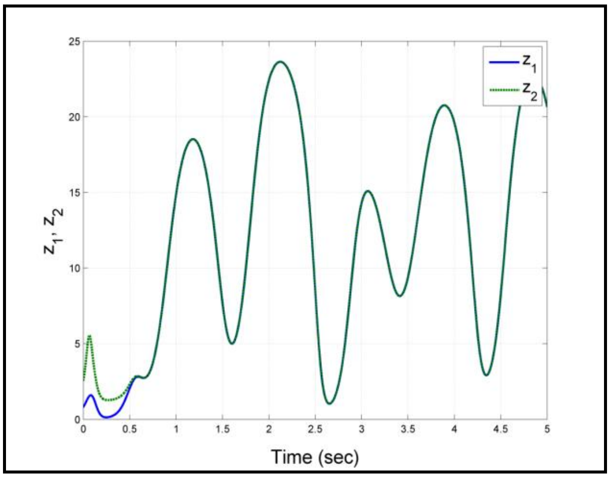

Figure7. Complete synchronization of the states $z_1(t), z_2(t)$

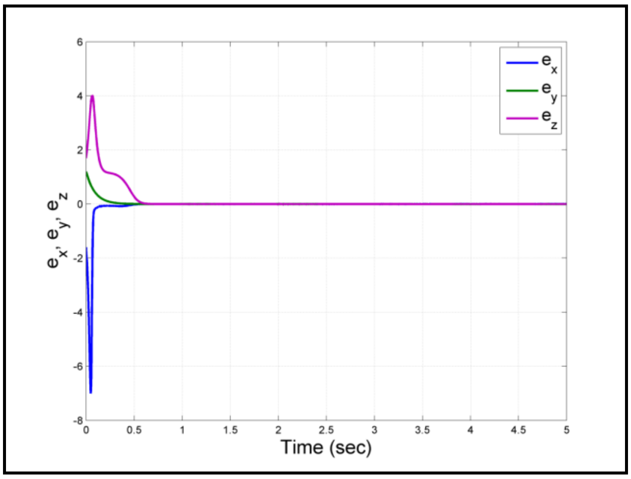

Figure 8. Time-history of the chaos synchronization errors $e_x(t), e_y(t), e_z(t)$

Conclusions

In this paper, new results have been derived for the analysis and adaptive synchronization of a chemical chaotic attractor discovered by Haung (2005). First, the paper discussed the qualitative properties, Lyapunov exponents, stability of equilibrium point at the originand phase portraits of the chemical chaotic attractor discovered by Haung. Then this paperderived new results for the adaptive synchronizer for the global chaos synchronization of the states of the identical chemical chaotic reactors. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

References

1. Azar, A. T., and Vaidyanathan, S., Chaos Modeling and Control Systems Design, Studies in Computational Intelligence, Vol. 581, Springer, New York, USA, 2015.
2. Azar, A. T., and Vaidyanathan, S., Computational Intelligence Applications in Modeling and Control, Studies in Computational Intelligence, Vol. 575, Springer, New York, USA, 2015.
3. Lorenz, E. N., Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, 1963, 20, 130-141.
4. Rössler, O. E., An equation for continuous chaos, Physics Letters A, 1976, 57, 397-398.
5. Arneodo, A., Coullet, P., and Tresser, C., Possible new strange attractors with spiral structure, Communications in Mathematical Physics, 1981, 79, 573-579.
6. Sprott, J. C., Some simple chaotic flows, Physical Review E, 1994, 50, 647-650.
7. Chen, G., and Ueta, T., Yet another chaotic attractor, International Journal of Bifurcation and Chaos, 1999, 9, 1465-1466.
8. Lü, J., and Chen, G., A new chaotic attractor coined, International Journal of Bifurcation and Chaos, 2002, 12, 659-661.
9. Cai, G., and Tan, Z., Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain Systems, 2007, 1, 235-240.
10. Tigan, G., and Opris, D., Analysis of a 3D chaotic system, Chaos, Solitons and Fractals, 2008, 36, 1315-1319.
11. Sundarapandian, V., and Pehlivan, I., Analysis, control, synchronization and circuit design of a novel chaotic system, Mathematical and Computer Modelling, 2012, 55, 1904-1915.
12. Sundarapandian, V., Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, 2013, 6, 45-52.
13. Vaidyanathan, S., A new six-term 3-D chaotic system with an exponential nonlinearity, Far East Journal of Mathematical Sciences, 2013, 79, 135-143.
14. Vaidyanathan, S., Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters, Journal of Engineering Science and Technology Review, 2013, 6, 53-65.
15. Vaidyanathan, S., A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East Journal of Mathematical Sciences, 2014, 84, 219-226.
16. Vaidyanathan, S., Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2014, 22, 41-53.
17. Vaidyanathan, S., and Madhavan, K., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system, International Journal of Control Theory and Applications, 2013, 6, 121-137.
18. Vaidyanathan, S., Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities, European Physical Journal: Special Topics, 2014, 223, 1519-1529.
19. Vaidyanathan, S., Volos, C., Pham, V.-T., Madhavan, K., and Idowu, B. A., Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities, Archives of Control Sciences, 2014, 24, 257-285.
20. Vaidyanathan, S., Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control, International Journal of Modelling, Identification and Control, 2014, 22, 207-217.
21. Pehlivan, I., Moroz, I. M., and Vaidyanathan, S., Analysis, synchronization and circuit design of a novel butterfly attractor, Journal of Sound and Vibration, 2014, 333, 5077-5096.
22. Jafari, S., and Sprott, J. C., Simple chaotic flows with a line equilibrium, Chaos, Solitons and Fractals, 2013, 57, 79-84.
23. Pham, V.-T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristic neural network, Optoelectronics and Advanced Materials – Rapid Communications, 2014, 8, 1157-1163.
24. Shimizu, K., Sekikawa, M., and Inaba, N., Mixed-mode oscillations and chaos from a simple second-order oscillator under weak periodic perturbation, Physics Letters A, 2011, 375, 1566-1569.
25. Behnia, S., Afrang, S., Akhshani, A., and Mabhouti, Kh., A novel method for controlling chaos in external cavity semiconductor laser, Optik-International Journal for Light and Electron Optics, 2013, 124, 757-764.
26. Yuan, G., Zhang, X., and Wang, Z., Generation and synchronization of feedback-induced chaos in

