

# Simulation Of Vertical Gas-Solid Flow: Comparison Of Correlations For Particle-Wall Friction And Drag Coefficient

**K.S. Rajan**

**School of Chemical & Biotechnology, SASTRA University, Tirumalaisamudram  
Thanjavur, India – 613401.**

**Corres.author: ksrajan@chem.sastra.edu  
Ph: 914362 304270**

**Abstract:** Pneumatic conveying is a widely used operation in power plants, mineral industries for transportation of dust, ash, mineral etc. It forms an easy and eco-friendly method for the dry transportation of particulate solids. Pressure drop plays an important role in the economic design of the pneumatic conveying system. A review of correlations available for estimation of drag coefficient and the particle-wall friction has been made. The ability of correlations to predict the pressure drop for the data available in the literature has been studied using a one-dimensional, two-fluid model. The simulation, along with statistical analysis indicates the suitability of few correlations for drag coefficient estimation for a wide range of particle size and solid mass flow rates. For the estimation of particle-wall friction, a correlation suitable for large particles at large solid feed rate needs to be developed from experimental results.

**Key words:** Pneumatic conveying, drag coefficient, particle-wall friction, pressure drop.

## INTRODUCTION

The experimental and computational study of two phase gas-solid flow in various configurations is of interest to researchers owing to its wide-range of applications [1-6]. The conventional approach to the simulation of two phase gas-solid flows is the use of one dimensional mass, momentum and energy conservation differential Equations for the gas phase and the solid phase using the cross-section averaged velocities, densities and volume fraction of the phases. These set of differential Equations are converted to system of algebraic (difference) Equations, the solution of which would provide the cross sectional averaged information about velocity, volume fraction and density of the phases [5-6, 7]. This approach is simple, and provides the macroscopic information about the system. While the use of cross-section averaged one dimensional

equations offer the advantage of computationally less rigorous and hence easier and faster, the simulations carried out using multi-dimensional models offer higher insights into the system. However the success of the one dimensional and multi-dimensional model depends on the accuracy of the empirical correlations used and their suitability for the simulation problem under question. The present study is aimed at comparing the existing correlations for the simulation and suggests the more suitable of the available correlations for the particular case. Proposal of a new correlation requires the rigorous analysis of more experimental data under widely different operating conditions and is not attempted here.

## MODEL

A one dimensional model for dilute phase pneumatic conveying has been developed using the momentum, mass and energy conservation Equations for both the phases and solved using the method reported earlier [4, 5]. The conservation Equations contain source terms that utilize mass, momentum and energy interaction between the gas and particles. Since the model was developed for dilute phase conveying, all types of particle-particle interactions are ignored. The model does not account for mass transfer between the phases. Hence only the sources due to the following were considered: particle-wall friction, gas-wall friction, gas-particle heat transfer, drag due to slip between the phases & gravitational force for both the phases. The detailed model, its assumptions, constitutive Equations, solution methodology and its performance can be obtained from our earlier work by neglecting the terms that contain radial derivative [4]. Evaluation of source terms requires a number of constitutive Equations for various parameters and is available in [4]. Gas-wall friction was determined by the well known Blasius Equation, which has been widely accepted to represent the gas-wall friction accurately. In the present work, the applicability of various correlations for drag coefficient and particle-wall friction has been tested by comparing the simulation results with the published experimental data [8, 9].

## ESTIMATION OF DRAG COEFFICIENT

A list of correlations that has been used widely for determining the single-particle drag coefficient in the simulation of fast fluidized beds, circulating fluidized beds, risers & pneumatic conveying by various researchers [10-17] is given in Table 1. These have been used in various one and two-dimensional simulation of gas-solid flow problems. The ranges of Reynolds number for which the correlations are applicable are also presented in Table 1.

While considering the effects of presence of multiple particles, the drag force per unit length that exists between gas and solid phases is given as [18]

$$F_d = \frac{3ACdA_s \dots_g (u_g - u_p)^2}{4A_g^{2.65}}$$

(1)

where Cd represents the single particle drag coefficient.

