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Computing Equilibrium Constants of Chemical Reactions – A New Approach

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Abstract: Exponential curves of the form: $10^{a} T^{b} \exp(c/T)$ have been fitted to the published data of equilibrium constants of formation of chemical species, available at regular temperature intervals. A method of evaluating the equilibrium constants of chemical reactions as a function of temperature, from the curve fit coefficients of the equilibrium constants of formation of the involved chemical species, is presented.

Keywords: Chemical thermodynamics, reaction equilibrium constants.

Introduction:

The equilibrium constants of the chemical reactions based on partial pressures (K_{pj}) and on mole concentrations (K_{cj}) are very much needed for the analysis of the oxidation mechanisms of fuels and for the calculation of equilibrium compositions and flame temperatures of fuel-oxidiser mixtures. The value of the equilibrium constant of a chemical reaction indicates whether a reaction will proceed to completion or not. Generally a reaction with an equilibrium constant value greater than 1000 is assumed to proceed to completion. A reaction with an equilibrium constant value less than 0.001 is assumed not to occur at all. Small values of equilibrium constants indicate that a reaction will not proceed to any appreciable degree. It is also known that the equilibrium constant of an endothermic reaction increases with increase in temperature and that of an exothermic reaction at two different temperatures, it possible to find the enthalpy of reaction applicable for that temperature range.

Generally equilibrium constants of chemical reactions are evaluated assuming ideal gas behavior for all constituent species. The equilibrium constant of a reaction involving ideal gases depends only on the reaction temperature and is independent of the pressure. It depends only on the stoichiometric relation between the reactants and the products and it is independent of the amount of various constituents initially present in the mixture.

This paper presents a method of an obtaining an expression for the equilibrium constants of chemical reactions as a function of temperature from the curve fit coefficients of equilibrium constants of formation of the chemical species involved.

Equilibrium Constants of Formation of Chemical Species

The equilibrium constants of formation of chemical species (K_{pi}) are available in Thermo Chemical Tables [1] but only at regular temperature intervals. Log K_{pi} values have been published for most of the chemical species in the temperature range 100-6000 K at 100 degree intervals. Such tables are most useful for quick hand calculations carried out at the tabulated temperatures or with linear interpolation, at intermediate temperatures.

Log K_{pi} values stand for the logarithms of the thermodynamic equilibrium constants based on partial pressures (K_{pi}) for the reaction in which the given species is formed from its elements in their most stable form

at the indicated temperature [2]. The equilibrium constants of formation of chemical species based on mole concentrations (K_{ci}) can be obtained using K_{pi} and the change in the number of moles of species as the transformation takes place from reactants to products for the reaction in which the given species is formed from its elements in their most stable form.

Nomenclature

- Equilibrium constant based on partial pressures (atm)
- Equilibrium constant based on mole constrations (mol/cc)
- Species index
- Reaction index
- Summation over products
- j Summation over reactants
- Standard state Gibbs function change of the reaction (kJ/kmol)
- Standard state enthalpy change of the reaction (kJ/kmol)
- We WY Standard state entropy change of the reaction (kJ/kmol K) Absolute temperature (K)
- 1 GR Universal gas constant (kJ/kmol K)
- AH" Universal gas constant (cc atm/mol K)

stoichiometric coefficient ASR a, b, c $T = m, n, \delta$ $\overline{R} = \overline{a}, \overline{b}, \overline{c}$ $\overline{R} = \overline{m}, \overline{m}, \overline{n}, \overline{b}$ constants \overline{R}_{p} A, B, C, D chemical species

Equilibrium Constants of Chemical Reactions

The equilibrium constant of a chemical reaction based on partial pressures (K_{pi}) can easily be calculated from the equilibrium constants of formation of the species (K_{pi}) involved, as given below:

$$\log K_{pj} = \underset{p}{\xi} \gg_i \log K_{pi} - \underset{q}{\xi} \gg_i \log K_{pi}$$
(1)

When the log K_{pi} valves of the species are not readily available, the reaction equilibrium constants based on partial pressures (K_{pi}) can be computed from the standard state Gibbs function change for the reaction using the expression:

$$|n K_{pj} = -\Delta G_{R}^{\circ} / \overline{R}T$$
$$= -(\Delta H_{R}^{\circ} - T\Delta S_{R}^{\circ}) / \overline{R}T$$
(2)

