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Kalman Filter based State Feedback Control of Shell and Tube Heat Exchanger

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Abstract: Temperature of fluid stream through tube is a key variable in shell and tube heat exchanger. However, this variable is not easy to measure straightly. Therefore, a state observer is designed to estimate the tube temperature variable for direct control of shell and tube heat exchanger. A dynamical state space model is used to describe the heat exchanger process. Kalman filter based observer is designed to generate the estimates of states for desirable control system. Kalman filter given the good performance in trimming the noise in the process and congregating to the actual states from improper initial states. A SFC (State Feedback Controller) is designed to control the temperature of fluid stream flowing out through the heat exchanger. A PID (Proportional Integral and Derivative) controller is also designed for temperature control. The simulations of state feedback and PID controller are performed using MATLAB. Obtained transient results are compared and an analysis is conducted.
Keywords: Heat exchanger, State feedback controller, State estimation, Kalman filter.

I. Introduction

Heat exchangers [1] are commonly used in petrochemical industries, gas industries and nuclear power plants. These are used for transferring heat between two or more medium with temperature differences like liquids and gases. The added advantage of these heat exchangers are compact structure, high efficiency and low cost [2]. An accurate model describing the dynamic behavior of the system is needed for an optimal operation and control. The controlled variables of the shell and tube heat exchanger are cold fluid temperature and hot fluid temperature. The manipulated variables are inlet flow rate of cold fluid and hot fluid.

State feedback control is an advanced control design method [3], which uses the states in the designed control laws under a premise that all the state variables should be available for measurement [4]. It is not possible or costly to measure all the state variables. For those cases a state observer is necessary to give good estimates of the system state variables and to implement state feedback control law. Considering the above requirements, to estimate immeasurable states and to mimic measurement and process noises. For a continuous system, a popular technique to estimate the system states is Kalman filter [5-6].A state space model is derived by using system identification. This model is used to design the state feedback and PID controllers. The analysis on the performance of these controllers is conducted. In addition, to test the efficiency of the estimator, closed loop analysis is also taken in the state feedback control.

II. Experimental Setup and The Heat Exchanger Model

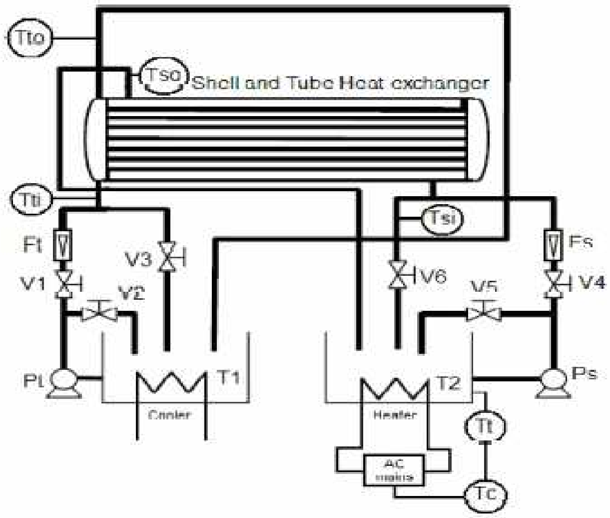

Fig.1. Heat exchanger setup

T_{si} –Temperature of shell inlet.
T_{so}- Temperature of shell outlet.
Tti -Temperature of tube inlet.
Tto -Temperature of tube outlet.
Fs - Flow rate of shell inlet.
Ft - Flow rate of tube inlet.
T2 - Shell tank.
T1 - Tube tank.
Pt - Tube pump.
Ps - Shell pump.
VI-V6 – Valves.