semiconductor ring lasers by injection-locking, Optik-International Journal for Light and Electron Optics, 2014, 125, 1950-1953.
27. Kyriazis, M., Applications of chaos theory to the molecular biology of aging, Experimental Gerontology, 1991, 26, 569-572.
28. Gaspard, P., Microscopic chaos and chemical reactions, Physica A: Statistical Mechanics and Its Applications, 1999, 263, 315-328.
29. Li, Q. S., and Zhu, R., Chaos to periodicity and periodicity to chaos by periodic perturbations in the Belousov-Zhabotinsky reaction, Chaos, Solitons & Fractals, 2004, 19, 195-201.
30. Villegas, M., Augustin, A., Gilg, A., Hmaidi, A., and Wever, U., Application of the polynomial chaos expansion to the simulation of chemical reactors with uncertainties, Mathematics and Computers in Simulation, 2012, 82, 805-817.
31. Aihira, K., Takabe, T., and Toyoda, M., Chaotic neural networks, Physics Letters A, 1990, 144, 333-340.
32. Huang, W. Z., and Huang, Y., Chaos of a new class of Hopfield neural networks, Applied Mathematics and Computation, 2008, 206, 1-11.
33. Lankalapalli, S., and Ghosal, A., Chaos in robot control equations, International Journal of Bifurcation and Chaos, 1997, 7, 707-720.
34. Nakamura, Y., and Sekiguchi, A., The chaotic mobile robot, IEEE Transactions on Robotics and Automation, 2001, 17, 898-904.
35. Shi, Z., Hong, S., and Chen, K., Experimental study on tracking the state of analog Chua's circuit with particle filter for chaos synchronization, Physics Letters A, 2008, 372, 5575-5580.
36. Matouk, A. E., and Agiza, H. N., Bifurcations, chaos and synchronization in ADVP circuit with parallel resistor, Journal of Mathematical Analysis and Applications, 2008, 341, 259-269.
37. Vaidyanathan, S., and Rajagopal K., Anti-synchronization of Li and T chaotic systems by active nonlinear control, Communications in Computer and Information Science, 2011, 198, 175-184.
38. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control, Communications in Computer and Information Science, 2011, 204, 84-93.
39. Sarasu, P., and Sundarapandian, V., Active controller design for generalized projective synchronization of four-scroll chaotic systems, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 26-33.
40. Vaidyanathan, S., and Rajagopal, K., Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control, International Journal of Systems Signal Control and Engineering Applications, 2011, 4, 55-61.
41. Vaidyanathan, S., Hybrid chaos synchronization of Liu and Lü systems by active nonlinear control, Communications in Computer and Information Science, 2011, 204, 1-10.
42. Sarasu, P., and Sundarapandian, V., The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control, International Journal of Soft Computing, 2011, 6, 216-223.
43. Vaidyanathan, S., and Pakiriswamy, S., The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems, Communications in Computer and Information Science, 2011, 245, 231-238.
44. Sundarapandian, V., and Karthikeyan, R., Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control, Journal of Engineering and Applied Sciences, 2012, 7, 254-264.
45. Karthikeyan, R., and Sundarapandian, V., Hybrid chaos synchronization of four-scroll systems via active control, Journal of Electrical Engineering, 2014, 65, 97-103.
46. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of Lu and Pan chaotic systems by adaptive nonlinear control, International Journal of Soft Computing, 2011, 6, 111-118.
47. Sundarapandian, V., and Karthikeyan, R., Adaptive anti-synchronization of uncertain Tigan and Li systems, Journal of Engineering and Applied Sciences, 2012, 7, 45-52.
48. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of three-scroll chaotic systems via adaptive control, European Journal of Scientific Research, 2012, 72, 504-522.
49. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control, International Journal of Soft Computing, 2012, 7, 28-37.
50. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of two-scroll systems via adaptive control, International Journal of Soft Computing, 2012, 7, 146-156.

