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A Novel Chemical Chaotic Reactor System and its Output Regulation via Integral Sliding Mode Control

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. In this paper, a new chemical chaotic reactor is derived by modifying the chemical chaotic reactor system obtained by the Huang (2005). This paper gives a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, new results are obtained for the output regulation of the novel chemical chaotic reactor system. MATLAB plots have been shown to illustrate the phase portraits of the novel chemical chaotic attractor and the output regulation of the novel chemical chaotic reactor system via integral sliding mode control.
Keywords: Chaos, chaotic systems, chemical reactor, chemical engineering, sliding mode control, output regulation, chaos control, stability.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal of anti-synchronization is to device a feedback mechanism so that the state trajectories of the master and slave systems are equal in magnitude and opposite in sign asymptotically. Because of the butterfly effect which causes exponential divergence of two trajectories of the system starting from nearby initial conditions, the anti-synchronization of chaotic systems is seemingly a challenging research problem.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-

100], intelligent control [101-110], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [111-128], biology [129-160], memristors [161-163], electrical circuits [164], etc.

This paper investigates first the qualitative properties of a chemical chaotic reactor model discovered by Huang in 2005 [165]. Huang derived the chemical reactor model by considering reactor dynamics with five steps (2 reversible and 3 non-reversible). Then a novel chemical chaotic reactor model is derived. The qualitative properties of the novel chemical chaotic reactor model are described. This paper also derives new results for the global chaos control of the novel chemical chaotic reactor model via integral sliding mode method. MATLAB plots are shown to illustrate the phase portraits and global chaos control of the novel chemical chaotic reactor.

2. Huang's Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics of Huang and Yang [165] consist of the following five steps given below.

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2X \tag{1a}$$

$$X + Y \xrightarrow{k_2} 2Y \tag{1b}$$

$$A_5 + Y \xrightarrow{k_3} A_2 \tag{1c}$$

$$X + Z \xrightarrow{k_4} A_3 \tag{1d}$$

$$A_4 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftarrows}} 2Z \tag{1e}$$

Equations (1a) and (1e) indicate reversible steps, while equations (1b), (1c) and (1d) indicate non-reversible steps of the Huang chemical reactor [165]. In (1), A_1, A_4, A_5 are initiators and A_2, A_3 are products. The intermediates whose dynamics are followed are X, Y and Z.

Assuming an ideal mixture and a well-stirred reactor, the macroscopic rate equations for the Huang's chemical reactor can be written in non-dimensionalized form as

$$\begin{cases} \dot{x} = a_1 x - k_{-1} x^2 - xy - xz \\ \dot{y} = xy - a_5 y \\ \dot{z} = a_4 z - xz - k_{-5} z^2 \end{cases} \tag{2}$$

In (2), x, y, z are the mole fractions of X, Y and Z. Also, the rate constants k_1, k_3 and k_5 are incorporated in the parameters a_1, a_4 and a_5.

To simplify the notations, we rename the constants and express the chemical reactor system (2) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \tag{3}$$

The system (3) is *chaotic* when the system parameters are chosen as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5 \tag{4}$$

For numerical simulations, we take the initial conditions

$$x(0) = 1.8, \quad y(0) = 2.5, \quad z(0) = 0.6 \tag{5}$$

The 3-D phase portrait of the chemical chaotic reactor (2) is depicted in Figure 1.

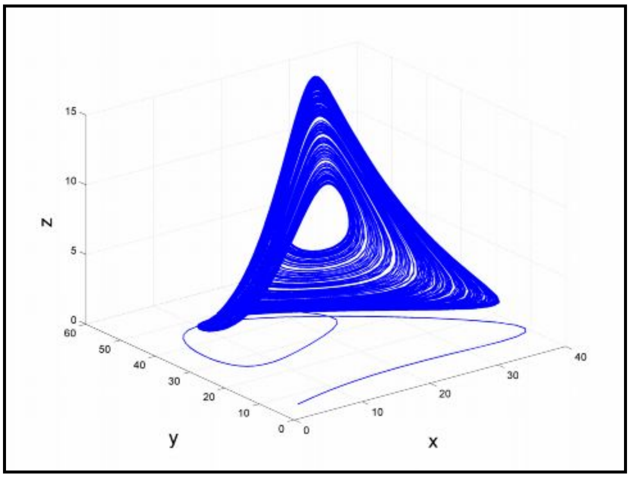

Figure 1. The 3-D phase portrait of the Huang chemical chaotic reactor

The Lyapunov exponents of the Huang's chemical chaotic attractor (3) are derived in MATLAB as

$$L_1 = 0.4001, \quad L_2 = 0, \quad L_3 = -11.8762 \tag{6}$$

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0337 \tag{7}$$

3. A Novel Chemical Chaotic Reactor System

In this section, we propose a novel chemical chaotic reactor system by modifying Huang's system (3) as

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz \\ \dot{y} = xy + rx - cy \\ \dot{z} = bz - xz - qz^2 \end{cases} \tag{8}$$

The novel 3-D system (8) is *chaotic* when the parameter values are taken as

$$a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5, \quad r = 0.01 \tag{9}$$

For numerical simulations, we take the initial conditions

$$x(0) = 0.1, \quad y(0) = 0.2, \quad z(0) = 0.1 \tag{10}$$

The 3-D phase portrait of the novel chemical chaotic reactor (8) is depicted in Figure 2. The 2-D projections of the strange attractor of the novel chemical reactor (8) on the (x, y), (y, z) and (x, z) coordinate planes are depicted in Figures 3-5, respectively.