## ESTIMATION OF PARTICLE-WALL FRICTION

Among several equations available for prediction of particle-wall friction and initial solids velocity, choice of an appropriate equation is difficult [19]. From the mass flow rates of gas and solids, pressure drop and porosity data from pneumatic conveying experiments, the particle-wall friction have been calculated and provided in the literature using an Equation of the form,

$$f_p = \frac{\Delta P_s}{L} \left( \frac{2D}{\dots_p u_p^2} \right);$$

(2)

where  $f_p$  represents friction factor for solids. The detailed experiment and the calculation of solids friction factor from a typical pneumatic conveying experiment are available in the literature [20]. A list of correlations that have been developed from such pneumatic conveying studies and published in the literature [20-24] is given in Table 2 which covers the wide range of particles sizes, mass flow rate of gas and particles, void fraction etc.

## RESULTS & DISCUSSION

Pressure drop is an important parameter in the design of the pneumatic conveying systems for disposal of dust, fly ash etc. The requirements of the blower/vacuum equipment are determined from the pressure drop along the conveying line. Pressure drop in a pneumatic conveying is due to the cumulative effects of particle acceleration, gas-wall friction, gas-solid friction, drag and gravitational forces [9]. During the particle acceleration, the particles gain momentum from gas by virtue of which their velocity increases to a large value from its initial velocity (a very low value). High pressure drops are experienced in the acceleration region, due to high relative velocity in that region. After the acceleration region, the gas and solid phase velocities practically remain constant. The length of acceleration region depends on the relative amounts of gas and solid flow, particle size etc.

**Table 1: Correlations for estimation of single particle drag coefficient.**

Correlation	Reference No.	Equation No.
$Cd = \frac{64}{f Re_p} \left(1 + \frac{64}{2f}\right); Re_p < 0.01$ $Cd = \frac{64}{f Re_p} (1 + 10^x); 0.01 < Re_p < 1.5$ $x = -0.883 + 0.906 \log(Re_p) - 0.025[\log(Re_p)]^2$ $Cd = \frac{64}{f Re_p} (1 + 0.138 Re_p^{0.792}); 1.5 < Re_p < 133$ $\log(Cd) = 2.0351 - 1.66(L) + 0.3958(L^2) - 0.0306(L^3);$ $40 < Re_p < 1000$ $Re_p = \frac{\dots_g (u_g - u_s) D_p}{\dots_g}$	[10]	(3)
$Cd = \frac{24}{Re_p}; Re_p < 1$ $Cd = 0.44; Re_p > 1000$	[11]	(4)
$Cd = \left(0.63 + \frac{4.8}{Re_p^{0.5}}\right)^2$	[12]	(5)
$Cd = \left(0.336 + \frac{17.3}{Re}\right)$ $Re = \frac{A_g \dots_g (u_g - u_s) D_p}{A_g}$	[13]	(6)
$Cd = \left(0.4 + \frac{24}{Re_p} + \frac{4}{Re_p^{0.5}}\right)$	[14]	(7)
$Cd = \left(0.28 + \frac{21}{Re_p} + \frac{6}{Re_p^{0.5}}\right)$	[15]	(8)
$Cd = 18 \left[1 + 0.125 (Re_p^{0.72})\right]; Re_p < 1000;$ $Cd = 0.445; Re_p > 1000$	[16]	(9)
$Cd = 18 \left[1 + 0.1 (Re_p^{0.75})\right]$	[17]	(10)

**Table 2: Correlations for estimation of particle-wall friction**

Correlation	Reference No.	Equation No.
$f_p = 0.0054 \left(\frac{W \dots_g}{\dots_p}\right)^{-0.115} \left(\frac{u_g D_p}{u_p D}\right)^{0.339}; W = m_s/m_g$	[21]	(11)
$f_p = 0.0017 \left(\frac{v_p u_t}{v_g^4 u_g}\right) \left(\frac{u_t v_p}{u_g - u_p}\right)^{1.5}$	[20]	(12)
$f_p = 12.2 \left(\frac{v_p}{v_g u_p}\right)$	[20]	(13)
$f_p = \frac{55.5 D^{1.1}}{u_p^{0.65} D_p^{0.26} \dots_p^{0.91}}$	[20]	(14)