Fig. 1 shows the experimental setup of the one side, two pass, cross current shell and tube Heat Exchanger. The heat exchanger consists of tubes that are fixed inside the shell side. The medium flowing through the shell and the tube side will not come in direct contact, but heat exchange takes place between the walls of the shell and tube. Tank T2 contains hot water, which is used as inlet to the shell side of the heat exchanger. Tank Tl contains cold diesel, which is used as the inlet of tube side of heat exchanger. Our aim is to control the Tube outlet temperature (Tto) by varying the Shell inlet flow (Fs). In order to implement state feedback controller, the model of the system needs to be derived. The model is found out by system identification. The inlet flow rate of Shell side (Fs) is kept constant and the outlet temperature (Tto) is continuously monitored. Obtained readings are taken.
A state space model is derived using system identification from the set of input output data. The derived system is given as:

$$A = \begin{bmatrix} -0.00014 & 0.00057 \\ 0.00067 & 0.00269 \end{bmatrix}$$
$$B = [1.486e\text{-}07 \ -1.0232e\text{-}06]^T$$
$$C = [411.920.13]$$

III. Observer Design

The observer is designed using Kalman filter. The system noise and the measurement noise in this process are considered as white noise with zero mean. Let us consider a linear time invariant state space model [7] given as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \boldsymbol{\phi}\mathbf{d} + \boldsymbol{\zeta}\mathbf{w}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\eta} \quad (1)$$

Here,**x-** A state vector of (n×1) dimension

u -Manipulated input vector of (m×1) dimension

y - Output vector of (p×1) dimension

w - System noise vector of (nw×1) dimension

- is measurement noise vector of (p×1) dimension

A, B, C, and ϕ are constant matrices.

The system and measurement noises have the covariance of Qand R respectively. Observability is a very important issue to consider in the observer design. To design an observer for a system [8], it is mandatory that the system should be observable. The observability check can be done using the observability matrix. A system is observable if and only if the observability matrix should have a rank **n**.

$$O = [C\ CA\ CA^2\ CA^3 \ldots \ldots CA^{n-1}]^T \quad (2)$$

After observability check, Kalman filter to be designed.

IV. Kalman Filter

The Kalman filter algorithm uses a series of measurements collected over time, containing random noise. It gives estimates of unknown variables [9]. The algorithm works in two step process. That is prediction and update. Let us consider a state space model:

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k$$
$$\mathbf{z}_k = H_k \mathbf{x}_k + \mathbf{v}_k$$

$$\mathbf{w}_k \sim N(0, Q_k) \quad \mathbf{v}_k \sim N(0, R_k) \qquad (3)$$

Where, H, B and F are system matrices.Q and R are covariance matrices.

If the input is Gaussian for a linear model, the state and output also be Gaussian [10].

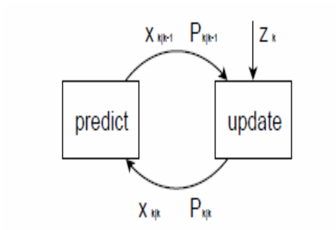

Fig.2. Kalman filter loop

It will estimate the next state before taking the measurements,

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k$$
$$\mathbf{z}_k = H_k \mathbf{x}_k + \mathbf{v}_k \qquad (4)$$

Update state after measurements are taken

$$K_k = P_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R_k\right)^{-1}$$
$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{z}_k - h(\hat{\mathbf{x}}_{k|k-1}, 0))$$
$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \qquad (5)$$

Here,

K is the Kalman gain matrix to be utilized in the update step.

P is the covariance matrix having information concerning exactness of the state estimation [11].

V. State Variable Feedback Controller Design

The designed Kalman filter gives the estimation of output and the states. Several control schemes can be made with Kalman filter. The SFC is one of the achievable control schemes. The control law for SFC as follows:

$$u = -Kx + K_r r \qquad (6)$$

Where, $\mathbf{K_r}$ is considered to get a steady state gain of unity between set point and actual response is given as:

$$K_r = -1 / (C\,(A-BK)^{-1}B) \qquad (7)$$

Here, **K** is the feedback gain matrix (FGM), it can be found by pole placement method [12].

In pole placement design the first step is to decide on the closed loop pole locations [13]. It is useful to remember that the required control action is related to how much far the open loop pole moved by the feedback. If a zero is nearer to the poles, the system may become uncontrollable [14]. To move those poles, a large control gains are needed. Dominant second order poles is an approach to choose the desired pole locations in the pole placement design. In this method we can deduce the rise time, overshoot and settling time directly from the pole locations. Several steps involved in finding the feedback gain that forces the roots of A-BK to μ_1, μ_2 μ_3 …. μ_n.

First step is that the controllability of the system to be checked. It can be done by using the given law:

$$M = [B\ AB A^2B\ \ldots\ \ldots\ A^{n-1}B] \qquad (8)$$

Here, M is controllability matrix.

