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Implementation of Robust Virtual Feedback Model Predictive Controller for Chemical Reactor

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Abstract: Model Predictive Control (MPC) schemes are now widely used in process industries for the control of key unit operations. In this paper, a state estimation based model predictive controller for nonlinear process has been proposed. The model predictive controller is designed by considering a state space model and an extended Kalman filter to predict the future behaviour of the system. The efficacy of the proposed MPC scheme has been demonstrated by conducting experimentalstudies on a continuous stirred tank reactor, a SISO system.The analysis of the extensive dynamic closed loop studies revealed that, the MPC scheme formulated produces satisfactory performance for servo operation.
Key words – Model Predictive Control, State Estimation, Extended Kalman Filter, Continuous Stirred Tank Reactor.

I. Introduction

Model Predictive Control is an increasingly significant and popular control approach because of its use of a possibly nonlinear multivariable process model and its ability to handle constraints on inputs, states and outputs. It uses open loop constrained optimization of finite horizon control criteria in a receding horizon approach. A model is used to predict the future behaviour of the system up to the horizon, starting from its current state and a constrained optimization based on the prediction yields an optimal open loop control sequence over the complete horizon. Only the first element in this sequence is applied to the plant. New measurements available at the next sample time permit the calculation of an updated initial state value and the optimization is then resolved [7]. As it is known, all the states of a system are not measurable in practice, state estimation is presented. When the prediction of future behaviour is done based on state estimation, there exist many advantages which include accurate result, suppression of noise, increased robustness and the estimator acts as model based filter. In state estimation based MPC, the current control action is obtained online by solving a finite horizon open loop optimal control problem from the state estimate of the system.

The need to achieve control of nonlinear process has led to more general MPC formulation. The survey on nonlinear control of chemical processes by Bequette [3] summarizes different NMPC algorithms. The advantage of using state estimation in model predictive control has been reported by Ricker [4].

The main contribution of this paper is to demonstrate the development of model predictive control scheme for a continuous stirred tank reactor using extended Kalman filter [5]. The organisation of this paper is as follows. Section 2 discusses about the modeling of CSTR. Section 3 presents the conventional MPC algorithm. Section 4 describes the state estimation based MPC. Extensive experimental results and analysis are presented in Section 5 and the conclusion in Section 6.

II. Model Identification of Cstr

System Identification means to find a function that will map the input and output time series on to the parameter space such that some objective function $\epsilon(y-\hat{y})$ is satisfied.The knowledge of the model is necessary for the design of soft sensing technique and model based control system.

System identification is an experimental approach [2] for determining the dynamic model of a system. It includes four steps:

i. Input/output data acquisition under an experimentation protocol
ii. Selection or estimation of the "model" structure
iii. Estimation of the model parameters
iv. Validation of the identified model (structure and values of the parameters)

A complete identification operation must necessarily comprise the four stages indicated above. The specific methods used at each stage depend on the type of model desired (parametric or non-parametric, continuous-time or discrete-time) and on the experimental conditions (for example: hypothesis on the noise, open loop or closed loop identification). The validation is the mandatory step to decide if the identified model is acceptable or not.As there does not exist a unique parameter estimation algorithm and a unique experimental protocol that always lead to a good identified model, the models obtained may not always pass the validation test. In this case, it is necessary to reconsider the estimation algorithms, the model complexity or the experimental conditions. System identification should be then considered as an iterative procedure as illustrated in Figure 1.

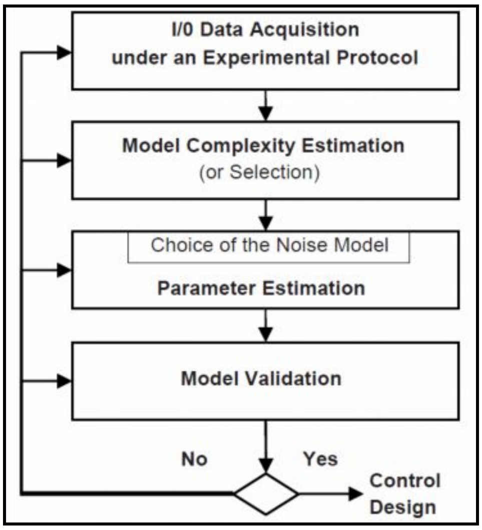


Figure 1.Flow diagram of System Identification

The system shown in Figure 2 was taken for system identification. The input was the coolant flow rate. The output variable was the temperature of the reactor.

