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Global Chaos Synchronization of Mathieu-Van der Pol Chaotic Systems via Adaptive Control Method

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we describe the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system (2009). We also derive new results for the global chaos synchronization of the identical Mathieu-Van der Pol chaotic systems with unknown parameters via adaptive control method. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos synchronization of the Mathieu-Van der Pol chaotic systems with unknown system parameters via adaptive control method.

Keywords: Chaos, chaotic systems, chaos control, Mathieu equation, Van der Pol equation, adaptive control.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the control, regulation and synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to device a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. In the control and regulation of chaotic systems, state feedback control laws are devised so as to regulate the state trajectories of the system to track the reference input signals.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we describe the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system obtained by Zheng-Ming Ge and Shih-Yu Li [143].

We also derive new results for the global chaos synchronization of the identical Mathieu-Van der Pol chaotic systems with unknown parameters via adaptive control method. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos synchronization of the identical Mathieu-Van der Pol chaotic systems with unknown system parameters via adaptive control method.

2. Mathieu-Van der Pol Chaotic System

Mathieu equation and Van der Pol equation are famous non-autonomous systems described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\left[a + b\sin(\omega t)\right]x_1 - \left[a + b\sin(\omega t)\right]x_1^3 - cx_2 + d\sin(\omega t) \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + q\sin(\omega t) \end{cases} \tag{2}$$

Exchanging $\sin(\omega t)$ in Eq. (1) with x_3 and $\sin(\omega t)$ in Eq. (2) with x_1, Zheng-Ming Ge and Shih-Yu Li obtained the Mathieu-Van der Pol system [143], which is a 4-D autonomous system described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + qx_1 \end{cases} \tag{3}$$

In Eq. (3), x_1, x_2, x_3, x_4 are the states and a, b, c, d, p, q are constant, positive, parameters.

In [130], it was shown that the Mathieu-Van der Pol system (3) is *chaotic* when the system parameters are chosen as

$$a = 10,\ b = 3,\ c = 0.4,\ d = 70,\ p = 5,\ q = 0.1 \tag{4}$$

For numerical simulations, we take the initial conditions

$$x_1(0) = 1.5,\ x_2(0) = 1.5,\ x_3(0) = 1.5,\ x_4(0) = 1.5 \tag{5}$$

Figures 1-4 show the 3-D projections of the Mathieu-Van der Pol 4-D chaotic system (3) on the (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