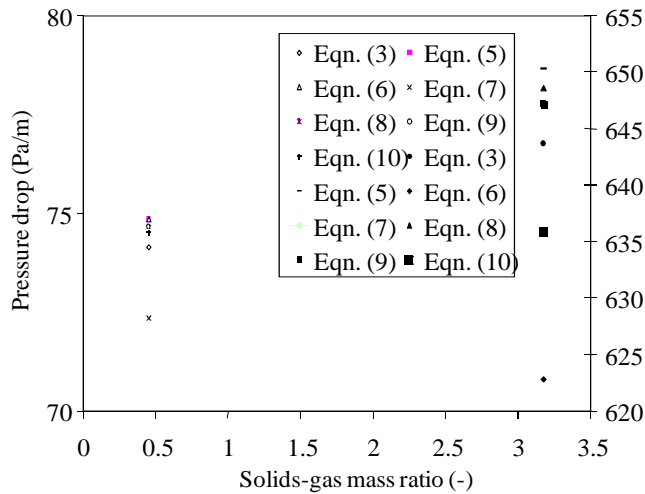
$f_p = A \left( \frac{v_p}{v_g} \right) \left( \frac{u_t v_p}{u_g - u_p} \right)^B ;$ $A = 0.0126 \text{ \& } B = -0.979 ; u_g > u_t$ $A = 0.041 \text{ \& } B = -1.021 ; u_g < u_t$	[22]	(15)
$f_p = 0.048(u_p)^{-1.22} ; D_p = 20\mu\text{m}, u_p = 3.5\text{-}22.9\text{m/s}$	[20]	(16)
$f_p = 0.074u_p^{-0.75} ; u_p = 3.0\text{-}6.7\text{m/s}$	[20]	(17)
$f_p = 0.0285u_p^{-1}(gD)^{0.5} ; D = 26.5 - 46.8 \text{ mm}$ $u_p = 1\text{-}10 \text{ m/s}$	[23]	(18)
$\lambda_z = 0.082 \mu^{-0.3} Fr^{-0.86} Fr_s^{0.25} (D/d_p)^{0.1} ; d_p > 500\mu\text{m}$ $\lambda_z = 2.1 \mu^{-0.3} Fr^{-1} Fr_s^{0.25} (D/d_p)^{0.1} ; d_p < 500\mu\text{m}$ $\Delta P = \lambda_z \rho_g U_g^2 L W / (2D) ; W = m_s / m_g ; Fr = u^2 / gD$ $Fr_s = u_t^2 / gD$	[24]	(19)

### COMPARISON OF CORRELATIONS FOR DRAG COEFFICIENT

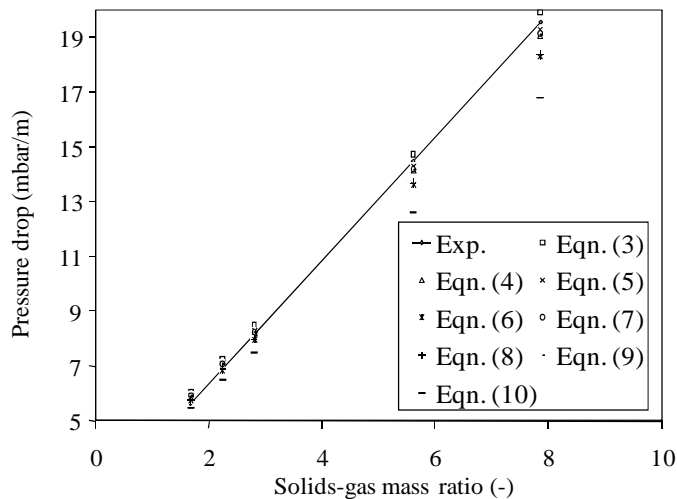
Correlations in the Table 1 were tested for their applicability to predict the pressure drop. Some literature studies show the comparison of drag coefficients predicted from different correlations for a wide range of particle Reynolds number ( $Re_p$ ). Since, pressure drop is more important from the economic design aspects, these correlations were tested for their applicability to predict pressure drop. This may yield a wide choice of expressions suitable to design/simulate dilute phase pneumatic conveying. The experimental data were so chosen from the literature such that the simulations were carried out with fine and coarse particles, with a wide range of solid to gas mass ratios.