The Lyapunov exponents of the novel chemical chaotic reactor (8) are obtained as

$$L_1 = 0.4354, \quad L_2 = 0, \quad L_3 = -11.9273 \tag{11}$$

Also, the Lyapunov dimension of the novel chemical chaotic reactor (8) is obtained as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0365 \tag{12}$$

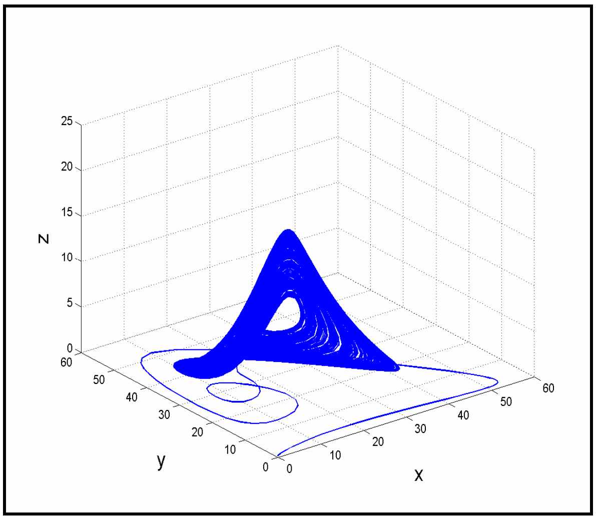

Figure 2. The 3-D phase portrait of the novel chemical chaotic reactor (8)

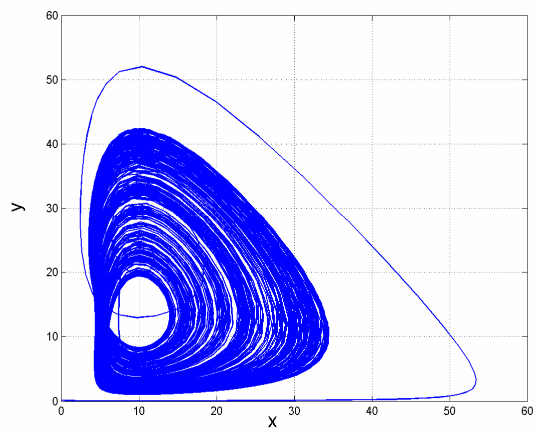

Figure 3. The 2-D projection of the novel chemical chaotic reactor (8) on the (x, y) plane

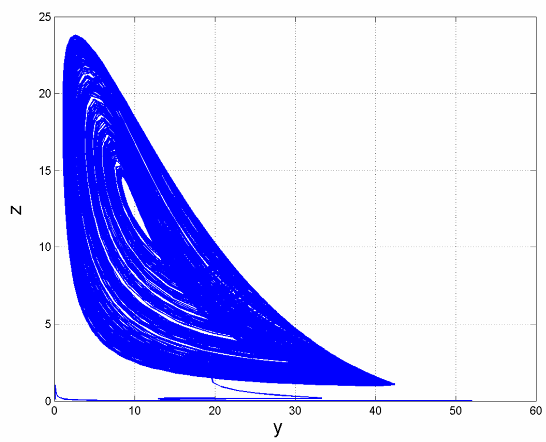

Figure 4. The 2-D projection of the novel chemical chaotic reactor (8) on the (y, z) plane

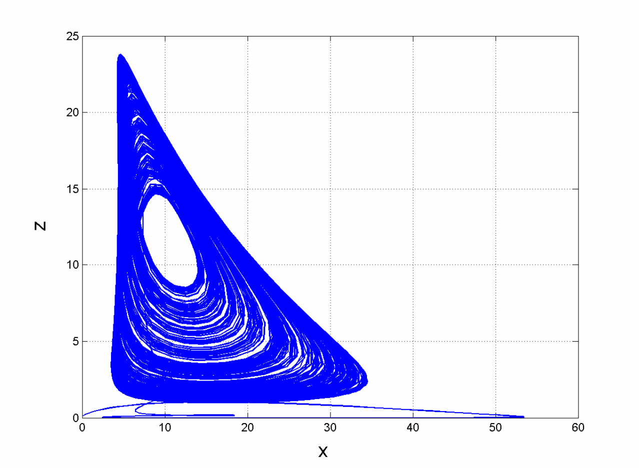

Figure 5. The 2-D projection of the novel chemical chaotic reactor (8) on the (x, z) plane

4. Output Regulation of the Novel Chemical Chaotic Reactor

In this section, we use integral sliding mode control method to regulate the states of the novel chemical chaotic reactor with unknown system parameters. We use Lyapunov stability theory to prove the main result derived in this section.

Thus, we consider the novel chemical reactor dynamics given by

$$\begin{cases} \dot{x} = ax - px^2 - xy - xz + u_x \\ \dot{y} = xy + rx - cy + u_y \\ \dot{z} = bz - xz - qz^2 + u_z \end{cases} \tag{13}$$

In (13), x, y, z are the states of the system. The design goal is to find feedback controls u_x, u_y, u_z so that the system states x, y, z are regulated to the reference values $\alpha_x, \alpha_y, \alpha_z$, respectively.