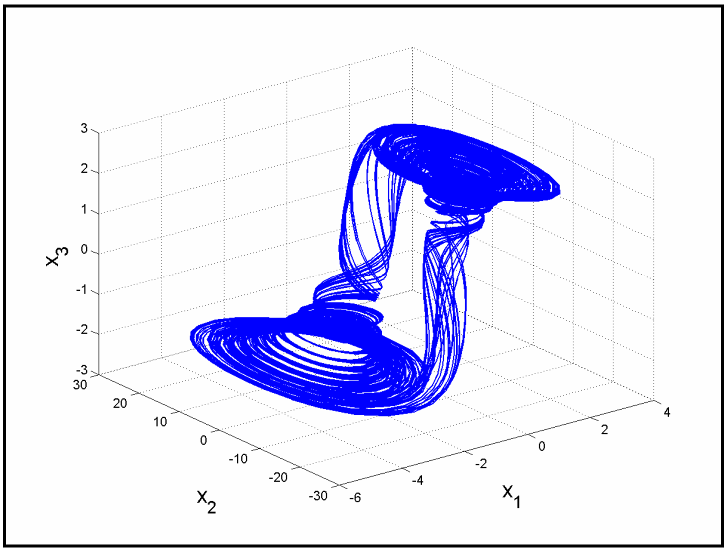

Figure 1. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_2, x_3) **space**

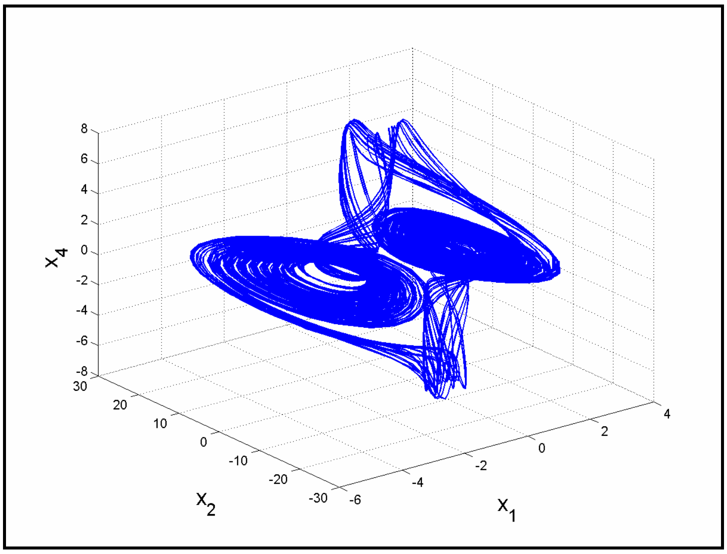

Figure 2. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_2, x_4) **space**

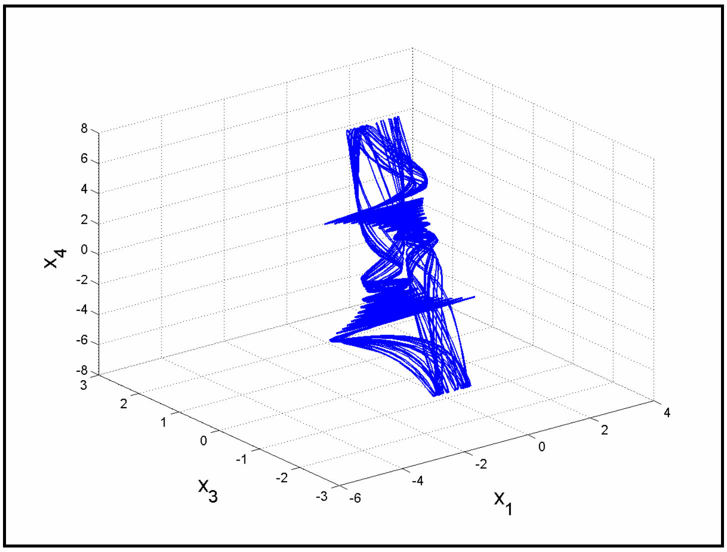

Figure 3. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_3, x_4) **space**

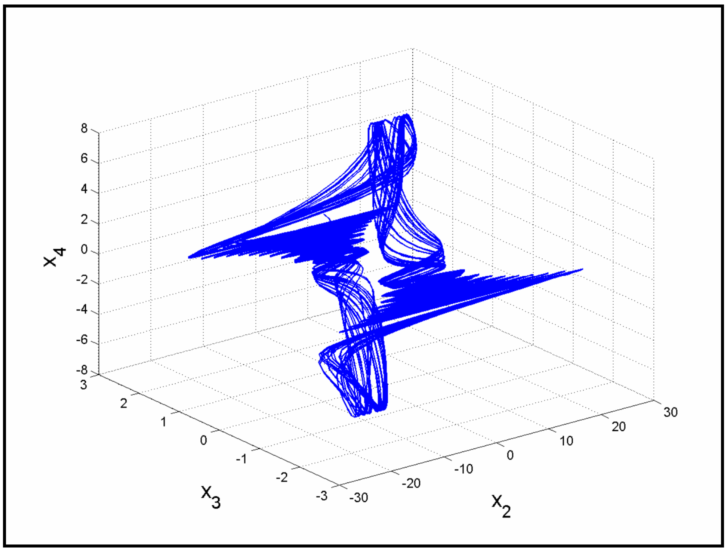

Figure 4. The 3-D projection of the Mathieu-Van der Pol system on the (x_2, x_3, x_4) space

The Lyapunov exponents of the Mathieu-Van der Pol system (3) are numerically found as

$$L_1 = 0.05225,\ \ L_2\ \ \ 0,\ \ L_3\ \ \ -0.49938,\ \ L_4\ \ \ -7.27089 \tag{6}$$

From the LE spectrum (6), it is immediate that the Mathieu-Van der Pol system (3) is a chaotic system and the Maximal Lyapunov Exponent (MLE) of the Mathieu-Van der Pol system (1) is $L_1 = 0.05225$.

Since the sum of the Lyapunov exponents in (6) is negative, it follows that the Mathieu-Van der Pol system (3) is dissipative.

Also, the Lyapunov dimension of the Mathieu-Van der Pol system (3) is derived as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1046 \tag{7}$$

3. Global Chaos Synchronization of the Mathieu-Van der Pol Systems via Adaptive Control

In this section, we use adaptive control method to achieve global chaos synchronization of the Mathieu-Van der Pol chaotic systems with unknown parameters. We use Lyapunov stability theory [144] to prove the main adaptive control result derived in this section using estimates of the unknown system parameters.