Poikoainen et al. [9] presented the experimental data for the pressure drop in the pneumatic conveying of ceramic particles of 64  $\mu\text{m}$  diameter through a duct of 19.2 cm diameter, at different solids feed rates and gas velocities. The pressure drop data were measured after 4 meters from the bottom of the conveying duct, thereby eliminating the pressure drop in the acceleration zone. A plot of solids friction factor and solids velocity, from their experimental data was also presented. Their data was utilized for

comparison of different expressions for drag coefficient. Simulations were done using the correlations listed in Table 1 for a particular value of gas and solid flow rates, thus for a fixed value of solid-gas mass ratio to predict the pressure drop for 5 m length after the acceleration zone. The simulations were repeated for different solid-gas mass ratios. The numerical value of wall-particle friction factors were taken from their work. It was observed that almost all the correlations in the Table 1 predicted the pressure drop very closer to the experimental value, with a very little absolute error of less than 1%. This behavior can be explained as follows: Since the simulated and reported pressure drops do not include the acceleration region, the velocities of both the phases in the portion of the conveying duct under simulation (after acceleration region) are fairly constant. Also with finer particles, the slip velocity (difference between the gas and solid velocity) is less, further reducing the pressure drop due to drag. To compare the true performance of correlations, the simulations were performed for 1m length of the acceleration region from the bottom to predict the pressure drop for the solid-gas mass ratios of 0.415 & 3.178. The results are shown in the Figure 1.



**Figure 1: Comparison of pressure drop predicted for acceleration region using the Equations (3) to (10) for the dilute phase conveying of 64 ~m ceramic particles through a duct of 0.192 m.**



**Figure 2: Comparison of pressure drop predicted using the Equations (3) to (10) and the experimental data of [8] for the dilute phase conveying of 5 mm polythene particles through a tube of 5.34 cm.**

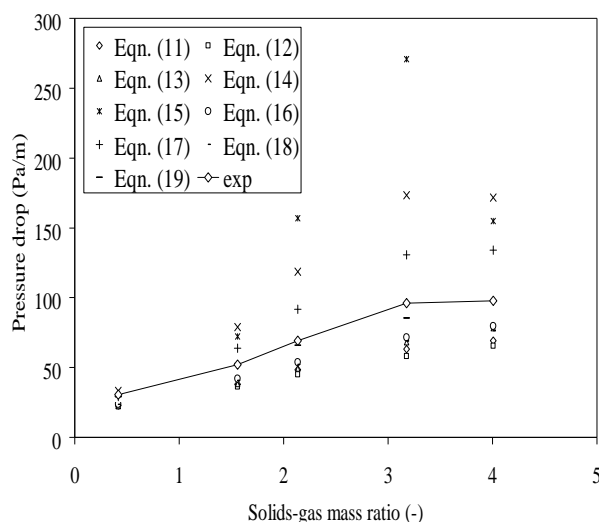
It can be seen from the **Figure 1** that the pressure drop in the acceleration region increases with the increase in solids-gas flow ratio. This is attributed to the fact that the presence of more solids results in their decreased acceleration, thereby increasing the slip velocity and hence the high pressure drop in the acceleration region. The RMS errors in the pressure drop prediction among the different correlations were 0.9 and 9.92 for the solid-gas mass ratio of 0.4175 and 3.178 respectively. Equations (3), (4), (6), (7) & (8) were found to predict pressure drop closer to each other with much lesser RMS errors. Drag force increases with the particle size, due to slower acceleration or larger slip velocities. Hence, the comparison of these correlations for a larger particle size was necessary to complete the

