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Expected Time to attain the Threshold Level using Multisource of HIV Transmission – Shock Model Approach

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Abstract: The important characteristic of the threshold for a person is the time to attain the break down point. The assumptions made are somewhat artificial, but are made because of lack of detailed real-world information. In this paper, a person affected having three sources two Exponential and one Erlang distribution with independent thresholds is considered and the mean and variance of the time to cross the threshold are obtained using the shock model and cumulative damage process. The analytical results are numerically illustrated by assuming specific distributions.
Key Word: Exponential, Erlang, HIV, Mean, Shock model, Threshold, Variance.

Introduction

It has been widely recognized that the amount of people infected with human immunodeficiency virus (HIV) has been increasing in recent years, especially in developing countries. Thus, there is an urgent need for rapid diagnosis, monitoring and antiretroviral therapy. HIV is a retrovirus that targets the CD4C T lymphocytes, which are the most abundant white blood cells of the immune system. Although HIV infects other cells also, it wreaks the most havoc on the CD4C T cells by causing their decline and destruction, thus decreasing the resistance of the immune system. A number of mathematical models have been proposed to understand HIV dynamics, disease progression, anti-retroviral response etc. The spreading of HIV has taken different directions since the virus was first discovered, or more correctly, our perception of how and where HIV is spreading has changed over the years. Initially, men were infected to a greater extent than women but this has evened out and now women are identified as the most affected group together with youth and poor.

In this paper a mathematical model is developed to obtain the expected time and Variance to reach the threshold level, in the context of HIV/AIDS with the assumptions that the times between decision epochs are independent and identically distributed (i.i.d) random variable, the number of exits at each period time are i.i.d. random variables and that the threshold level is a random variable following two Exponential distribution and one Erlang distribution. One can see for more detail in Esary *et al,* [1] Nowak and May[2] and Stilianakis *et al.*[3] *For* obtaining mean and variance of expected time one can refer Kannan *et al*[4] and Palanivel *et al.*[5]

Assumption of the Model

- An uninfected partner has sexual contacts with an infected person, unsterile needles for drug abuse and Transfusion of infected blood products.
- On every occasion of the above three behaviors there is a random amount of transmission of HIV, which together contributes to the antigenic diversity.
- The damages due to the events namely sexual contacts, sharing of needles and blood products are statistically independent.
- If the cumulative damage due to successive events crosses the antigenic diversity threshold level the seroconversion takes place. The inter-arrival times between contacts, sharing needles and blood products are statistically independent.

Notation

X_i : a continuous random variable denoting the amount of damage caused to the component on the ith occasion of contacts (Shock), 1, 2….k and $X_i's$ are i.i.d

Y_1, Y_2, Y_3: Continuous random variable denoting the threshold levels for the three components.

$g(.)$: The probability density function of X.

$g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: the k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{j=1}^{k} X_i$

$f(.)$: p.d.f. of random variable denoting between three modes of contact with the corresponding c.d.f. $F(.)$.

$F_k(.)$: k-fold convolution of $F(.)$.

$S(.)$: Survival function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$ [i.e., Probability of exactly k contact]

$L(t)$: $1 - S(t)$.

Result

Let $\bar{H}(x) = 1 - H(x)$

$$= 1 - \left(1 - e^{-\lambda_1 x}\right)\left(1 - e^{-\lambda_2 x}\right)\left(1 - e^{-\lambda_3 x} + \lambda_3 e^{\lambda_3 x}\right)$$

$$\bar{H}(x) = e^{-\lambda_1 x} + e^{-\lambda_2 x} + e^{-\lambda_3 x} - e^{-(\lambda_1+\lambda_2)x} - \lambda_3 e^{\lambda_3 x} + e^{-(\lambda_2+\lambda_3)x} + \lambda_3 e^{-(\lambda_2-\lambda_3)x}$$
$$- e^{-(\lambda_1+\lambda_3)x} + \lambda_3 e^{-(\lambda_1-\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_3)x} - \lambda_3 e^{-(\lambda_1+\lambda_2-\lambda_3)x}$$

$P(X_1 + X_2 + \cdots + X_k < Y)$ = P [the system does not fail, after k epochs of exits].