As the master system, we consider the Mathieu-Van der Pol system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + bx_3)(x_1 + x_1^3) - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + qx_1 \end{cases} \tag{8}$$

In (8), x_1, x_2, x_3, x_4 are the states and $a, b, c,\ d, p, q$ are constant, unknown, parameters.
As the slave system, we consider the Mathieu-Van der Pol system with controls given by

$$\begin{cases} \dot{y}_1 = y_2 + u_1 \\ \dot{y}_2 = -(a + by_3)(y_1 + y_1^3) - cy_2 + dy_3 + u_2 \\ \dot{y}_3 = y_4 + u_3 \\ \dot{y}_4 = -y_3 + p(1 - y_3^2)y_4 + qy_1 + u_4 \end{cases} \tag{9}$$

In (9), y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are adaptive controls to be determined.

The complete synchronization error between the Mathieu-Van der Pol systems (8) and (9) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4 \end{cases} \tag{10}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 + u_1 \\ \dot{e}_2 = -a(e_1 + y_1^3 - x_1^3) - b(y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3) - ce_2 + de_3 + u_2 \\ \dot{e}_3 = e_4 + u_3 \\ \dot{e}_4 = -e_3 + p(e_4 - y_3^2 y_4 + x_3^2 x_4) + qe_1 + u_4 \end{cases} \tag{11}$$

Now, we consider the adaptive controller defined by

$$\begin{cases} u_1 = -e_2 - k_1 e_1 \\ u_2 = \hat{a}(t)(e_1 + y_1^3 - x_1^3) + \hat{b}(t)(y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3) + \hat{c}(t)e_2 - \hat{d}(t)e_3 - k_2 e_2 \\ u_3 = -e_4 - k_3 e_3 \\ u_4 = e_3 - \hat{p}(t)(e_4 - y_3^2 y_4 + x_3^2 x_4) - \hat{q}(t)e_1 - k_4 e_4 \end{cases} \tag{12}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (12) into (11), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -[a - \hat{a}(t)](e_1 + y_1^3 - x_1^3) - [b - \hat{b}(t)](y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3) \\ \qquad -[c - \hat{c}(t)]e_2 + [d - \hat{d}(t)]e_3 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \\ \dot{e}_4 = [p - \hat{p}(t)](e_4 - y_3^2 y_4 + x_3^2 x_4) + [q - \hat{q}(t)]e_1 - k_4 e_4 \end{cases} \tag{13}$$

We define the parameter estimation errors as follows:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_d(t) = d - \hat{d}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{14}$$

Using (14), the closed-loop system (13) can be simplified as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -e_a(e_1 + y_1^3 - x_1^3) - e_b(y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3) - e_c e_2 + e_d e_3 - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \\ \dot{e}_4 = e_p(e_4 - y_3^2 y_4 + x_3^2 x_4) + e_q e_1 - k_4 e_4 \end{cases} \tag{15}$$

Differentiating (14) with respect to time, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_d(t) = -\dot{\hat{d}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{16}$$

Next, we consider the candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b, e_c, e_d, e_p, e_q) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_p^2 + e_q^2\right) \tag{17}$$

Differentiating (17) along the trajectories of (15) and (16), we get the following dynamics

$$\begin{aligned} \dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[-e_2\left(e_1 + y_1^3 - x_1^3\right) - \dot{\hat{a}}\right] \\ + e_b\left[-e_2\left(y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3\right) - \dot{\hat{b}}\right] + e_c\left[-e_2^2 - \dot{\hat{c}}\right] \\ + e_d\left[e_2 e_3 - \dot{\hat{d}}\right] + e_p\left[e_4\left(e_4 - y_3^2 y_4 + x_3^2 x_4\right) - \dot{\hat{p}}\right] + e_q\left[e_1 e_4 - \dot{\hat{q}}\right] \end{aligned} \tag{18}$$

In view of (18), we take the following parameter update law:

$$\begin{cases} \dot{\hat{a}} = -e_2\left(e_1 + y_1^3 - x_1^3\right) \\ \dot{\hat{b}} = -e_2\left(y_3 y_1 - x_3 x_1 + y_3 y_1^3 - x_3 x_1^3\right) \\ \dot{\hat{c}} = -e_2^2 \\ \dot{\hat{d}} = e_2 e_3 \\ \dot{\hat{p}} = e_4(e_4 - y_3^2 y_4 + x_3^2 x_4) \\ \dot{\hat{q}} = e_1 e_4 \end{cases} \tag{19}$$

Next, we state and prove the main result of this section.