Thus, the regulation errors are defined by

$$\begin{cases} e_x(t) = x(t) - \alpha_x \\ e_y(t) = y(t) - \alpha_y \\ e_z(t) = z(t) - \alpha_z \end{cases} \tag{14}$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_x = ax - px^2 - xy - xz + u_x \\ \dot{e}_y = xy + rx - cy + u_y \\ \dot{e}_z = bz - xz - qz^2 + u_z \end{cases} \tag{15}$$

Based on the sliding mode control theory [166], the integral sliding surface of each error variable is defined as follows:

$$\begin{cases} s_x = \left[\dfrac{d}{dt} + \lambda_x\right]\left[\displaystyle\int_0^t e_x(\tau)d\tau\right] = e_x + \lambda_x \displaystyle\int_0^t e_x(\tau)d\tau \\ s_y = \left[\dfrac{d}{dt} + \lambda_y\right]\left[\displaystyle\int_0^t e_y(\tau)d\tau\right] = e_y + \lambda_y \displaystyle\int_0^t e_y(\tau)d\tau \\ s_z = \left[\dfrac{d}{dt} + \lambda_z\right]\left[\displaystyle\int_0^t e_z(\tau)d\tau\right] = e_z + \lambda_z \displaystyle\int_0^t e_z(\tau)d\tau \end{cases} \tag{16}$$

The derivative of each equation in (16) yields

$$\begin{cases} \dot{s}_x = \dot{e}_x + \lambda_x e_x \\ \dot{s}_y = \dot{e}_y + \lambda_y e_y \\ \dot{s}_z = \dot{e}_z + \lambda_z e_z \end{cases} \tag{17}$$

The Hurwitz condition is satisfied if $\lambda_x, \lambda_y, \lambda_z$ are positive constants.

Based on the exponential reaching law [166], we set

$$\begin{cases} \dot{s}_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ \dot{s}_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \\ \dot{s}_z = -\eta_z \operatorname{sgn}(s_z) - k_z s_z \end{cases} \tag{18}$$

Comparing equations (17) and (18), we get

$$\begin{cases} \dot{e}_x + \lambda_x e_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ \dot{e}_y + \lambda_y e_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \\ \dot{e}_z + \lambda_z e_z = -\eta_z \operatorname{sgn}(s_z) - k_z s_z \end{cases} \tag{19}$$

Using Eq. (15), we can rewrite Eq. (19) as follows:

$$\begin{cases} ax - px^2 - xy - xz + u_x + \lambda_x e_x = -\eta_x \operatorname{sgn}(s_x) - k_x s_x \\ xy + rx - cy + u_y + \lambda_y e_y = -\eta_y \operatorname{sgn}(s_y) - k_y s_y \\ bz - xz - qz^2 + u_z + \lambda_z e_z = -\eta_z \operatorname{sgn}(s_z) - k_z s_z \end{cases} \tag{20}$$

From Eq. (20), the control laws are obtained as follows:

$$\begin{cases} u_x = -ax + px^2 + xy + xz - \lambda_x e_x - \eta_x \operatorname{sgn}(s_x) - k_x s_x \\ u_y = -xy - rx + cy - \lambda_y e_y - \eta_y \operatorname{sgn}(s_y) - k_y s_y \\ u_z = -bz + xz + qz^2 - \lambda_z e_z - \eta_z \operatorname{sgn}(s_z) - k_z s_z \end{cases} \tag{21}$$

Next, we state and prove the main result of this section.

Theorem 1. The states x, y, z of the novel chemical chaotic reactor (13) are regulated to track the constant reference signals $\alpha_x, \alpha_y, \alpha_z$, respectively as $t \to \infty$ for all initial conditions $(x(0), y(0), z(0)) \in R^3$ by the integral sliding mode control law (21), where the constants $\lambda_x, \lambda_y, \lambda_z$, η_x, η_y, η_z, k_x, k_y, k_z are all positive.

Proof. This result is proved using Lyapunov stability theory [167].

We consider the following quadratic Lyapunov function

$$V(s_x, s_y, s_z) = \frac{1}{2}\left(s_x^2 + s_y^2 + s_z^2\right) \tag{22}$$

where s_x, s_y, s_z are as defined in (16).

The time-derivative of V is obtained as

$$\dot{V} = s_x \dot{s}_x + s_y \dot{s}_y + s_z \dot{s}_z \tag{23}$$

Substituting from Eq. (18) into (23), we get

$$\dot{V} = s_x[-\eta_x \operatorname{sgn}(s_x) - k_x s_x] + s_y[-\eta_y \operatorname{sgn}(s_y) - k_y s_y] + s_z[-\eta_z \operatorname{sgn}(s_z) - k_z s_z] \quad (24)$$

Simplifying Eq. (24), we obtain

$$\dot{V} = -\eta_x |s_x| - k_x s_x^2 - \eta_y |s_y| - k_y s_y^2 - \eta_z |s_z| - k_z s_z^2 \quad (25)$$

Since $k_x, k_y, k_z > 0$ and $\eta_x, \eta_y, \eta_z > 0$, it follows from (25) that $\dot{V}$ is a negative definite function.

Thus, by Lyapunov stability theory [167], it follows that $(s_x, s_y, s_z) \to (0,0,0)$ as $t \to \infty$.

Hence, it is immediate that $(e_x, e_y, e_z) \to (0,0,0)$ as $t \to \infty$.

This completes the proof. ■

5. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (13) when the integral sliding mode controller (21) is implemented.

The parameter values of the novel chemical reactor (13) are taken as in the chaotic case, viz.

$$a = 30, \; b = 16.5, \; c = 10, \; p = 0.5, \; q = 0.5, \; r = 0.01 \quad (26)$$

We take the sliding constants as

$$\lambda_x = \lambda_y = \lambda_z = 0.1, \eta_x = \eta_y = \eta_z = 0.1, k_x = k_y = k_z = 25 \quad (27)$$

We take the initial conditions of the novel chemical reactor (13) as

$$x(0) = 2.6, \; y(0) = 5.9, \; z(0) = 0.8 \quad (28)$$

We take the constant reference signals as

$$\alpha_x = 1, \; \alpha_y = 2, \; \alpha_z = 3 \quad (29)$$

Figure 6 shows the output regulation of the novel chemical reactor (13).

Figure 7 shows the time-history of the output regulation errors e_x, e_y, e_z.