The reaction equilibrium constant based on mole concentrations (K_{ci}) can be computed using K_{pi} and the change in the number of moles of species as the transformation takes place from reactants to products. The reaction equilibrium constants K_{pj} and K_{cj} are related by the expression:

$$-(\xi_{p}\gamma_{i}-\xi_{p}\gamma_{i})$$

$$K_{ej} = K_{pj} (\overline{R}_{p}T)$$
(3)

Hence, the reaction equilibrium constant K_{cj} will be equal to K_{pi} when the number of moles of the product species are equal to the number of moles of the reactant species.

The reaction equilibrium constants based on partial pressuers (K_{pj}) and on mole concentrations (K_{cj}) are not readily available in literature for most of the chemical reactions. They have to be computed as stated earlier using the equilibrium constants of formation of the species involved or the standard state Gibbs function change for the reaction. A continuous representation of the reaction equilibrium constant as a function of temperature will be more useful than the values at regular temperature intervals. Such approximations are more accurate and efficient, both as regards speed and computer storage, than methods of interpolation between tabulated values.

Continuous Representation of Equilibrium Constants

A method of obtaining expressions for the reaction equilibrium constants K_{pj} and K_{cj} as a function of temperature from the equilibrium constants of formation of the chemical species involved has been devised. This involves the fitting of exponential curves of the form: $10^a T^b \exp(c/T)$ to the equilibrium constants of formation of the species which are available at regular temperature intervals in Thermo Chemical Tables [1]. Table I gives the curve fit coefficients a, b and c of the equilibrium constants of formation based on partial pressures (K_{pi}) for 48 C-H-O-N-S chemical species for two different temperature ranges, 1000-3000K and 3000-6000K. These constants were obtained by applying the method of Least Squares to the published data available at regular temperature intervals. Tables II gives, similarly obtained, curve fit coefficients m, n and s of the equilibrium constants of formation based on mole concentrations (K_{ci}) for the same chemical species and for the same temperature ranges. It is to be noted that the curve fit coefficient c for K_{pi} has the same value as the curve fit coefficient s for K_{ci} for any chemical species for a given temperature range.

S.	Chemical	Tempera	ture Range	e 1000 –	Temperature Range 3000 –			
Ν	Species		3000 K		6000 K			
0.		a	b	c	а	b	c	
1	C_2H_4	-3.9749	-0.0965	-4475	-3.2034	-0.2780	-5434	
2	C_2H_2	3.8304	-0.3013	-27125	3.2224	-0.1381	-26833	
3	C_2N_2	2.8618	-0.1405	-37718	4.0745	-0.4492	-38692	
4	C ₂ H	9.7175	-0.8780	-57955	8.7116	-0.6147	-57324	
5	CH ₄	-5.6173	-0.0524	10982	-5.3799	-0.0971	10432	
6	CH ₃	-0.0057	-0.3534	-16740	1.0215	-0.6091	-17698	
7	CH ₂ O	-0.8549	-0.3249	14799	0.4806	-0.6589	13595	
8	CH ₂	2.6300	-0.2090	-45963	3.4990	-0.4252	-46774	
9	СНО	4.9931	-0.7922	1236	6.5273	-1.1819	-11	
10	CO ₂	0.8905	-0.2399	47224	2.1213	-0.5535	46248	
11	COS	0.7463	-0.0622	24305	1.2786	-0.1977	23879	
12	CS_2	0.5809	-0.0663	-1215	1.2547	-0.2387	695	
13	СН	6.9943	-0.3427	-71649	7.6244	-0.4975	-72284	
14	СО	7.5207	-0.8445	12598	8.6178	-1.1248	11742	
15	CN	7.3329	-0.5981	-52837	4.9749	0.0088	-51112	
16	CS	7.1731	-0.7126	-20548	7.8577	-0.8880	-21070	
17	С	9.6076	-0.3776	-86753	9.8707	-0.4473	-86903	
18	C(solid)	0.0000	0.0000	0	0.0000	0.0000	0	
19	H ₂ O	-1.7700	-0.3365	29574	-2.0438	-0.2597	29629	
20	H_2S	-2.9799	0.1124	11043	-3.0565	0.1360	11013	
21	H ₂	0.0000	0.0000	0.0000	0.0000	0.0000	0	
22	HCN	2.3634	-0.2048	-16207	2.6223	-0.2665	-16510	