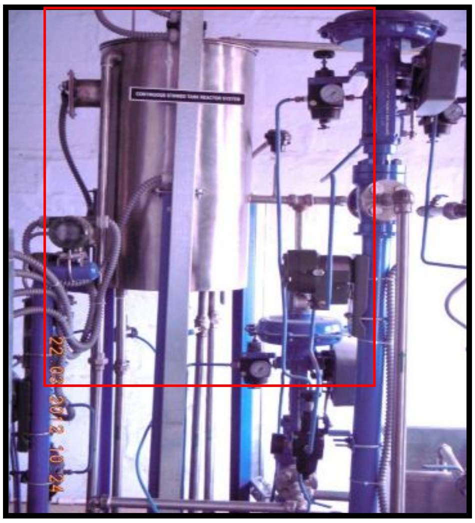
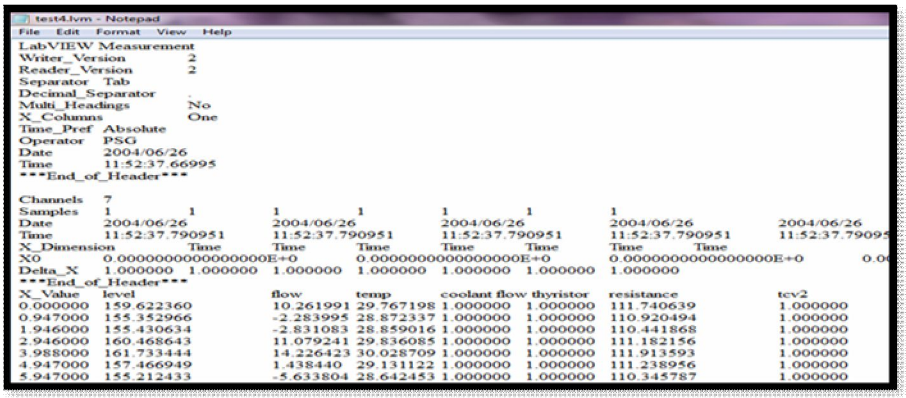


Figure 2. SISO process

The Input and output data were collected for the above process using Compact RIO as shown in Figure 3. The average feed flow was maintained as constant value. The temperature data was collected till it reaches steady state. A step change was given in coolant flow rate and again the temperature was measured once it reaches steady state. All the values were obtained in terms of (1-5) V in order to normalize them within a single unit range.



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X_Columns	One
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Operator	PSG
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X_Dimension	Time	Time	Time	Time	Time	Time	Time
X0	0.0000000000000000E+0		0.0000000000000000E+0		0.0000000000000000E+0		0.00
Delta_X	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
***End_of_Header***
X_Value	level		flow	temp	coolant flow	thyristor	resistance		tcv2
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0.947000	155.352966		-2.283995	28.872337	1.000000	1.000000	110.920494		1.000000
1.946000	155.430634		-2.831083	28.859016	1.000000	1.000000	110.441868		1.000000
2.946000	160.468643		11.079241	29.836085	1.000000	1.000000	111.182156		1.000000
3.988000	161.733444		14.226423	30.028709	1.000000	1.000000	111.913593		1.000000
4.947000	157.466949		1.438440	29.131122	1.000000	1.000000	111.238956		1.000000
5.947000	155.212433		-5.633804	28.642453	1.000000	1.000000	110.345787		1.000000
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Figure 3. View of Sample Data Collection Using Compact RIO Modules

The parametric model approach was used. Recursive least square (RLS) method was used for estimating the parameters. After the completion of parameter estimation, results were validated against a new set of data for same operating condition. The Actual plant output vs. Identified Model response is shown in Figure 4.