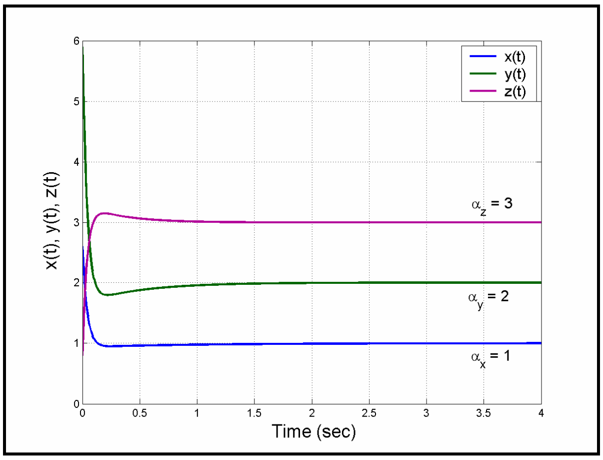

Figure 6. Output regulation of the novel chemical chaotic reactor

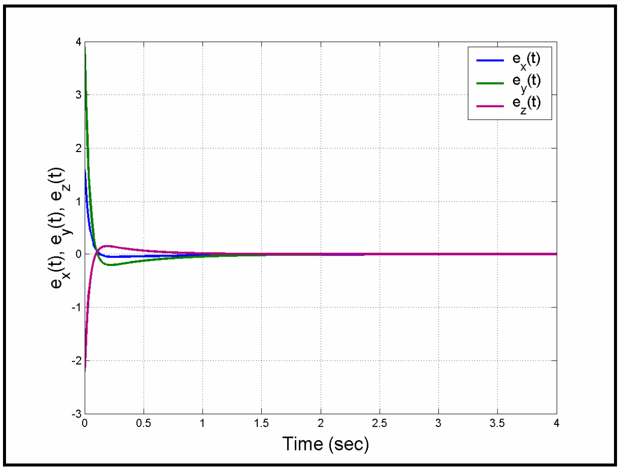

Figure 7. Time-history of the regulation errors $e_x(t), e_y(t), e_z(t)$

6. Conclusions

In this paper, a new chemical chaotic reactor is derived by modifying the chemical chaotic reactor system obtained by the Huang (2005). We gave a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, new results were obtained for the output regulation of the novel chemical chaotic reactor system. MATLAB plots were shown to illustrate the phase portraits of the novel chemical chaotic attractor and the output regulation of the novel chemical chaotic reactor system via integral sliding mode control method.

References

1. Azar, A. T., and Vaidyanathan, S., Chaos Modeling and Control Systems Design, Studies in Computational Intelligence, Vol. 581, Springer, New York, USA, 2015.
2. Azar, A. T., and Vaidyanathan, S., Computational Intelligence Applications in Modeling and Control, Studies in Computational Intelligence, Vol. 575, Springer, New York, USA, 2015.
3. Lorenz, E. N., Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, 1963, 20, 130-141.
4. Rössler, O. E., An equation for continuous chaos, Physics Letters A, 1976, 57, 397-398.
5. Arneodo, A., Coullet, P., and Tresser, C., Possible new strange attractors with spiral structure, Communications in Mathematical Physics, 1981, 79, 573-579.
6. Sprott, J. C., Some simple chaotic flows, Physical Review E, 1994, 50, 647-650.
7. Chen, G., and Ueta, T., Yet another chaotic attractor, International Journal of Bifurcation and Chaos, 1999, 9, 1465-1466.
8. Lü, J., and Chen, G., A new chaotic attractor coined, International Journal of Bifurcation and Chaos, 2002, 12, 659-661.
9. Cai, G., and Tan, Z., Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain Systems, 2007, 1, 235-240.
10. Tigan, G., and Opris, D., Analysis of a 3D chaotic system, Chaos, Solitons and Fractals, 2008, 36, 1315-1319.
11. Sundarapandian, V., and Pehlivan, I., Analysis, control, synchronization and circuit design of a novel chaotic system, Mathematical and Computer Modelling, 2012, 55, 1904-1915.
12. Sundarapandian, V., Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, 2013, 6, 45-52.
13. Vaidyanathan, S., A new six-term 3-D chaotic system with an exponential nonlinearity, Far East Journal of Mathematical Sciences, 2013, 79, 135-143.
14. Vaidyanathan, S., Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters, Journal of Engineering Science and Technology Review, 2013, 6, 53-65.
15. Vaidyanathan, S., A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East Journal of Mathematical Sciences, 2014, 84, 219-226.
16. Vaidyanathan, S., Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2014, 22, 41-53.