analysis. Woodhead et al. [8] presented the pressure drop data for the pneumatic conveying of 5 mm polythene particles through a duct of 5.34 cm at different gas velocities and suspension densities. The pressure drop included the acceleration region also. Capes and Nakamura Equation [20] was used to estimate the particle-wall friction. Simulations were performed using the correlations for various solids-gas mass ratios from 1.7 to 7.8 and the results are plotted in **Figure 2**. **Figure 2** shows the measured pressure drop and the pressure drop predicted from correlations as a function of the solids-gas mass ratio. The deviations are large at the larger solids-gas ratio, due to higher slip and higher drag. A statistical analysis was performed and the RMS errors for these

correlations were estimated. The Equations (3), (4), (5), (7) & (9) had RMS errors lesser than 0.35, while the other correlations showed RMS errors in the range of 0.65-1.55 for the prediction of pressure drop. Hence, it can be concluded that the Equations (3), (4), (5), (7) & (9) can be satisfactorily utilized for the design and simulation of pneumatic conveying of large particles (5 mm) and for wide range of solids-gas mass ratios (1.7–7.8). The best correlation under review for the simulation of data in [8] is the correlation of Kaskas [14] with a RMS error of 0.22 for pressure drop prediction.

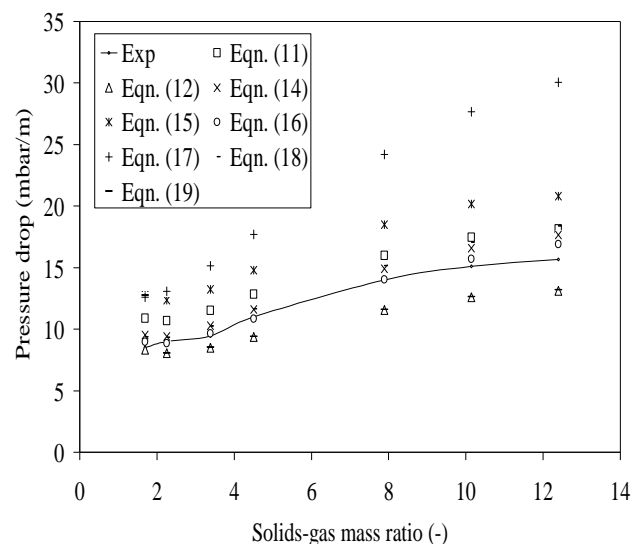
From the simulations carried out in the present study and the comparison with experimental data in [8] and [9], it can be concluded that the Equations (3), (4) & (7) can be used for the simulation of pressure drop in pneumatic conveying of particles in the size range 64  $\mu$ m to 5 mm and solids-gas mass ratio in the range of 0.4 – 7.8.

**COMPARISON OF CORRELATIONS FOR WALL-PARTICLE FRICTION** Correlations in Table 2 were tested for their applicability to predict the pressure drop through estimation of wall-particle friction. The simulations were performed for the conveying of ceramic spherical particles of 64  $\mu$ m through the duct of 19.2 cm for various solids-gas mass ratios. The correlations in the Table 2 were used to predict



**Figure 3: Comparison of pressure drop predicted using the Equations (11) to (19) and the experimental data of [9] for the dilute phase conveying of 64  $\mu$ m ceramic particles through a duct of 0.192m.**

the particle-wall friction in the model for each solids-gas mass ratio. For the estimation of drag coefficient, the Equation 5 was used, as proposed in the previous section of this article. The results were compared with the experimental data in [9]. The range of solids-gas mass ratio for the simulation is 0.4-4. The results are plotted in Figure 3, as pressure drop-solids-gas mass ratio relationship predicted using different correlations for wall-particle friction. It can be seen from the Figure 3 that most of the correlations predict the pressure drop with large deviation from the experimental value. It can also be seen that the Equation (19) predicts the pressure drop closely with the experimental data, when compared with the other correlations. The RMS error for the data predicted by Equation (19) was 5 Pa/m, while the other correlations had RMS errors in the range of 16-90 Pa/m for the prediction of pressure drop. Since the prediction of Equations (11), (12), (16) & (18) are fairly closer to each other and have the same behavior as that of the experimental data; they can be improved by altering the empirical constants in these Equations. This also indicates the strong effect of particle size and mass flow ratios on the wall-particle friction. Hence Equation (19) and the Equations (11), (12), (16) & (18) (with different constants) may be used for design and simulation for particles in the micron size range.