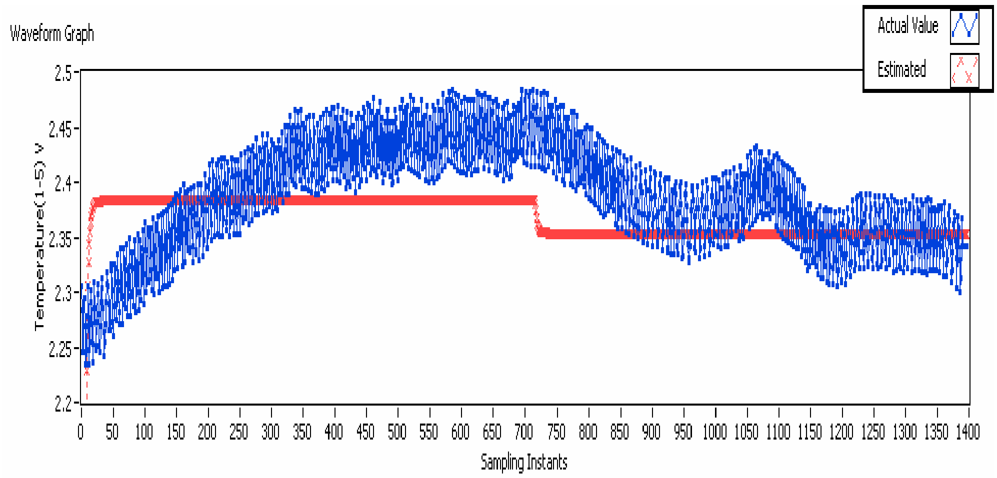

Figure 4. Comparison of Open loop response of Actual plant and Identified Model

Thus the Mathematical model of CSTR real process was obtained in the form of discrete state space form and is shown here.

$$x(k+1) = [0.9997]x(k) + [1.885e^{-5}]u(k) \quad (1)$$

$$y(k) = [9.94281]x(k) \quad (2)$$

where,

u(k) is Coolant flow (manipulated variable)

y(k) is Temperature (output variable)

III. Conventional Mpc Algorithm

The model predictive control is a strategy that is based on the explicit use of some kind of system model to predict the controlled variables over a certain time horizon, the prediction horizon. The control strategy can be described as follows [7]:

1. At each sampling time, the value of the controlled variable y(t+k) is predicted over the prediction horizon k=1,……N_2. This prediction depends on the future values of the control variable u(t+k) within a control horizon k=1,…N_u, where $N_u \leq N_2$. If $N_u < N_2$, then u(t+k)=u(t+N_u), k=N_u+1,…N_2.
2. A reference trajectory r(t+k), k=1,….N_2 is defined which describes the desired system trajectory over the prediction horizon.
3. The vector of future controls u(t+k) is computed such that a cost function, usually a function of the errors between the reference trajectory and the predicted output of the model, is minimised.
4. Once minimisation is achieved, the first optimised control action is applied to the plant and the plant outputs are measured. Use this measurement of the plant states as the initial states of the model to perform the next iteration.

Steps 1 to 4 are repeated at each sampling instant; this is called receding horizon strategy.

The above steps are can be expressed by the following equations:

$$\min\left(\sum_{i=1}^{p}[x_d(k+1)-x(k+i)]^2\right)$$

subject to

$$u_{min} < u < u_{max} \quad \forall k$$

where k is the time step, u(k) is the control vector at time k, $x_d(k)$ and x(k) are the desired output (reference) and predicted output vector of the model at time k respectively, p is the prediction time horizon. The block diagram of a model predictive controller is shown in Figure 5.