17. Vaidyanathan, S., and Madhavan, K., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system, International Journal of Control Theory and Applications, 2013, 6, 121-137.
18. Vaidyanathan, S., Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities, European Physical Journal: Special Topics, 2014, 223, 1519-1529.
19. Vaidyanathan, S., Volos, C., Pham, V. T., Madhavan, K., and Idowu, B. A., Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities, Archives of Control Sciences, 2014, 24, 257-285.
20. Vaidyanathan, S., Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control, International Journal of Modelling, Identification and Control, 2014, 22, 207-217.
21. Vaidyanathan, S., and Azar, A.T., Analysis and control of a 4-D novel hyperchaotic system, Studies in Computational Intelligence, 2015, 581, 3-17.
22. Vaidyanathan, S., Volos, C., Pham, V.T., and Madhavan, K., Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation, Archives of Control Sciences, 2015, 25, 135-158.
23. Vaidyanathan, S., Volos, C., and Pham, V.T., Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation, Archives of Control Sciences, 2014, 24, 409-446.
24. Vaidyanathan, S., A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control, International Journal of Control Theory and Applications, 2013, 6, 97-109.
25. Vaidyanathan, S., Analysis, properties and control of an eight-term 3-D chaotic system with an exponential nonlinearity, International Journal of Modelling, Identification and Control, 2015, 23, 164-172.
26. Vaidyanathan, S., Azar, A.T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23, 267-277.
27. Vaidyanathan, S., Qualitative analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with a quartic nonlinearity, International Journal of Control Theory and Applications, 2014, 7, 1-20.
28. Vaidyanathan, S., Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities, International Journal of Control Theory and Applications, 2014, 7, 35-47.
29. Vaidyanathan, S., and Pakiriswamy, S., A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control, Journal of Engineering Science and Technology Review, 2015, 8, 52-60.
30. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, adaptive control and adaptive synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 181-191.
31. Vaidyanathan, S., Rajagopal, K., Volos, C.K., Kyprianidis, I.M., and Stouboulos, I.N., Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW, Journal of Engineering Science and Technology Review, 2015, 8, 130-141.
32. Pham, V.T., Volos, C.K., Vaidyanathan, S., Le, T.P., and Vu, V.Y., A memristor-based hyperchaotic system with hidden attractors: Dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8, 205-214.
33. Pham, V.T., Volos, C.K., and Vaidyanathan, S., Multi-scroll chaotic oscillator based on a first-order delay differential equation, Studies in Computational Intelligence, 2015, 581, 59-72.
34. Vaidyanathan, S., Volos, C.K., Kyprianidis, I.M., Stouboulos, I.N., and Pham, V.T., Analysis, adaptive control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation, Journal of Engineering Science and Technology Review, 2015, 8, 24-36.
35. Vaidyanathan, S., Volos, C.K., and Pham, V.T., Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium, Journal of Engineering Science and Technology Review, 2015, 8, 232-244.
36. Sampath, S., Vaidyanathan, S., Volos, C.K., and Pham, V.T., An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation, Journal of Engineering Science and

Technology Review, 2015, 8, 1-6.
37. Vaidyanathan, S., A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters, Journal of Engineering Science and Technology Review, 2015, 8, 106-115.
38. Vaidyanathan, S., Pham, V.-T., and Volos, C. K., A 5-D hyperchaotic Rikitake dynamo system with hidden attractors, European Physical Journal: Special Topics, 2015, 224, 1575-1592.
39. Pham, V.-T., Vaidyanathan, S., Volos, C. K., and Jafari, S., Hidden attractors in a chaotic system with an exponential nonlinear term, European Physical Journal: Special Topics, 2015, 224, 1507-1517.
40. Vaidyanathan, S., Hyperchaos, qualitative analysis, control and synchronisation of a ten-term 4-D hyperchaotic system with an exponential nonlinearity and three quadratic nonlinearities, International Journal of Modelling, Identification and Control, 2015, 23, 380-392.
41. Vaidyanathan, S., and Azar, A. T., Analysis, control and synchronization of a nine-term 3-D novel chaotic system, Studies in Computational Intelligence, 2015, 581, 19-38.
42. Vaidyanathan, S., and Volos, C., Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system, Archives of Control Sciences, 2015, 25, 279-299.
43. Vaidyanathan, S., Analysis, control and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities, Kyungpook Mathematical Journal, 2015, 55, 563-586.
44. Pehlivan, I., Moroz, I. M., and Vaidyanathan, S., Analysis, synchronization and circuit design of a novel butterfly attractor, Journal of Sound and Vibration, 2014, 333, 5077-5096.
45. Pham, V. T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristic neural network, Optoelectronics and Advanced Materials – Rapid Communications, 2014, 8, 1157-1163.
46. Sundarapandian, V., Output regulation of Van der Pol oscillator, Journal of the Institution of Engineers (India): Electrical Engineering Division, 88, 20-24, 2007.
47. Sundarapandian, V., Output regulation of the Lorenz attractor, Far East Journal of Mathematical Sciences, 2010, 42, 289-299.
48. Vaidyanathan, S., and Rajagopal, K., Anti-synchronization of Li and T chaotic systems by active nonlinear control, Communications in Computer and Information Science, 2011, 198, 175-184.
49. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control, Communications in Computer and Information Science, 2011, 198, 10-17.
50. Vaidyanathan, S., Output regulation of the unified chaotic system, Communications in Computer and Information Science, 2011, 198, 1-9.
51. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control, 2011, 198, 84-93.
52. Vaidyanathan, S., Hybrid chaos synchronization of Liu and Lu systems by active nonlinear control, Communications in Computer and Information Science, 2011, 204, 1-10.
53. Sarasu, P., and Sundarapandian, V., Active controller design for generalized projective synchronization of four-scroll chaotic systems, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 26-33.
54. Vaidyanathan, S., and Rasappan, S., Hybrid synchronization of hyperchaotic Qi and Lu systems by nonlinear control, Communications in Computer and Information Science, 2011, 131, 585-593.
55. Vaidyanathan, S., and Rajagopal, K., Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 55-61.
56. Vaidyanathan, S., Output regulation of Arneodo-Coullet chaotic system, Communications in Computer and Information Science, 2011, 133, 98-107.
57. Sarasu, P., and Sundarapandian, V., The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control, International Journal of Soft Computing, 2011, 6, 216-223.
58. Vaidyanathan, S., and Pakiriswamy, S., The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems, Communications in Computer and Information Science, 2011, 245, 231-238.
59. Sundarapandian, V., and Karthikeyan, R., Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control, Journal of Engineering and Applied Sciences, 2012, 7, 254-264.
60. Vaidyanathan, S., and Pakiriswamy, S., Generalized projective synchronization of double-scroll chaotic