In general, assuming that the threshold Y follows Exponential and Erlang Distribution with parameter θ, it can be shown that..

$$P(X_i < Y) = \int_0^\infty g_k(x)\bar{H}(x)dx$$

$$= \int_0^\infty g_k(x)[e^{-\lambda_1 x} + e^{-\lambda_2 x} + e^{-\lambda_3 x} - e^{-(\lambda_1+\lambda_2)x} - \lambda_3 e^{\lambda_3 x} - e^{-(\lambda_2+\lambda_3)x}$$
$$+ \lambda_3 e^{-(\lambda_2-\lambda_3)x} - e^{-(\lambda_1+\lambda_3)x} + \lambda_3 e^{-(\lambda_1-\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_3)x}$$
$$+ \lambda_3 e^{-(\lambda_1+\lambda_2-\lambda_3)x}]dx$$

If the cumulative damage in any one of the damage system cross its threshold level, the transfer of HIV is not possible. The probability that the infected person survey even after the cumulative loss of damage in the respective sources after decision is given by

$$P(T > t) = \sum_{k=0}^{\infty} P[\text{there are exactly k instants of exit in}(0,t)]$$
$$* P[\text{the system does not fail in } (0,t)]$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left[\sum_{i=1}^{k} X_i < \max\,(Y_1, Y_2, Y_3)\right]$$

It is also known from renewal theory that

$$P(\text{exactly k policy decesions in } (0,t]) = F_k(t) - F_{K+1}(t) \qquad \text{with} \quad F_0(t) = 1$$

$$= \sum_{k=0}^{\infty} V_k(t) P(X_i < Y)$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \int_0^{\infty} g_k(x) \{ e^{-\lambda_1 x} + e^{-\lambda_2 x} + e^{-\lambda_3 x} - e^{-(\lambda_1+\lambda_2)x} - e^{-(\lambda_2+\lambda_3)x}$$
$$- e^{-(\lambda_1+\lambda_3)x} + e^{-(\lambda_1+\lambda_2+\lambda_3)x}$$
$$- \left[e^{\lambda_3 x} + e^{-(\lambda_2-\lambda_3)x} + e^{-(\lambda_1-\lambda_3)x} - e^{-(\lambda_1+\lambda_2-\lambda_3)x} \right] \lambda_3 \}$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1)]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_2)]^k$$
$$+ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_3)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1+\lambda_2)]^k$$
$$- \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_2+\lambda_3)]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1+\lambda_3)]^k$$
$$+ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1+\lambda_2+\lambda_3)]^k$$
$$- \left\{ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_3)]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_2-\lambda_3)]^k \right.$$
$$+ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1-\lambda_3)]^k$$
$$\left. - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)][g^*(\lambda_1+\lambda_2-\lambda_3)]^k \right\} \lambda_3$$

$$= [1 - (1 - g^*(\lambda_1))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1)]^{k-1} + [1 - (1 - g^*(\lambda_2))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2)]^{k-1}$$
$$+ [1 - (1 - g^*(\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_3)]^{k-1}$$
$$- [1 - (1 - g^*(\lambda_1+\lambda_2))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2)]^{k-1}$$
$$- [1 - (1 - g^*(\lambda_2+\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2+\lambda_3)]^{k-1}$$
$$- [1 - (1 - g^*(\lambda_1+\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_3)]^{k-1}$$
$$+ [1 - (1 - g^*(\lambda_1+\lambda_2+\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2+\lambda_3)]^{k-1}$$
$$- \left\{ [1 - (1 - g^*(\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_3)]^{k-1} \right.$$
$$+ [1 - (1 - g^*(\lambda_2-\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2-\lambda_3)]^{k-1}$$
$$+ [1 - (1 - g^*(\lambda_1-\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1-\lambda_3)]^{k-1}$$
$$\left. - [1 - (1 - g^*(\lambda_1+\lambda_2-\lambda_3))] \sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2-\lambda_3)]^{k-1} \right\} \lambda_3$$

Now

$L(T) = 1 - S(t)$

Taking Laplace transform of $L(T)$, we get

$$
\begin{aligned}
L(T) = 1 - \Big[& \left[1-\left(1-g^*(\lambda_1)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1)]^{k-1} + \left[1-\left(1-g^*(\lambda_2)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2)]^{k-1} \\
& + \left[1-\left(1-g^*(\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_3)]^{k-1} \\
& - \left[1-\left(1-g^*(\lambda_1+\lambda_2)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2)]^{k-1} \\
& - \left[1-\left(1-g^*(\lambda_2+\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2+\lambda_3)]^{k-1} \\
& - \left[1-\left(1-g^*(\lambda_1+\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_3)]^{k-1} \\
& + \left[1-\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2+\lambda_3)]^{k-1} \\
& - \Big\{ \left[1-\left(1-g^*(\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_3)]^{k-1} \\
& + \left[1-\left(1-g^*(\lambda_2-\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_2-\lambda_3)]^{k-1} \\
& + \left[1-\left(1-g^*(\lambda_1-\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1-\lambda_3)]^{k-1} \\
& - \left[1-\left(1-g^*(\lambda_1+\lambda_2-\lambda_3)\right)\right]\sum_{k=1}^{\infty} F_k(t)[g^*(\lambda_1+\lambda_2-\lambda_3)]^{k-1} \Big\} \lambda_3
\end{aligned}
$$