**Figure 4: Comparison of pressure drop predicted using the Equations (11) to (19) and the experimental data of [8] for the dilute phase conveying of 5 mm polythene particles through a tube of 5.34 cm.**

To study the performance of these correlations for large sized particles, simulations were

performed for the conveying of ceramic spherical particles of 5 mm diameter through the tube of 5.34 cm for various solids-gas mass ratios in the range of 1.7 – 12.4. Figure 4 shows the comparison of pressure drops predicted by different correlations and the experimental data. Again, the predictions from different correlations are different to a large extent. Statistical analysis shows that the correlation of Capes and Nakamura [20] predicts the pressure drop with a RMS error of 0.67 mbar/m. Equations (11), (12), (13), (17) & (18) had RMS errors between 1.3 and 2.9 mbar/m for pressure drop prediction. Since the pressure drop values are less, higher deviations will result in too an erroneous prediction. The deviations are more pronounced at higher solids-gas mass ratios, indicating that a correlation suitable for the simulation of large particle flow at high solids-gas mass ratios needs to be developed.

## **CONCLUSIONS**

The prediction of pressure drop is critical for the design and simulation of pneumatic conveying equipments. The economy of a pneumatic

conveying operation for dust disposal depends on the pressure drop in the conveying line. Since the pressure drop estimation involves determination of number of forces acting on the solids and the gas, various correlations proposed in the literature for the prediction of drag coefficient and wall-particle friction were tested. Some of the existing correlations for the prediction of drag coefficient have been found to be satisfactory for particles of small and large size (64  $\mu$ m & 5 mm) for a wide range of mass flow ratios (0.4 – 7.8). The existing correlations for the prediction of wall-particle friction estimate the pressure drop with a large error, for the large particles at high solids-gas mass flow ratios, indicating the necessity for more experimental investigations in these operating conditions.

## **Acknowledgements:**

The author acknowledges Dr. Visa Poikoinen for providing the experimental data for the simulation and for his valuable suggestions through electronic mail.

## **NOMENCLATURE**

### **ALPHABETS**

A	Cross-sectional Area (m <sup>2</sup> )
C <sub>d</sub>	Drag Coefficient (-)
D	Diameter of duct (m)
D <sub>p</sub>	Diameter of particle (m)
f <sub>p</sub>	Particle-wall friction factor (-)
m	Mass flow rate (kg/s)
$\Delta P$	Pressure drop (Pa)
Re <sub>p</sub>	Particle Reynolds number (-)
Re	Reynolds number (-)
u	Actual velocity (m/s)
W	Solids-gas mass flow ratio (-)
L	Length (m)
F <sub>d</sub>	Drag force per unit length (N/m)
Fr	Froude number (-)

### **GREEK SYMBOLS**

$\rho$	Density (kg/m <sup>3</sup> )
$\mu$	Viscosity (kg/m.s)
$\varepsilon$	Volume fraction of phase (-)
$\lambda$	Friction factor in Equation 19 (-)