systems using active feedback control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 84, 111-118.
61. Pakiriswamy, S., and Vaidyanathan, S., Generalized projective synchronization of three-scroll chaotic systems via active control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 85, 146-155.
62. Karthikeyan, R., and Sundarapandian, V., Hybrid chaos synchronization of four-scroll systems via active control, Journal of Electrical Engineering, 2014, 65, 97-103.
63. Vaidyanathan, S., Azar, A. T., Rajagopal, K., and Alexander, P., Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control, International Journal of Modelling, Identification and Control, 2015, 23, 267-277.
64. Yassen, M. T., Chaos synchronization between two different chaotic systems using active control, Chaos, Solitons and Fractals, 2005, 23, 131-140.
65. Jia, N., and Wang, T., Chaos control and hybrid projective synchronization for a class of new chaotic systems, Computers and Mathematics with Applications, 2011, 62, 4783-4795.
66. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control, Communication in Computer and Information Science, 2011, 205, 193-202.
67. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control, International Journal of Soft Computing, 2011, 6, 111-118.
68. Vaidyanathan, S., Adaptive controller and synchronizer design for the Qi-Chen chaotic system, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunication Engineering, 2012, 85, 124-133.
69. Sundarapandian, V., Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lectures on Electrical Engineering, 2013, 131, 319-327.
70. Vaidyanathan, S., Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, 2013, 177, 1-10.
71. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control, Communications in Computer and Information Science, 2011, 205, 193-202.
72. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of Lü and Pan systems by adaptive nonlinear control, European Journal of Scientific Research, 2011, 64, 94-106.
73. Sundarapandian, V., and Karthikeyan, R., Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control, International Journal of Systems Signal Control and Engineering Application, 2011, 4, 18-25.
74. Sundarapandian, V., and Karthikeyan, R., Adaptive anti-synchronization of uncertain Tigan and Li systems, Journal of Engineering and Applied Sciences, 2012, 7, 45-52.
75. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of three-scroll chaotic systems via adaptive control, European Journal of Scientific Research, 2012, 72, 504-522.
76. Vaidyanathan, S., and Rajagopal, K., Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control, International Journal of Soft Computing, 2012, 7, 28-37.
77. Sarasu, P., and Sundarapandian, V., Generalized projective synchronization of two-scroll systems via adaptive control, International Journal of Soft Computing, 2012, 7, 146-156.
78. Sarasu, P., and Sundarapandian, V., Adaptive controller design for the generalized projective synchronization of 4-scroll systems, International Journal of Systems Signal Control and Engineering Application, 2012, 5, 21-30.
79. Vaidyanathan, S., Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control, International Journal of Control Theory and Applications, 2012, 5, 41-59.
80. Vaidyanathan, S., and Pakiriswamy, S., Generalized projective synchronization of six-term Sundarapandian chaotic systems by adaptive control, International Journal of Control Theory and Applications, 2013, 6, 153-163.
81. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback, Malaysian Journal of Mathematical Sciences, 2013, 7, 219-246.
82. Suresh, R., and Sundarapandian, V., Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback, Far East Journal of Mathematical Sciences, 2013, 73, 73-95.
83. Rasappan, S., and Vaidyanathan, S., Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems

via backstepping control with novel feedback, Archives of Control Sciences, 2012, 22, 343-365.

84. Rasappan, S., and Vaidyanathan, S., Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design, Kyungpook Mathematical Journal, 2014, 54, 293-320.
85. Vaidyanathan, S., and Rasappan, S., Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback, Arabian Journal for Science and Engineering, 2014, 39, 3351-3364.
86. Vaidyanathan, S., Idowu, B. A., and Azar, A. T., Backstepping controller design for the global chaos synchronization of Sprott's jerk systems, Studies in Computational Intelligence, 2015, 581, 39-58.
87. Vaidyanathan, S., Volos, C. K., Rajagopal, K., Kyprianidis, I. M., and Stouboulos, I. N., Adaptive backstepping controller design for the anti-synchronization of identical WINDMI chaotic systems with unknown parameters and its SPICE implementation, Journal of Engineering Science and Technology Review, 2015, 8, 74-82.
88. Vaidyanathan, S., and Sampath, S., Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control, Communications in Computer and Information Science, 2011, 205, 156-164.
89. Sundarapandian, V., and Sivaperumal, S., Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, 2011, 6, 224-231.
90. Vaidyanathan, S., and Sampath, S., Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, 2012, 9, 274-279.
91. Vaidyanathan, S., Analysis and synchronization of the hyperchaotic Yujun systems via sliding mode control, Advances in Intelligent Systems and Computing, 2012, 176, 329-337.
92. Vaidyanathan, S., and Sampath, S., Sliding mode controller design for the global chaos synchronization of Coullet systems, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 84, 103-110.
93. Vaidyanathan, S., and Sampath, S., Hybrid synchronization of hyperchaotic Chen systems via sliding mode control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, 2012, 85, 257-266.
94. Vaidyanathan, S., Global chaos control of hyperchaotic Liu system via sliding control method, International Journal of Control Theory and Applications, 2012, 5, 117-123.
95. Vaidyanathan, S., Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, 2012, 5, 15-20.
96. Vaidyanathan, S., Global chaos synchronisation of identical Li-Wu chaotic systems via sliding mode control, International Journal of Modelling, Identification and Control, 2014, 22, 170-177.
97. Vaidyanathan, S., and Azar, A. T., Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems, Studies in Computational Intelligence, 2015, 576, 527-547.
98. Vaidyanathan, S., and Azar, A. T., Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems, Studies in Computational Intelligence, 2015, 576, 549-569.
99. Vaidyanathan, S., Sampath, S., and Azar, A. T., Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system, International Journal of Modelling, Identification and Control, 2015, 23, 92-100.
100. Li, H., Liao, X., Li, C., and Li, C., Chaos control and synchronization via a novel chatter free sliding mode control strategy, Neurocomputing, 2011, 74, 3212-3222.
101. Inbarani, H. H., Bagyamathi, M., and Azar, A. T., A novel hybrid feature selection method on rough set and improved harmony search, Neural Computing and Applications, 2015, 26 (8), 1859-1880.
102. Anter, A. M., Hassanien, A. E., ElSoud, M.A., and Azar, A. T., Automatic liver parenchyma segmentation system from abdominal CT scans using hybrid techniques, International Journal of Biomedical Engineering and Technology, 2015, 17 (2), 148-167.
103. Azar, A. T., and Hassanien, A. E., Dimensionality reduction of medical big data using neural-fuzzy classifier, Soft Computing, 2015, 19 (4), 1115-1127.
104. Ding, S., Shi, Z., Chen, K., and Azar, A. T., Mathematical modeling and analysis of soft computing, Mathematical Problems in Engineering, 2015, art no. 578321.
105. Mekki, H., Boukhetala, D., and Azar, A. T., Sliding modes for fault tolerant control, Studies in Computational Intelligence, 2015, 576, 407-433.
106. Azar, A. T., and Serrano, F. E., Design and modeling of anti wind up PID controllers, Studies in