51. Vaidyanathan, S., Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control, International Journal of Control Theory and Applications, 2012, 5, 41-59.
52. Vaidyanathan, S., Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, 2013, 177, 1-10.
53. Sundarapandian, V., Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lecture Notes in Electrical Engineering, 2013, 131, 319-327.
54. Pham, V.T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristive neural network, Optoelectronics and Advanced Materials: Rapid Communications, 2014, 8, 1157-1163.
55. Pham, V.T, Volos, Ch.K., and Vaidyanathan, S., Multi-scroll chaotic oscillator based on a first-order delay differential equation, Studies in Computational Intelligence, 2015, 581, 59-72.
56. Vaidyanathan, S., Volos, Ch. K., Kyprianidis, I. M., Stouboulos, I. N., and Pham, V.-T., Analysis, adaptive control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 24-36.
57. Vaidyanathan, S., and Pakiriswamy, S., A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control, Journal of Engineering Science and Technology Review, 2015, 52-60.
58. Vaidyanathan, S., Rajagopal, K., Volos, Ch. K., Kyprianidis, I. M., and Stouboulos, I. N., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW, Journal of Engineering Science and Technology Review, 2015, 8, 130-141.
59. Vaidyanathan, S., Volos, Ch. K., and Pham, V.-T., Analysis, adaptive control and synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 181-191.
60. Vaidyanathan, S., Volos, Ch. K., and Pham, V.-T., Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium, Journal of Engineering Science and Technology Review, 2015, 8, 232-244.
61. Vaidyanathan, S., and Sampath, S., Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control, Communications in Computer and Information Science, 2011, 205, 156-164.
62. Sundarapandian, V., and Sivaperumal, S., Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, 2011, 6, 224-231.
63. Vaidyanathan, S., and Sampath, S., Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, 2012, 9, 274-279.
64. Vaidyanathan, S., Global chaos control of hyperchaotic Liu system via sliding control method, International Journal of Control Theory and Applications, 2012, 5, 117-123.
65. Vaidyanathan, S., Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, 2012, 5, 15-20.
66. Vaidyanathan, S., Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control, International Journal of Modelling, Identification and Control, 2014, 22, 170-177.
67. Vaidyanathan, S., and Azar, A.T., Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems, Studies in Computational Intelligence, 2015, 576, 527-547.
68. Vaidyanathan, S., and Azar, A.T., Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems, Studies in Computational Intelligence, 2015, 576, 549-569.
69. Suresh, R., and Sundarapandian, V., Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with a recursive feedback, Far East Journal of Mathematical Sciences, 2013, 73, 73-95.
70. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback, Malaysian Journal of Mathematical Sciences, 2013, 7, 219-246.
71. Rasappan, S., and Vaidyanathan, S., Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design, Kyungpook Mathematical journal, 2014, 54, 293-320.
72. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback, Arabian Journal for Science and Engineering, 2014, 39, 3351-3364.

73. Vaidyanathan, S., Idowu, B.A., and Azar, A.T., Backstepping controller design for the global chaos synchronization of Sprott's jerk systems, Studies in Computational Intelligence, 2015, 581, 39-58.
74. Haung, Y., Chaoticity of some chemical attractors: a computer assisted proof, Journal of Mathematical Chemistry, 2005, 38, 107-117.
75. Hahn, W., Stability of Motion, Springer, New York, USA,1967.

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