Table I Curve Fit Coefficients for the Equilibrium Constants of Formation of Chemical Species Based on Partial Pressures (K_{pi}) for the Temperature Ranges 1000–3000 K and 3000–6000 K $K_{pi} = 10^{a} T^{b} \exp(c/T)$

23	HNO ₃	-14.7981	1.1468	17501	-12.7165	0.6178	15819
24	HNO	-2.7376	0.0469	-11547	-1.4973	-0.2688	-12536
25	HS	0.7252	0.0232	-9546	0.9271	-0.0269	-9738
26	Н	1.2980	0.4907	-26290	2.7447	0.1181	-27347
27	O ₃	-4.3048	0.2195	-17069	-2.2871	-0.2986	-18578
28	O ₂	0.0000	0.0000	0	0.0000	0.0000	0
29	0	2.4878	0.2675	-30160	3.2390	0.0723	-30667
30	N_2H_4	-17.2380	1.4591	-8849	-17.1606	1.4543	-9253
31	N_2O_5	-25.3228	2.1889	-618	-20.0002	0.8215	-4588
32	N_2O_4	-22.3888	2.1099	271	-19.1575	1.2841	-2237
33	N_2H_2	-7.8695	0.4917	-24225	-7.7119	0.4588	-24515
34	N ₂ O	-5.9403	0.6177	-9403	-5.0069	0.3792	-10130
35	N ₂	0.0000	0.0000	0	0.0000	0.0000	0
36	NH ₃	-6.8142	0.1855	6976	-9.4266	0.8601	8847
37	NO ₃	-10.7105	0.8492	-7992	-7.9408	0.1386	-10078
38	NH ₂	-1.7246	-0.1309	-19633	-1.2861	-0.2376	-20097
39	NO ₂	-3.6245	0.0889	-3784	-1.9436	-0.3414	-5070
40	NH	0.9414	0.0277	-40706	1.0283	0.0068	-40805
41	NO	0.7178	-0.0148	-10906	1.4961	-0.2148	-11484
42	NS	0.4238	0.0832	-23967	0.6094	0.0351	-24097
43	Ν	2.0965	0.3722	-56972	1.8552	0.4288	-56665
44	S_2O	-3.7317	0.1152	22400	-2.6593	-0.1597	21589
45	S_2	0.0000	0.0000	0	0.0000	0.0000	0
46	SO ₃	-10.9731	0.6745	55961	-8.6626	0.0839	54175
47	SO_2	-4.4810	0.1918	43751	-3.5263	-0.0532	43035
48	S	2.1022	0.2972	-25889	1.6259	0.4186	-25512

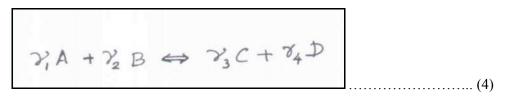
Table II Curve Fit Coefficients for the Equilibrium Constants of Formation of Chemical Species Based on Mole Concentration (K_{ci}) for the Temperature Ranges 1000–3000 K and 3000–6000 K $K_{ci} = 10^{m} T^{n} \exp(s/T)$