Theorem 1. The adaptive control law (12) and the parameter update law (19) achieve global and exponential synchronization of the identical 4-D Mathieu-Van der Pol chaotic systems (8) and (9), where k_1, k_2, k_3, k_4 are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [144].

The quadratic Lyapunov function V defined by (17) is positive definite on R^{10}.

Substituting the parameter update law (19) into (18), we get the time derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{20}$$

which is negative semi-definite on R^{10}.

Thus, by Barbalat's lemma in Lyapunov stability theory [144], it follows that the closed-loop error dynamics (15) is globally exponentially stable.

This completes the proof. ■

4. Numerical Simulations

We use the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ to solve the system of differential equations (8) and (9), when the adaptive control law (12) and parameter update law (19) are implemented.

We take the parameter values of the Mathieu-Van der Pol chaotic systems as in the chaotic case, viz.

$$a = 10,\ \ b = 3,\ \ c = 0.4,\ \ d = 70,\ \ p = 5,\ \ q = 0.1 \qquad (21)$$

We take the gain constants as

$$k_1 = 6,\ \ k_2 = 6,\ \ k_3 = 6,\ \ k_4 = 6 \qquad (22)$$

We take the initial values of the master system (8) as

$$x_1(0) = 2.4,\ \ x_2(0) = 3.1,\ \ x_3(0) = 17.8,\ \ x_4(0) = 20.5 \qquad (23)$$

We take the initial values of the slave system (9) as

$$y_1(0) = 8.5,\ \ y_2(0) = 4.3,\ \ y_3(0) = 12.7,\ \ y_4(0) = 2.8 \qquad (24)$$

We take the initial values of the parameter estimates as

$$\hat{a}(0) = 6,\ \ \hat{b}(0) = 11,\ \ \hat{c}(0) = 3,\ \ \hat{d}(0) = 4,\ \ \hat{p}(0) = 16,\ \ \hat{q}(0) = 5 \qquad (25)$$

Figures 5-8 show the complete synchronization of the Mathieu-Van der Pol chaotic systems (8) and (9).

Figure 9 shows the time-history of the synchronization errors e_1, e_2, e_3, e_4.