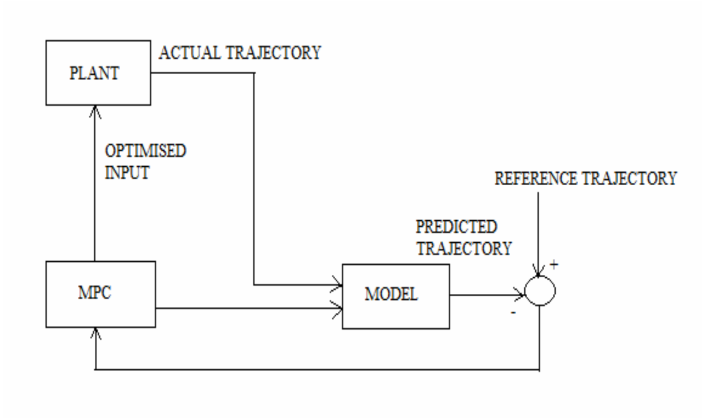

Figure 5.Block Diagram of MPC controller

As the control variables in a MPC controller are calculated based on the predicted output, the model thus needs to be able to reflect the dynamic behaviour of the system as accurately as possible.

IV. State Estimation Based Mpc

A. Block Diagram

Conventional MPC techniques are based on the use of linear models. The need to achieve tighter control of strong non-linear process has led to more general MPC formulation in which nonlinear dynamic model is used for prediction [1]. When the prediction of future behaviour is done based on state estimation, there exist many advantages which include accurate result, suppression of noise, increased robustness and the estimator acts as model based filter [4]. In state estimation based MPC, the current control action is obtained online by solving a finite horizon open loop optimal control problem from the state estimate of the system. Figure 6 shows the block diagram of state estimation based MPC.

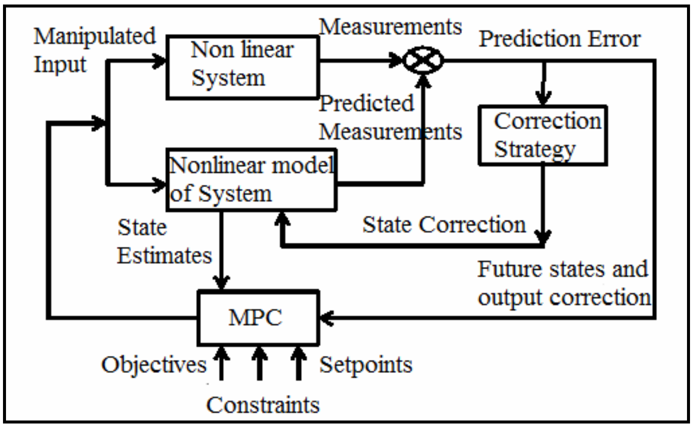

Figure 6.Block Diagram of State Estimation (Virtual Feedback) Based MPC

B. Extended Kalman Filter

In this section, the EKF algorithm to estimate the internal states and future behaviour of the system has been proposed. The well-known Kalman filter solves the state estimation problem in a stochastic linear system [8]. The extended Kalman Filter (EKF) is probably the most widely used nonlinear filter. For nonlinear problems, the Kalman Filter is not strictly applicable since linearity plays an important role in its derivation and performance as an optimal filter. The EKF attempts to overcome this difficulty by using a linearized approximation where the linearization is performed about the current state estimate [8].The basic framework for the EKF involves the estimation of the state of a nonlinear dynamic system given by (3) and (4)

$$x(k) = \left[x(k+1) + \int_{t_{k-1}}^{t_k} F[x(\tau), u(k)]\,d\tau\right] + w(k) \quad (3)$$
$$y(k) = H[x(k)] + v(k) \quad (4)$$

In the above equation, x(k)represents the unobserved state of the system, u(k) is a known exogenous input and y(k) is the only observed signal. We have assumed w(k) and v(k) as zero mean Gaussian white noise sequences with covariance matrices Q and R respectively. The symbols F and H represent an n-dimensional function vector and are assumed known. EKF involves the recursive estimation of the mean and covariance of the state under maximum likelihood condition. The function Fcanbe used to compute the predicted state from the previous estimate and similarly the function H can be used to compute the predicted measurement from the predicted state. However, F and H cannot be applied to the covariance directly. Instead a matrix of partial derivatives (Jacobian) is computed at each time step with current predicted state and evaluated. This process essentially linearizes the non-linear function around the current estimate.