Fuzziness and Soft Computing, 2015, 319, 1-44.
107. Azar, A. T., and Serrano, F. E., Adaptive sliding mode control of the furuta pendulum, Studies in Computational Intelligence, 2015, 576, 1-42.
108. Azar, A. T., and Serrano, F. E., Deadbeat control for multivariable systems with time varying delays, Studies in Computational Intelligence, 2015, 581, 97-132.
109. Azar, A. T., and Serrano, F. E., Robust IMC-PID tuning for cascade control systems with gain and phase margin specifications, Neural Computing and Applications, 2014, 25(5), 983-995.
110. Ding, S., Shi, Z., and Azar, A. T., Research and development of advanced computing technologies, Scientific World Journal, 2015, art. No. 239723.
111. Vaidyanathan, S., Adaptive synchronization of chemical chaotic reactors, International Journal of ChemTech Research, 2015, 8 (2), 612-621.
112. Vaidyanathan, S., Adaptive control of a chemical chaotic reactor, International Journal of PharmTech Research, 2015, 8 (3), 377-382.
113. Vaidyanathan, S., Dynamics and control of Brusselator chemical reaction, International Journal of ChemTech Research, 2015, 8 (6), 740-749.
114. Vaidyanathan, S., Anti-synchronization of Brusselator chemical reaction systems via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 759-768.
115. Vaidyanathan, S., Dynamics and control of Tokamak system with symmetric and magnetically confined plasma, International Journal of ChemTech Research, 2015, 8 (6), 795-803.
116. Vaidyanathan, S., Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control, International Journal of ChemTech Research, 2015, 8 (6), 818-827.
117. Vaidyanathan, S., A novel chemical chaotic reactor system and its adaptive control, International Journal of ChemTech Research, 2015, 8 (7), 146-158.
118. Vaidyanathan, S., Adaptive synchronization of novel 3-D chemical chaotic reactor systems, International Journal of ChemTech Research, 2015, 8 (7), 159-171.
119. Vaidyanathan, S., Global chaos synchronization of chemical chaotic reactors via novel sliding mode control method, International Journal of ChemTech Research, 2015, 8 (7), 209-221.
120. Vaidyanathan, S., Sliding mode control of Rucklidge chaotic system for nonlinear double convection, International Journal of ChemTech Research, 2015, 8 (8), 25-35.
121. Vaidyanathan, S., Global chaos synchronization of Rucklidge chaotic systems for double convection via sliding mode control, International Journal of ChemTech Research, 2015, 8 (8), 61-72.
122. Vaidyanathan, S., Anti-synchronization of chemical chaotic reactors via adaptive control method, International Journal of ChemTech Research, 2015, 8 (8), 73-85.
123. Vaidyanathan, S., Adaptive synchronization of Rikitake two-disk dynamo chaotic systems, International Journal of ChemTech Research, 2015, 8 (8), 100-111.
124. Vaidyanathan, S., Adaptive control of Rikitake two-disk dynamo system, International Journal of ChemTech Research, 2015, 8 (8), 121-133.
125. Vaidyanathan, S., State regulation of Rikitake two-disk dynamo chaotic system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 374-386.
126. Vaidyanathan, S., Anti-synchronization of Rikitake two-disk dynamo chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 393-405.
127. Vaidyanathan, S., Global chaos control of Mathieu-Van der Pol system via adaptive control method, International Journal of ChemTech Research, 2015, 8 (9), 406-417.
128. Vaidyanathan, S., Global chaos synchronization of Mathieu-Van der Pol chaotic systems via adaptive control method, International Journal of ChemTech Research, 2015, 8 (10), 148-162.
129. Garfinkel, A., Spano, M.L., Ditto, W.L., and Weiss, J.N., Controlling cardiac chaos, Science, 1992, 257, 1230-1235.
130. May, R.M., Simple mathematical models with very complicated dynamics, Nature, 261, 259-267.
131. Vaidyanathan, S., Adaptive backstepping control of enzymes-substrates system with ferroelectric behaviour in brain-waves, International Journal of PharmTech Research, 2015, 8 (2), 256-261.
132. Vaidyanathan, S., Adaptive biological control of generalized Lotka-Volterra three species biological system, International Journal of PharmTech Research, 2015, 8 (4), 622-631.
133. Vaidyanathan, S., 3-cells cellular neural network (CNN) attractor and its adaptive biological control, International Journal of PharmTech Research, 2015, 8 (4), 632-640.
134. Vaidyanathan, S., Adaptive synchronization of generalized Lotka-Volterra three species biological systems, International Journal of PharmTech Research, 2015, 8 (5), 928-937.