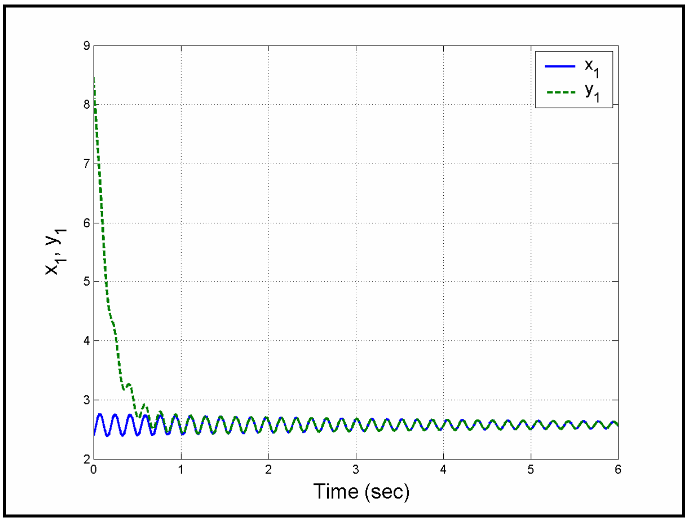

Figure 5. Synchronization of the states x_1 and y_1

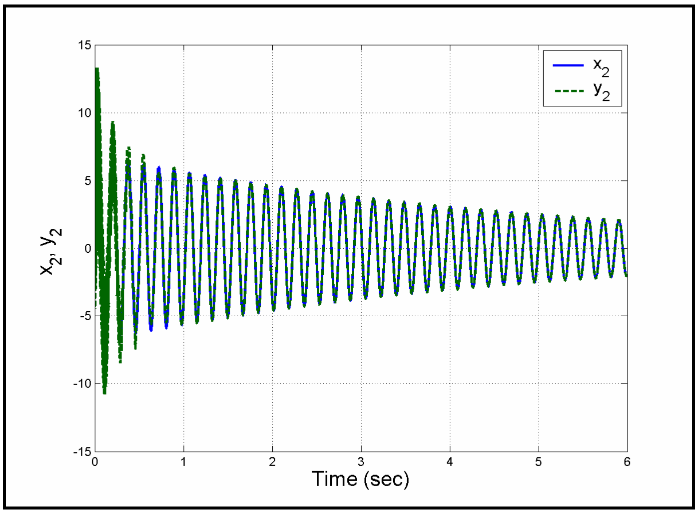

Figure 6. Synchronization of the states x_2 and y_2

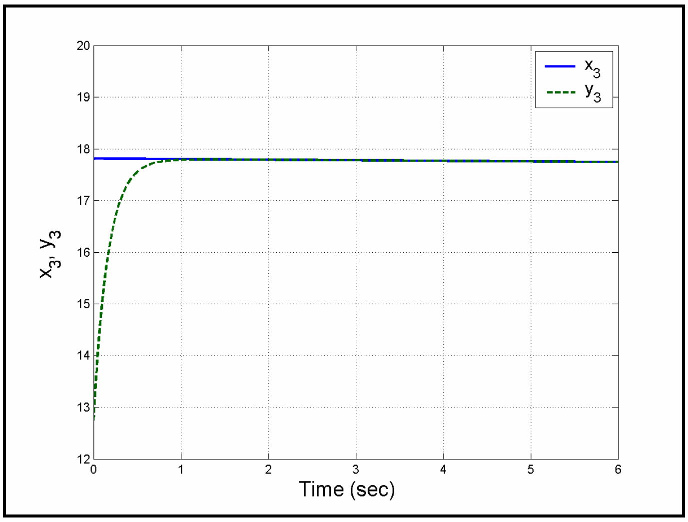

Figure 7. Synchronization of the states x_3 and y_3

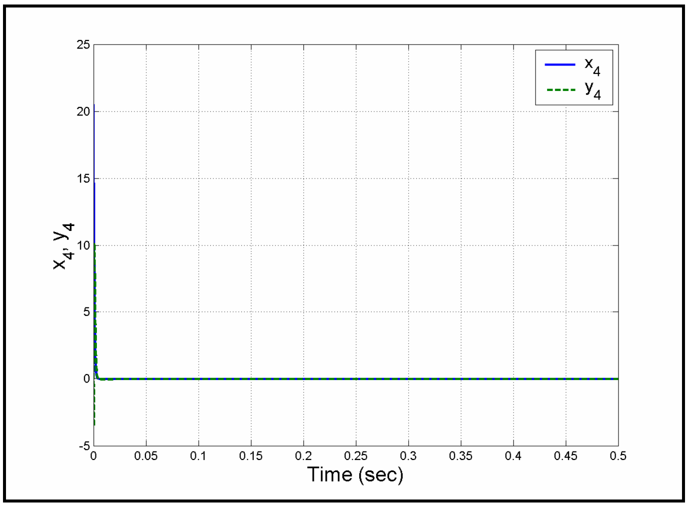

Figure 8. Synchronization of the states x_4 and y_4

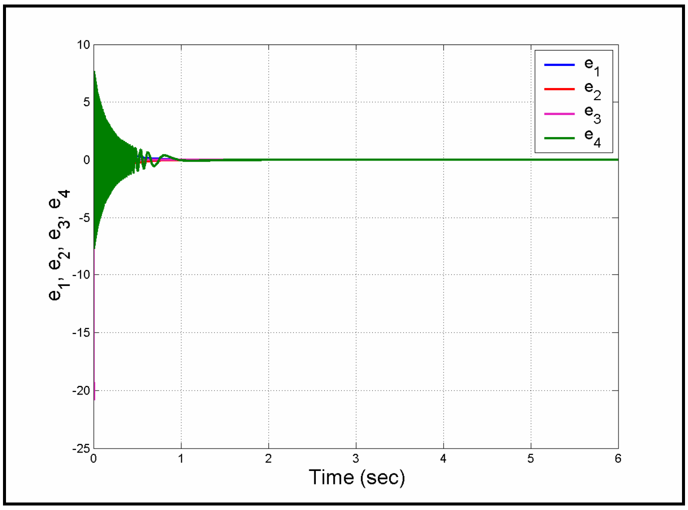

Figure 9. Time-history of the synchronization errors e_1, e_2, e_3, e_4

5. Conclusions

Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we described the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system (2009). We also derived new results for the global chaos synchronization of the Mathieu-Van der Pol chaotic system with unknown parameters via adaptive control method. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos synchronization of the Mathieu-Van der Pol chaotic system with unknown system parameters via adaptive control method.

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