The predicted state estimates are obtained as

$$\hat{x}(k|k-1) = \hat{x}(k-1|k-1) + \int_{t_{k-1}}^{t_k} F[x(\tau), u(k-1)]\,d\tau \quad (5)$$

The covariance matrix of estimation errors in the predicted estimates is obtained as

$$P(k|k-1) = \varphi(k)P(k-1|k-1)\varphi(k)^T + Q \quad (6)$$

where $\varphi(k)$ is nothing but Jacobian matrix of partial derivatives of F with respect to x

$$\varphi(k) = \left[\frac{\partial F}{\partial x}\right]_{[\hat{x}(k-1|k-1), u(k-1)]} \quad (7)$$

Note that the extended Kalman filter (EKF) computes covariances using the linear propagation. The measurement prediction, computation of innovation and covariance matrix of innovation are as follows

$$\hat{y}(k|k-1) = H[\hat{x}(k|k-1)] \quad (8)$$
$$y(k|k-1) = y(k) - \hat{y}(k|k-1) \quad (9)$$
$$V(k) = C(k)P(k|k-1)C(k)^T + R \quad (10)$$

where C(k) is the Jacobian matrix of partial derivatives of H with respect to x.

$$C(k) = \left[\frac{\partial H}{\partial x}\right]_{[\hat{x}(k-1|k-1),u(k-1)]} \quad (11)$$

The Kalman gain is computed using the following equation

$$K(k) = P(k|k-1)C(k)^T V^{-1}(k) \quad (12)$$

The updated state estimates are obtained using the following equation

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)y(k|k-1) \quad (13)$$

The covariance matrix of estimation errors in the updated state estimates is obtained as

$$P(k|k) = [1 - K(k)C(k)]P(k|k-1) \quad (14)$$

C. Model Predictive Controller Based on EKF

The concept of model predictive control involves the repeated optimization of a performance objective such as (15) over a finite horizon extending up to a prediction horizon N_2. The control variable u(k+j), over the control horizon N_u , is obtained from solving the cost function [6].

$$J = \sum_{i=1}^{N_2} \|\hat{y}(t+i) - \omega(t+i)\|_Q^2 + \sum_{j=1}^{N_u} \|\Delta u(t+j-1)\|_R^2 \quad (15)$$

$\hat{y}(t+i)$ is the predicted outputs vector, $\omega(t+i)$ is the reference and Δu is the increment of input vectors. R and Q are the weighting matrices and N_2, N_u must be tuned as controller parameters.

The controller is based on a nonlinear state space model that utilizes an extended Kalman filter (EKF) for estimate of the system states. By using the estimated state variables of the system derivation of some equations, predicted outputs are obtained. The predicted output at j step ahead is as (16).

$$y(t+j) = CA^j x(t) + \sum_{i=0}^{j-1} CA^{j-i-1}Bu(t+i) + \sum_{i=0}^{j-1} CA^{j-i-1}Nw(t+i) + \sum_{i=0}^{j-1} CA^{j-i-1}N_0 \quad (16)$$

By applying the expectation function to (16), we have:

$$y(t+j) = CA^j E[x(t)] + \sum_{i=0}^{j-1} CA^{j-i-1}Bu(t+i) + \sum_{i=0}^{j-1} CA^{j-i-1}Nw(t+i) + \sum_{i=0}^{j-1} CA^{j-i-1}N_0 \quad (17)$$

$\hat{Y}$ is vector including one step ahead to N_2 step ahead predicted outputs.