S. N	Chemical Species	Tempera	ture Rango 3000 K	e 1000 –	Temperature Range 3000 – 6000 K			
0.		m	n	S	m	n	S	
1	C_2H_4	1.7675	2.9035	-4475	2.5390	2.7220	-5434	
2	C_2H_2	7.6587	1.6987	-27125	7.0507	1.8619	-26833	
3	C_2N_2	6.6900	1.8600	-37718	7.9028	1.5508	-38692	
4	C ₂ H	12.5886	0.6220	-57955	11.5828	0.8853	-57324	
5	CH ₄	-1.7890	1.9476	10982	-1.5516	1.9029	10432	
6	CH ₃	2.8655	1.1466	-16740	3.8927	0.8909	-17698	
7	CH ₂ O	2.0163	1.1751	14799	3.3517	0.8411	13595	
8	CH ₂	4.5441	0.7910	-45963	5.4131	0.5748	-46774	
9	СНО	6.9072	0.2078	1236	8.4414	-0.1819	-11	
10	CO ₂	2.8046	0.7601	47224	4.0354	0.4465	46248	
11	COS	2.6604	0.9378	24305	3.1928	0.8023	23879	
12	CS_2	2.4951	0.9337	-1215	3.1689	0.7613	695	
13	СН	7.9514	0.1573	-71649	8.5815	-0.0025	-72284	
14	СО	8.4778	-0.3445	12598	9.5749	-0.6248	11742	
15	CN	8.2899	-0.0981	-52837	5.9320	0.5088	-51112	
16	CS	8.1302	-0.2126	-20548	8.8147	-0.3880	-21070	
17	С	9.6076	-0.3776	-86753	9.8707	-0.4473	-86903	
18	C(solid)	0.0000	0.0000	0	0.0000	0.0000	0	
19	H ₂ O	-0.8130	0.1635	29574	-1.0867	0.2403	29629	
20	H_2S	-2.0229	0.6124	11043	-2.0994	0.6360	11013	
21	H ₂	0.0000	0.0000	0	0.0000	0.0000	0	

-	I						
22	HCN	4.2776	0.7952	-16207	4.5364	0.7335	-16510
23	HNO ₃	-11.9269	2.6468	17501	-9.8453	2.1178	15819
24	HNO	-1.7805	0.5469	-11547	-0.5402	0.2312	-12536
25	HS	0.7252	0.0232	-9546	0.9271	-0.0269	-9738
26	Η	0.3409	-0.0093	-26290	1.7876	-0.3819	-27347
27	O ₃	-3.3477	0.7195	-17069	-1.3301	0.2014	-18578
28	O ₂	0.0000	0.0000	0	0.0000	0.0000	0
29	0	1.5307	-0.2325	-30160	2.2819	-0.4277	-30667
30	N_2H_4	-13.4097	3.4591	-8849	-13.3323	3.4543	-9253
31	N_2O_5	-20.5374	4.6889	-618	-15.2149	3.3215	-4588
32	N_2O_4	-18.5605	4.1099	271	-15.3293	3.2841	-2237
33	N_2H_2	-5.9553	1.4917	-24225	-5.7978	1.4588	-24515
34	N ₂ O	-4.9833	1.1177	-9403	-4.0498	0.8792	-10130
35	N_2	0.0000	0.0000	0	0.0000	0.0000	0
36	NH ₃	-4.9000	1.1855	6976	-7.5124	1.8601	8847
37	NO ₃	-8.7964	1.8492	-7992	-6.0266	1.1386	-10078
38	NH ₂	-0.7675	0.3691	-19633	-0.3291	0.2624	-20097
39	NO ₂	-2.6674	0.5889	-3784	-0.9865	0.1586	-5070
40	NH	0.9414	0.0277	-40706	1.0283	0.0068	-40805
41	NO	0.7178	-0.0148	-10906	1.4961	-0.2148	-11484
42	NS	0.4238	0.0832	-23967	0.6094	0.0351	-24097
43	Ν	1.1395	-0.1278	-56972	0.8981	-0.0712	-56665
44	S ₂ O	-2.7746	0.6152	22400	-1.7022	0.3403	21589
45	S_2	0.0000	0.0000	0	0.0000	0.0000	0
46	SO ₃	-9.0590	1.6745	55961	-6.7485	1.0839	54175
47	SO_2	-3.5240	0.6918	43751	-2.5692	0.4468	43035
48	S	1.1451	-0.2028	-25889	0.6688	-0.0814	-25512

The exponential curves of the form used are found to fit to the published data excellently well. The equilibrium constants of formation of the species given in published thermochemical tables [1] can be readily obtained with the help these curve fit coefficients with an error less than 3%. The curve fit coefficients a,b and c or m,n and s of the equilibrium constants of formation of the chemical species are used to compute the reaction equilibrium constants K_{pi} and K_{ci} as indicated below:

Considering the chemical reaction of the form:



the equilibrium constants of formation based on partial pressures, of the chemical species A,B,C and D are expressed as:

$$K_{p}(A) = 10^{a_{1}} T^{b_{1}} exp(C_{1}/T)$$

$$K_{p}(B) = 10^{a_{2}} T^{b_{2}} exp(C_{2}/T)$$

$$K_{p}(C) = 10^{a_{3}} T^{b_{3}} exp(C_{3}/T)$$

$$K_{p}(D) = 10^{a_{4}} T^{b_{4}} exp(C_{4}/T)$$
(5)

Similarly the equilibrium constants of formation based on mole concentrations, of the species A, B, C and D are expressed as:

$$K_{c}(A) = 10^{m_{1}} T^{n_{1}} exp(s_{1}/T)$$

$$K_{c}(B) = 10^{m_{2}} T^{n_{2}} exp(s_{2}/T)$$

$$K_{c}(C) = 10^{m_{3}} T^{n_{3}} exp(s_{3}/T)$$

$$K_{c}(D) = 10^{m_{4}} T^{n_{4}} exp(s_{4}/T) \dots (6)$$

Then the reaction equilibrium constants K_{pj} and K_{cj} for the reaction (4) can be obtained from these curve fit coefficients of the equilibrium constants of formation of the species as given below. The reaction equilibrium constant based on partial pressures (K_{pj}) can be computed as:

Similarly, the reaction equilibrium constant based on mole concentration (K_{cj}) of the reaction (4) can be computed as:

$$\begin{aligned} \mathsf{Ke}_{j} &= 10^{\overline{m}} \mathsf{T}^{\overline{n}} \exp\left(\overline{s}/\mathsf{T}\right) \quad \text{where} \\ \overline{m} &= \frac{z}{7} \mathcal{V}_{i} m_{i} - \frac{z}{7} \mathcal{V}_{i} m_{i} = \mathcal{V}_{3} m_{3} + \mathcal{V}_{4} m_{4} - \mathcal{V}_{1} m_{1} - \mathcal{V}_{2} m_{2} \\ \overline{n} &= \frac{z}{7} \mathcal{V}_{i} n_{i} - \frac{z}{7} \mathcal{V}_{i} n_{i} = \mathcal{V}_{3} n_{3} + \mathcal{V}_{4} n_{4} - \mathcal{V}_{1} n_{1} - \mathcal{V}_{2} n_{2} \\ \overline{n} &= \frac{z}{7} \mathcal{V}_{i} \mathcal{N}_{i} - \frac{z}{7} \mathcal{V}_{i} \mathcal{N}_{i} = \mathcal{V}_{3} \mathcal{N}_{3} + \mathcal{V}_{4} \mathcal{N}_{4} - \mathcal{V}_{1} \mathcal{N}_{1} - \mathcal{V}_{2} n_{2} \\ \overline{\kappa} &= \frac{z}{7} \mathcal{V}_{i} \mathcal{S}_{i} - \frac{z}{7} \mathcal{V}_{i} \mathcal{S}_{i} = \mathcal{V}_{3} \mathcal{S}_{3} + \mathcal{V}_{4} \mathcal{S}_{4} - \mathcal{V}_{1} \mathcal{S}_{1} - \mathcal{V}_{2} \mathcal{S}_{2} \end{aligned}$$

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The coefficients \bar{a} , \bar{b} and \bar{c} or \bar{m} , \bar{n} and \bar{s} of the reaction equilibrium

constants can thus be readily computed for any reaction involving the particular

species. The units of $\bar{\alpha}$ is (atm) raised to the power $(\xi_{\vec{p}} \gamma_{i} - \xi_{\vec{p}} \gamma_{i})$ and that of \bar{m} , is (mol/cc) raised to the power $(\xi_{\vec{p}} \gamma_{i} - \xi_{\vec{p}} \gamma_{i})$ The units of \bar{b} and $\bar{\gamma}$ is dimensionless and that of \bar{c} and $\bar{\beta}$ is Kelvin.

The equilibrium constants of any number of chemical reactions (both in forward and reverse directions) can thus be evaluated with the help of these curve fit coefficients of equilibrium constant formation of chemical species involved. The comparison of the values of reaction equilibrium constants K_{pj} and K_{cj} computed using these curve fit coefficients a, b, c, m, n and s with the values obtained using the specific temperature log K_{pi} data of JANAF thermochemical tables [1] is found to be in very good agreement, with a deviation less than 5%. Table 3 shows the comparison of K_{pj} values obtained using the above two methods for three different temperatures for 15 chemical reactions normally encounted in reaction mechanisms.