For the system to be controllable, the matrix M should have the full rank. The second step is to determine the values of a_1, a_2, a_3….a_n from the characteristic equation,

$$|sI - A| = s^n + a_1 s^{n-1} + \ldots\ldots + a_{n-1} s + a_n \qquad (9)$$

Where $\mathbf{a_i}$'s represents the coefficients of the characteristic polynomial equation.

Third step is to govern the matrix T, the transformation matrix. It converts the state equation into controllable canonical form. If it is in that form, then T=I. The T matrix is given as:

$$T = MW \qquad (10)$$

Where,

W is given as:

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

Fourth step is to get the desired characteristic polynomial using desired eigenvalues and find the values of a_1, a_2,a_3,…..,a_n.

$$(s-\mu_1)(s-\mu_2)\ldots(s-\mu_n) = s^n + \acute{a}_1 s^{n-1} + \ldots + \acute{a}_{n-1}s + \acute{a}^n \qquad (11)$$

The last step is to find the FGM using the given equation,

$$K = [\acute{a}_n - a_n \,|\, \acute{a}_{n-1} - a_{n-1} \,|\, \ldots\ldots\ldots \,|\, \acute{a}_2 - a_2 \,|\, \acute{a}_1 - a_1]\, T^{-1} \qquad (12)$$

Ackermann's Formula to Determine the Matrix K:

A well-known formula to find the FGM is the Ackermann's formula. Consider the state space model

$$\dot{x} = Ax + Bu \qquad (13)$$

Here the SFC law is u =-Kx**.** If the above system is controllable, the feedback control law alters the system equation into $\dot{x}$ **=** (A – BK) x**.**

Let us describe $\tilde{A}$ =A-BK.Then the wanted characteristic polynomial will be

$|sI-A+BK|=|sI-\tilde{A}|=(s-\mu_1)(s-\mu_2)\ldots\ldots\ldots(s-\mu_n)$

$=s^n + á_1 s^{n-1} + \ldots + á_{n-1}s + á^n$(14)

By means of Cayley-Hamilton theorem, $\tilde{A}$ fulfills its characteristic polynomial,

$\phi(\tilde{A}) = \tilde{A}^n + á_1\tilde{A}^{n-1} + \ldots + á_{n-1}\tilde{A} + á_n I=0$ (15)

For simplicity, by considering the case n=3, and the following identical nesses:

I=I

$\tilde{A}$=A-BK

$\tilde{A}^2 = (A-BK)^2 = A^2 - ABK - BK\tilde{A}$

$\tilde{A}^3 = (A-BK)^3 = A^3 - A^2BK - ABK\tilde{A} - BK\tilde{A}^2$

By multiplying the equation by $á_3$, $á_2$, $á_1$ and $á_0$, and the results is added. Then the resulting equation is

$á_3I + á_2\tilde{A} + á_1\tilde{A}^2 + \tilde{A}^3 = á_3I + á_2(A - BK) + á_1(A^2 - ABK - BK\tilde{A}) + A^3 - A^2BK - ABK\tilde{A} - BK\tilde{A}^2 = á_3I + á_2A + á_1A^2 + A^3 - á_2BK - á_1ABK - á_1BK\tilde{A} - A^2BK - ABK\tilde{A} - BK\tilde{A}^2$ (16)

From equation (15), we have

$\phi(\tilde{A})=\tilde{A}^n+á_1\tilde{A}^{n-1}+\ldots+á_{n-1}\tilde{A}+á_nI=0$ and also we have

$\phi(\tilde{A})=\tilde{A}^n+á_1\tilde{A}^{n-1}+\ldots+á_{n-1}\tilde{A}+á_nI \neq 0$

By substituting the last two equation in (4), we will get

$\phi(\tilde{A}) = \phi(A) - á_2BK - á_1BK\tilde{A} - BK\tilde{A}^2 - á_1ABK - ABK\tilde{A} - A^2BK$

Since $\phi(A) = 0$**,** we obtain

$\phi(A)=B(á_2K+á_1K\tilde{A}+K\tilde{A}^2) + AB(á_1K+K\tilde{A}) + A^2BK=$

$$[B \mid AB \mid A^2B]\begin{bmatrix}\alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K\end{bmatrix}$$