$$\hat{Y} = \begin{bmatrix} \hat{y}(t+1) \\ \hat{y}(t+2) \\ \vdots \\ \hat{y}(t+N_2) \end{bmatrix} = \begin{bmatrix} CAE[x(t)] + CBu(t) + CNw(t) + CN_0 \\ CA^2E[x(t)] + \sum_{i=0}^{1} CA^{1-i}Bu(t+i) + \sum_{i=0}^{1} CA^{1-i}Nw(t+i) + \sum_{i=0}^{1} CA^{1-i}N_0 \\ \vdots \\ CA^{N_2}E[x(t)] + \sum_{i=0}^{N_2-1} CA^{1-i}Bu(t+i) + \sum_{i=0}^{N_2-1} CA^{1-i}Nw(t+i) + \sum_{i=0}^{N_2-1} CA^{1-i}N_0 \end{bmatrix} \quad (18)$$

Closed form of the above equation is as (19).

$$\hat{Y} = F\hat{x}(t) + GU + HW + H_0 \quad (19)$$

$$\hat{x}(t) = E[x(t)] \quad (20)$$

The estimates of the states are computed using extended Kalman filter and other parameters are defined as follows:

$$F = \begin{bmatrix} CA \\ CA^2 \\ \cdot \\ \cdot \\ CA^{N_2} \end{bmatrix}$$

G, H are block lower triangular matrices that nonzero elements are $(G)_{ij} = CA^{i-j}B$, $(H)_{ij} = CA^{i-j}N$ and H_0 is a vector which its j^th^ element would be $\sum_{i=0}^{j-1} CA^{i-j}N_0$

By considering the performance index,

$$\hat{Y}_{N12} = F_{N12}\hat{x}(t) + G_{N12u}U_{N_u} + H_{N12}W_{N12} + H_{0N12} \quad (21)$$
$$\hat{Y}_{N12} = [\hat{y}(t+1)^T \ldots\ldots \hat{y}(t+N_2)^T]^T$$
$$U_{N_u} = [u(t)^T \ldots\ldots u(t+N_u-1)^T]^T$$
$$W_{N12} = [w(t+1)^T \ldots\ldots w(t+N_2)^T]^T$$

And by solving $\frac{\partial J}{\partial u} = 0$,control law is obtained as in (22):

$$\Delta U_{N_u} = (G_{N12u}^T Q G_{N12u} + R)^{-1} G_{N12u}^T Q(\omega - F_{N12}\hat{x}(t) - H_{N12}W_{N12} - H_{0N12})$$
(22)$u(t) = u(t-1) + (\Delta U_{N_u})_{(1,1)}$

The response of this controller is appropriate if model of the system was well known. But in general case, uncertainties of the model may cause to bad behaviour of the controller and a robust controller must be designed. Also, the MPC schemes have been implemented in a moving horizon framework i.e. only the first move u(k|k) is implemented on the plant and the optimization problem is reformulated at the next sampling instant based on the updated information from the plant [1].

V. Experimental Results and Analysis

MPC scheme for CSTR has been developed with the sampling time of 1second, prediction horizon of $N_2 = 10$, and control horizon of $N_u = 2$. The error weighting matrix and the controller weighting matrix used in the MPC formulation are $W_E = 10$ and $W_U = 0.1$. The following constraint on the manipulated inputs (inflow rates) is imposed i.e. $1 \le u \le 5$.

A. Closed loop experimental study of CSTR

In order to access the tracking capability of the proposed EKF based MPC scheme, a set point variation is introduced in the temperature.

a. Closed loop response: Conventional MPC

Figure7 and Figure8 show the closed loop response of the system with conventional MPC and the corresponding variation in the control inputs respectively.

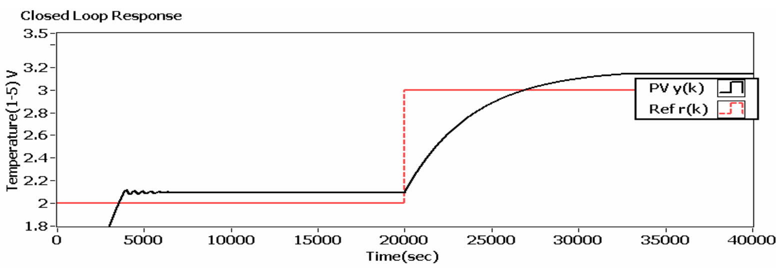

Figure 7.Closed Loop Response - Conventional MPC

Figure 7 shows the deviation of the output from the set point due to measurement noise in the system.