S.No.	Chemical Reaction	Reaction Equilibrium constants (K _{pi}) obtained using curve fit coefficients of K _{pi} of species			Reaction Equilibrium constants (K _{pj}) obtained using the specific temperature (Janaf table) data of log K _{pi} of species		
			3000K	6000K	1100K	3000K	6000K
1.	$CO_2 + H_2 \leftrightarrow CO + H_2O$	0.696	7.187	9.121	0.695	7.278	9.268
2.	$CO_2 + H_2 \leftrightarrow CO + \frac{1}{2}O_2$	6.00×10^{-11}	0.328	69.249	6.05×10^{-11}	0.345	69.823
3.	$CO + O_2 \leftrightarrow CO_2 + O$	2.591	0.344	0.284	2.600	0.344	0.283
4.	$C(s) + 2H_2 \leftrightarrow CH_4$	0.098	6.17x10 ⁻⁵	1.02×10^{-5}	0.096	6.04×10^{-5}	1.0x10 ⁻⁵
5.	$C(s) + \frac{1}{2} O_2 \leftrightarrow CO$	2.87×10^{11}	2.56×10^{6}	1.65×10^5	2.89×10^{10}	2.53×10^{6}	1.59×10^{5}
6.	$CH_4 + H_2O \leftrightarrow 3H_2 + CO$	25.07	1.88×10^{9}	1.23×10^{11}	26.24	1.90×10^9	1.25×10^{11}
7.	$O_2 \leftrightarrow O+O$	2.42×10^{-20}	0.013	384.510	2.48×10^{-20}	0.013	390.840
8.	$H_2O \leftrightarrow H_2 + \frac{1}{2}O_2$	8.62x10 ⁻¹¹	0.046	7.592	8.71x10 ⁻¹¹	0.045	7.534
9.	$H_2 \leftrightarrow H+H$	5.07×10^{-18}	0.025	264.780	5.15×10^{-18}	0.025	266.680
10.	$N_2 \leftrightarrow N+N$	8.73x10 ⁻⁴⁴	1.93×10^{-10}	0.056	8.71x10 ⁻⁴⁴	1.93×10^{-10}	0.056
11.	$\frac{1}{2}$ N ₂ + $\frac{1}{2}$ O ₂ \leftrightarrow NO	8.65x10 ⁻⁵	0.122	0.713	8.65x10 ⁻⁵	0.122	0.715
12.	$N + O_2 \leftrightarrow NO + O$	4.55×10^7	991.230	59.160	4.61×10^7	995.410	59.570
13.	$NO + N \leftrightarrow N_2 + O$	6.08×10^{15}	6.62×10^4	116.240	6.17×10^{15}	6.66×10^4	116.680
14.	$N + O_3 \leftrightarrow NO_2 + O$	6.02×10^{17}	1.14×10^{6}	1.20×10^3	6.09×10^{17}	1.15×10^{6}	1.20×10^3
15.	$SO_2 + O_2 \leftrightarrow SO_3 + O$	2.82×10^{-10}	1.01×10^{-4}	3.03x10 ⁻³	2.84×10^{-10}	1.01×10^{-4}	3.02×10^{-3}

Table III Comparison of Computed Reaction Equilibrium Constant (K_{pi}) Values

Conclusion:

A new method of computing equilibrium constants of chemical reactions as a function of temperature from the curve fit coefficients of equilibrium constants of formation of the chemical species involved, is presented. It is more accurate and useful than the methods of interpolation between tabulated values.

The combustion mechanism of many fuels are now theoretically analyzed by simulating the oxidation reactions in a high speed computer. An oxidation mechanism of a fuel may involve a large number of chemical reactions, but the chemical species involved will be relatively smaller in number. Hence, for ease of computation, it is suggested that the equilibrium constants of formation of the species be expressed as a function of temperature from which the reaction equilibrium constant can be computed using the method presented here. This method is more efficient in the computer analyses, both as regards speed and storage.

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