Pre multiplying the above equation by the inverse of the matrix, M in the both of the sides, we get

$$[B \mid AB \mid A^2B]^{-1}\phi(A) = \begin{bmatrix}\alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K\end{bmatrix}$$

Pre multiplying the above equation by [0 0 1], we get

$[0\ 0\ 1][B \mid AB \mid A^2B]^{-1}\phi(A) =$

$$[0 \quad 0 \quad 1]\begin{bmatrix}\alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K\end{bmatrix} = K$$

We can write the above equation as

$K=[0\ 0\ 1][B \mid AB \mid A^2B]^{-1}\phi(A)$

The generalized form of above equation for a positive integer n is given as:

$K= [0\ 0....0\ 1]\ [B \mid AB \mid A^2B \mid .... \mid A^{n-1}B]^{-1}\ \phi(A)$ - (17)

The above equation is the Ackermann's formula to obtain the FGM.

Obtained results for K and K_r from equations (7) and (17) are given as:

K =[5604.7 -0.3349]

K_r = 15.477

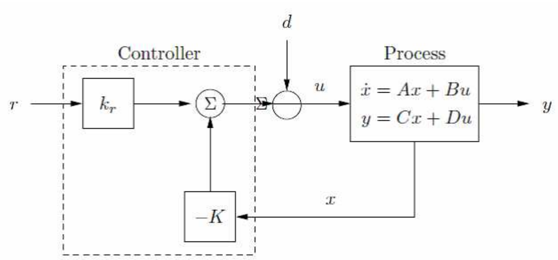

Fig.3. Block diagram for introducing the reference input with full state feedback with a composite gain.

VI. Results and Discussion

Simulation results for SFC and PID control of shell and tube heat exchanger is done using MATLAB Simulink. The controlled variables are cold fluid temperature and hot fluid temperature. The manipulated variables are inlet flow rate of cold fluid and hot fluid.

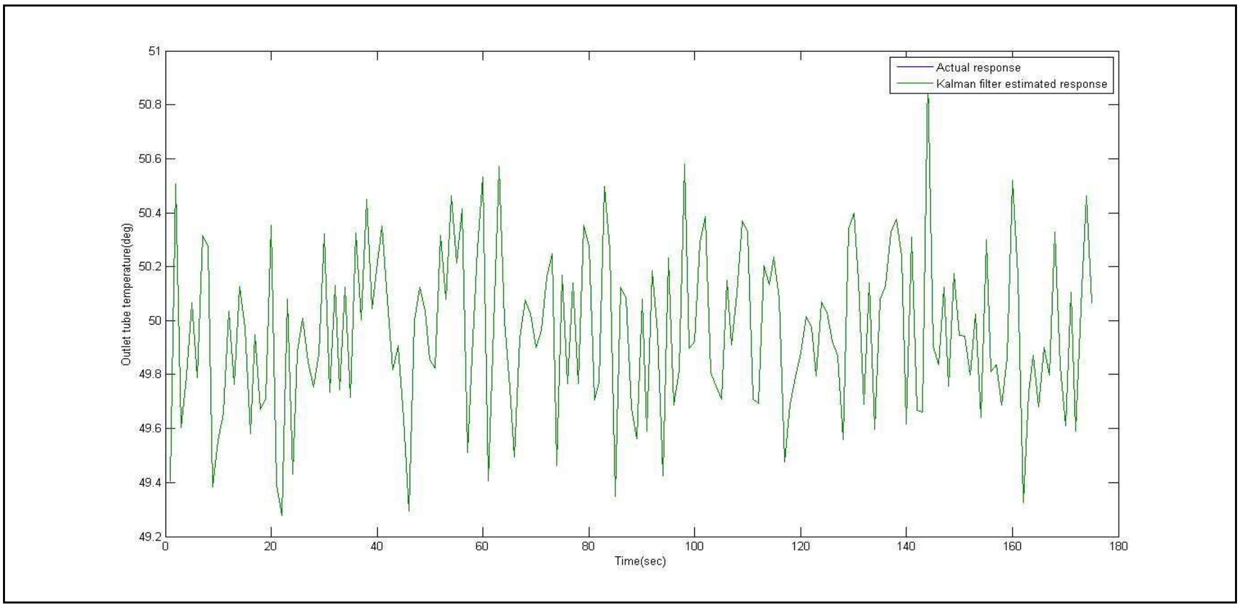

Fig.4. The effectualness of the Kalman filter in producing the best estimate of outlet tube temperature from noisy measurements.