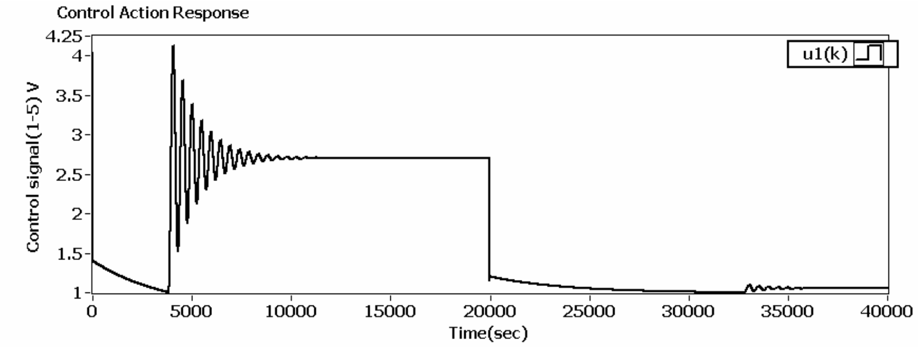

Figure 8.Control Input – Conventional MPC

b. Closed Loop response: State Estimation Based MPC

Figure 9 andFigure 10 show the closed loop response of the system with state estimation based MPC and the corresponding variation in the control inputs respectively.

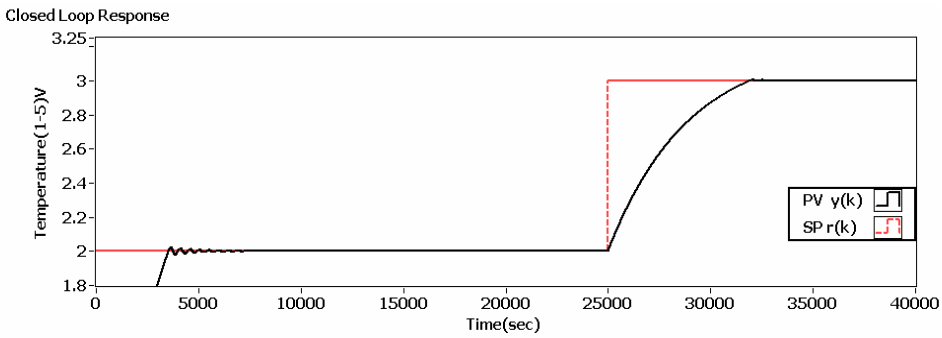

Figure 9.Closed Loop Response – State Estimation based MPC

By comparing Figure 7 and Figure 9, it can be seen, the state estimation based MPC filters the noise more effectively and the system settles with less offset than conventional MPC.

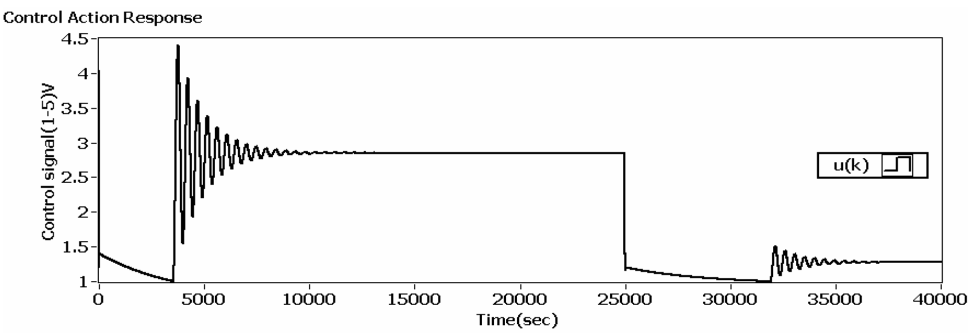

Figure 10.Control Input – State Estimation based MPC

From Figure8 andFigure 10 it can be clearly seen that the control inputs follow the constraints in both the cases.