### **SUBSCRIPTS & SUPERSSCRIPTS**

g	Gas phase
p	Particle
s	Solid phase
t	Terminal

**REFERENCES**

1. Rajan KS, Srivastava SN, Pitchumani B, Surendiran V. "Thermal conductance of pneumatic conveying preheater for air-gypsum and air-sand heat transfer", *Int J Therm Sci*, 2010; 49(1): 182-186.
2. Rajan KS, Dhasandhan K, Srivastava SN, Pitchumani B. "Studies on Gas-Solid Heat Transfer during Pneumatic Conveying", *Int J Heat Mass Transfer*, 2008; 51(11-12): 2801-2813.
3. Rajan KS, Dhasandhan K, Srivastava SN, Pitchumani B. "Experimental study of thermal effectiveness in pneumatic conveying heat exchanger", *Appl Therm Eng*, 2007; 27(8-9): 1345-1351.
4. Rajan KS, Pitchumani B, Srivastava S.N, Mohanty B. "Two dimensional simulation of gas-solid heat transfer during pneumatic conveying", *Int J Heat Mass Transfer*, 2007; 50 (5-6): 967-976.
5. Rajan KS, Srivastava S.N, Pitchumani B, Mohanty B. "Simulation of countercurrent gas-solid heat exchanger: Effect of solid loading ratio and particle size", *Appl Therm Eng*, 2007; 27(8-9): 1345-1351.
6. Rajan KS, Srivastava S.N, Pitchumani B, Mohanty B. "Simulation of gas-solid heat transfer during pneumatic conveying: Use of multiple gas inlets along the duct", *Int Commun Heat Mass Transfer*, 2006; 33(10): 1234-1242.
7. Skuratovsky I, Levy A, Borde I. "Two-Fluid, Two-Dimensional Model for Pneumatic Drying", *Drying Technol.*, 2003; 21 (9): 1645-1668.
8. Woodhead SR, Hettiaratchi K, Reed AR. "Comparison between pressure drop in horizontal and vertical pneumatic conveying pipelines", *Powder Technol*, 1998; 95: 67-73.
9. Poikoinen V, Rautiainen A, Stewart G, Sarkomaa P. "An experimental study of vertical pneumatic conveying", *Powder Technol*, 1999; 104: 139-150.
10. Levy A, Borde I. "Steady state one dimensional flow model for a pneumatic dryer", *Chem Eng Process.*, 1999; 38: 121-130.
11. Bolkan Y, Berruti F, Zhu J, Milne B. "Hydrodynamic Modeling of CFB Risers and Downers", *Int J Chem React Eng.*, 2003; 1:
12. Xu BH, Yu AB. "Numerical simulation of the gas-solid flow in a fluidized bed by combining discrete particle method with computational fluid dynamics", *Chem Eng Sci*, 1997; 52 (16): 2785-2809.
13. Benyahia S, Arastoopour H, Knowlton TM, Massah H. "Simulation of particles and gas flow behavior in the riser section of a circulating fluidized bed using the kinetic theory approach for the particulate phase", *Powder Technol*, 2000; 112: 24-33.
14. Kovacs L, Varadi S. "Two-Phase Flow in the Vertical Pipeline of Air Lift", *Periodica Polytechnica Ser. Mech. Eng.*, 1999; 43(1): 3-18.
15. Zhang Y, Reese JM. "Particle-gas turbulence interactions in a kinetic theory approach to granular flows", *Int J Multiphas Flow*, 2001; 27: 1945-1964.
16. Han T, Levy A, Kalman H, Peng Y, "Model for dilute gas-particle flow in constant-area lance with heating and friction", *Powder Technol*, 2000; 112: 283-288.
17. Cao J, Ahmadi G, "Gas-Particle Two-Phase Turbulent Flow in a Vertical Duct", *Int J Multiphas Flow*, 1995; 21(6): 1203-1228.
18. Levy A. "Two-fluid approach for plug flow simulations in horizontal pneumatic conveying", *Powder Technol*, 2000; 112: 263-272.
19. Dzido G, Palica MC, Raczek J. "Investigations of the acceleration region in the vertical pneumatic conveying", *Powder Technol*, 2002; 127: 99-106.
20. Rautiainen A, Sarkomaa P. "Solids friction factors in upward, lean gas-solids flows", *Powder Technol*, 1998; 95: 25-35.
21. Ozbelge TA, "Solids friction factor correlation for vertical upward pneumatic conveying", *Int J Multiphas Flow*, 1984; 10(4): 459-465.
22. Yang WC, "A correlation for solid friction factor in vertical pneumatic conveying lines", *AIChE J.*, 1978; 24: 548.
23. Wang FJ, Zhu JX, Beeckmans JM, "Pressure gradient and particle adhesion in the pneumatic transport of cohesive fine powders", *Int J Multiphas Flow*, 2000; 26: 245-265.
24. Gerchow FJ, Dilute Phase Pneumatic Conveying, Chapter 14 in *Encyclopedia of Fluid Mechanics*, Cheremisinoff N.P.(ed),4, Gulf Publishing, Houston, 1986.

\*\*\*\*\*