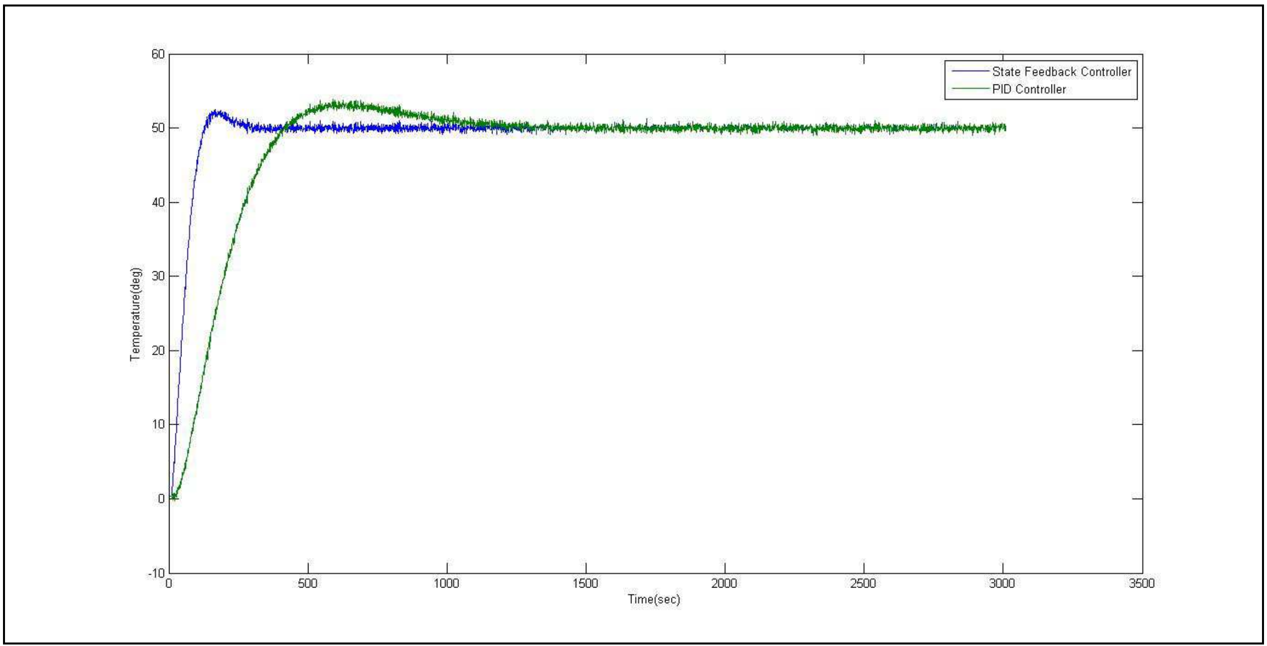

Fig.5. Comparision of responses of SFC and PID controller.

The SFC and PID controllers are designed and the simulations are carried out using MATLAB software.

The designed Kalman filter based observer given the best estimates of output and the states. The estimated Kalman filter output is presented in Fig.4.The performance of different controllers are evaluated using the step response of the system. The responses of both SFC and PID controllers are shown in Fig.5.

From the above it is observed that the overshoot has got reduced drastically by 33.33% when SFC was used when compared to conventional PID controller. The rise time of the system has improved by 62.61% and the settling time has got reduced by 75% in the case of SFC compared to conventional PID controller. The parameters are tabulated in Table 1.

Table 1

Controller	Rise Time (SEC)	Over Shoot (%)	Settling Time (SEC)
SFC	126	4	330
PID	337	6	1320

VII. Conclusion

In this paper SFC is designed for the control of temperature of a shell and tube heat exchanger and it is discussed by comparing with PID controller. The analysis of SFC and the PID control system model is designed by using SIMULINK. An analysis of the above results comes to a conclusion that Heat exchanger embedded with SFC will give superior results compared to a Heat exchanger embedded with conventional PID controller.

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