B. Performance index

Integral Square Error (ISE) is used as performance index to evaluate the performance of conventional and state estimation based MPC. Table 1 shows the comparative study.

Table 1 Comparison between conventional MPC and state estimation based MPC

Controller/ Performance index	**ISE**
Conventional MPC	**23875**
State Estimation based MPC	**15204.3**

From Table 1, it is clear that the state estimation based MPC shows minimum ISE than the conventional MPC. Moreover it shows that the state estimation based MPC has better noise suppressing capability and set point tracking than conventional MPC.

VI. Conclusion

In this paper, a MPC scheme based on EKF has been formulated and applied to CSTR. The EKF is used to predict the future behaviour of the system. Further, a comparative experimental study has been done between conventional MPC and state estimation based MPC formulations. The performance of both the controllers has been evaluated based on the performance index ISE. From the extensive experimental studies on CSTR and the performance index, it can be inferred that state estimation based MPC is able to achieve satisfactory performance following the constraints and can suppress noise.

Acknowledgment

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References

1. Geetha M., Jovitha Jerome and Devatha V "Design of State Estimation Based Model Predictive Controller for a Two Tank Interacting System", Procedia Engineering, Elsevier Vol. 64 , 2013, pp.244 – 253.
2. Manikandan P, Geetha M, Jovitha Jerome, "Weighted Fuzzy Fault Tolerant Model Predictive Control" Proceedings of 2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), July 6-11, 2014, Beijing, China, pp.83-90.
3. B.W. Bequette, "Nonlinear control of chemical processes: a review", *Industrial Engineering and Chemical Research*, 1991, vol.30, pp.1391-1413.
4. N.L. Ricker, "Model predictive control with state estimation", *Industrial Engineering and Chemistry Research*, 1990, vol.29, pp. 374-382.
5. Jay H. Lee, N.L. Ricker, "Extended Kalman Filter based nonlinear model predictive control", *Industrial Engineering and Chemistry Research, 1994,*vol.33, pp.1530-1541.
6. M. Khodabandeh and H.Bolandi, "Model Predictive Control with State Estimation and Adaptation Mechanism for a Continuous Stirred Tank Reactor", *International Conference on Control, Automation and Systems,*2007, pp. 17-20.
7. B. Wayne Bequette, "*Process Control: modelling, Design and Simulation*", Prentice-Hall India Pvt. Ltd., New Delhi, 2003.
8. Geetha M, Jovitha Jerome, Arun Kumar P," Critical Evaluation of non-linear Filter Configurations for the State Estimation of Continuous Stirred Tank Reactor", Applied soft computing, 2014, Vol.25,pp.452-460.
9. Geetha M, "Performance Analysis of soft sensing Technique for a Continuous Stirred Tank Reactor" International Journal of Electrical and Electronics Engineering (IJEEE) Vol.1, Issue 2 Nov 2012, pp. no: 11-18.
10. Geetha M, Naveen R, Jovitha Jerome and Suriya kumar V "Real-Time Implementation and Performance Analysis of State Estimation based Model Predictive Controller for CSTR Plant" Proc of 4th IEEE International Conference on Computing, Communication and Networking Technologies pp.13-18 July 2013.

11. Manikandan P, Geetha M, Jubi K, Jovitha Jerome., Fault Tolerant Fuzzy Gain Scheduling Proportional-Integral-Derivative Controller for Continuous Stirred Tank Reactor. Aust. J. Basic & Appl. Sci., 7(13): 84-93, 2013.
12. Geetha, M.; Manikandan, P.; Jerome, J., "Soft computing techniques based optimal tuning of virtual feedback PID controller for chemical tank reactor," Evolutionary Computation (CEC), 2014 IEEE Congress on , vol., no., pp.1922,1928, 6-11 July 2014.
13. Manikandan, Pandiyan, and Mani Geetha. "Takagi Sugeno fuzzy expert model based soft fault diagnosis for two tank interacting system." Archives of Control Sciences 24, no. 3 (2014).

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