135. Vaidyanathan, S., Synchronization of 3-cells cellular neural network (CNN) attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (5), 946-955.
136. Vaidyanathan, S., Chaos in neurons and adaptive control of Birkhoff-Shaw strange chaotic attractor, International Journal of PharmTech Research, 2015, 8 (5), 956-963.
137. Vaidyanathan, S., Adaptive chaotic synchronization of enzymes-substrates system with ferroelectric behaviour in brain waves, International Journal of PharmTech Research, 2015, 8 (5), 964-973.
138. Vaidyanathan, S., Lotka-Volterra population biology models with negative feedback and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (5), 974-981.
139. Vaidyanathan, S., Chaos in neurons and synchronization of Birkhoff-Shaw strange chaotic attractors via adaptive control, International Journal of PharmTech Research, 2015, 8 (6), 1-11.
140. Vaidyanathan, S., Lotka-Volterra two species competitive biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (6), 32-44.
141. Vaidyanathan, S., Coleman-Gomatam logarithmic competitive biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (6), 94-105.
142. Vaidyanathan, S., Output regulation of the forced Van der Pol chaotic oscillator via adaptive control method, International Journal of PharmTech Research, 2015, 8 (6), 106-116.
143. Vaidyanathan, S., Adaptive control of the FitzHugh-Nagumo chaotic neuron model, International Journal of PharmTech Research, 2015, 8 (6), 117-127.
144. Vaidyanathan, S., Global chaos synchronization of the forced Van der Pol chaotic oscillators via adaptive control method, International Journal of PharmTech Research, 2015, 8 (6), 156-166.
145. Vaidyanathan, S., Adaptive synchronization of the identical FitzHugh-Nagumo chaotic neuron models, International Journal of PharmTech Research, 2015, 8 (6), 167-177.
146. Vaidyanathan, S., Global chaos synchronization of the Lotka-Volterra biological systems with four competitive species via active control, International Journal of PharmTech Research, 2015, 8 (6), 206-217.
147. Vaidyanathan, S., Anti-synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 26-38.
148. Vaidyanathan, S., Active control design for the anti-synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 2015, 8 (7), 58-70.
149. Vaidyanathan, S., Anti-synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (7), 71-83.
150. Vaidyanathan, S., Sliding controller design for the global chaos synchronization of enzymes-substrates systems, International Journal of PharmTech Research, 2015, 8 (7), 89-99.
151. Vaidyanathan, S., Sliding controller design for the global chaos synchronization of forced Van der Pol chaotic oscillators, International Journal of PharmTech Research, 2015, 8 (7), 100-111.
152. Vaidyanathan, S., Lotka-Volterra two-species mutualistic biology models and their ecological monitoring, International Journal of PharmTech Research, 2015, 8 (7), 199-212.
153. Vaidyanathan, S., Active control design for the hybrid chaos synchronization of Lotka-Volterra biological systems with four competitive species, International Journal of PharmTech Research, 2015, 8 (8), 30-42.
154. Vaidyanathan, S., Hybrid chaos synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 48-60.
155. Vaidyanathan, S., Hybrid chaos synchronization of 3-cells cellular neural network attractors via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 61-73.
156. Vaidyanathan, S., A novel coupled Van der Pol conservative chaotic system and its adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 79-94.
157. Vaidyanathan, S., Global chaos synchronization of novel coupled Van der Pol conservative chaotic systems via adaptive control method, International Journal of PharmTech Research, 2015, 8 (8), 95-111.
158. Vaidyanathan, S., Global chaos synchronization of 3-cells cellular neural network attractors via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 118-130.
159. Vaidyanathan, S., Anti-synchronization of the generalized Lotka-Volterra three-species biological systems via adaptive control, International Journal of PharmTech Research, 2015, 8 (8), 144-156.
160. Vaidyanathan, S., Global chaos control of 3-cells cellular neural network attractor via integral sliding mode control, International Journal of PharmTech Research, 2015, 8 (8), 211-221.

161. Pham, V.-T., Volos, C. K., Vaidyanathan, S., and Vu, V. Y., A memristor-based hyperchaotic system with hidden attractors: dynamics, synchronization and circuital emulating, Journal of Engineering Science and Technology Review, 2015, 8, 205-214.
162. Volos, C. K., Kyprianidis, I. M., Stouboulos, I. N., Tlelo-Cuautle, E., and Vaidyanathan, S., Memristor: A new concept in synchronization of coupled neuromorphic circuits, Journal of Engineering Science and Technology Review, 2015, 8, 157-173.
163. Pham, V.-T., Volos, C., Jafari, S., Wang, X., and Vaidyanathan, S., Hidden hyperchaotic attractor in a novel simple memristive neural network, Optoelectronics and Advanced Materials, Rapid Communications, 2014, 8, 1157-1163.
164. Volos, C. K., Pham, V.-T., Vaidyanathan, S., Kyprianidis, I. M., and Stouboulos, I. N., Synchronization phenomena in coupled Colpitts circuits, Journal of Engineering Science and Technology Review, 2015, 8, 142-151.
165. Huang, Y., and Yang, X. S., Chaoticity of some chemical attractors: a computer assisted proof, Journal of Mathematical Chemistry, 2005, 38, 107–117.
166. Slotine, J. and Li, W., Applied Nonlinear Control, Prentice Hall, New Jersey, USA, 1991.
167. Khalil, H. K., Nonlinear Systems, Third Edition, Prentice Hall, New Jersey, USA, 2002.

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