$$
\begin{aligned}
L^*(S) = 1 - \Big\{ & \frac{\left(1-g^*(\lambda_1)\right)f^*(S)}{[1-g^*(\lambda_1)f^*(S)]} + \frac{\left(1-g^*(\lambda_2)\right)f^*(S)}{[1-g^*(\lambda_2)f^*(S)]} + \frac{\left(1-g^*(\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_3)f^*(S)]} \\
& - \frac{\left(1-g^*(\lambda_1+\lambda_2)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_2)f^*(S)]} - \frac{\left(1-g^*(\lambda_2+\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_2+\lambda_3)f^*(S)]} \\
& - \frac{\left(1-g^*(\lambda_1+\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_3)f^*(S)]} + \frac{\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_2+\lambda_3)f^*(S)]} \\
& - \Big[\frac{\left(1-g^*(\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_3)f^*(S)]} + \frac{\left(1-g^*(\lambda_2-\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_2-\lambda_3)f^*(S)]} + \frac{\left(1-g^*(\lambda_1-\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_1-\lambda_3)f^*(S)]} \\
& - \frac{\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)f^*(S)}{[1-g^*(\lambda_1+\lambda_2+\lambda_3)f^*(S)]} \Big] \lambda_3 \Big\}
\end{aligned}
$$

$$= 1 - \left\{ \frac{c(1-g^*(\lambda_1))}{[c+s-g^*(\lambda_1)c]} + \frac{c(1-g^*(\lambda_2))}{[c+s-g^*(\lambda_2)c]} + \frac{c(1-g^*(\lambda_3))}{[c+s-g^*(\lambda_3)c]} - \frac{c(1-g^*(\lambda_1+\lambda_2))}{[c+s-g^*(\lambda_1+\lambda_2)c]} \right.$$
$$- \frac{c(1-g^*(\lambda_2+\lambda_3))}{[c+s-g^*(\lambda_2+\lambda_3)c]} - \frac{c(1-g^*(\lambda_1+\lambda_3))}{[c+s-g^*(\lambda_1+\lambda_3)c]} + \frac{c(1-g^*(\lambda_1+\lambda_2+\lambda_3))}{[c+s-g^*(\lambda_1+\lambda_2+\lambda_3)c]}$$
$$- \left[\frac{c(1-g^*(\lambda_3))}{[c+s-g^*(\lambda_3)c]} + \frac{c(1-g^*(\lambda_2-\lambda_3))}{[c+s-g^*(\lambda_2-\lambda_3)c]} + \frac{(1-g^*(\lambda_1-\lambda_3))}{[c+s-g^*(\lambda_1-\lambda_3)c]} \right.$$
$$\left. \left. - \frac{c(1-g^*(\lambda_1+\lambda_2+\lambda_3))}{[c+s-g^*(\lambda_1+\lambda_2+\lambda_3)c]} \right] \lambda_3 \right\}$$

$$E(T) = -\frac{d}{ds} L^*(S) \; given \; s = 0$$

On simplification

$$= -\frac{c(1-g^*(\lambda_1))}{c^2[1-g^*(\lambda_1)]^2} - \frac{c(1-g^*(\lambda_2))}{c^2[1-g^*(\lambda_2)]^2} - \frac{c(1-g^*(\lambda_3))}{c^2[1-g^*(\lambda_3)]^2} + \frac{c(1-g^*(\lambda_1+\lambda_2))}{c^2[1-g^*(\lambda_1+\lambda_2)]^2}$$
$$+ \frac{c(1-g^*(\lambda_2+\lambda_3))}{c^2[1-g^*(\lambda_2+\lambda_3)]^2} + \frac{c(1-g^*(\lambda_1+\lambda_3))}{c^2[1-g^*(\lambda_1+\lambda_3)]^2}$$
$$- \frac{c(1-g^*(\lambda_1+\lambda_2+\lambda_3))}{c^2[1-g^*(\lambda_1+\lambda_2+\lambda_3)]^2}$$
$$- \left[-\frac{c(1-g^*(\lambda_3))}{c^2[1-g^*(\lambda_3)]^2} - \frac{c(1-g^*(\lambda_2-\lambda_3))}{c^2[1-g^*(\lambda_2-\lambda_3)]^2} - \frac{(1-g^*(\lambda_1-\lambda_3))}{c^2[1-g^*(\lambda_1-\lambda_3)]^2} \right.$$
$$\left. + \frac{c(1-g^*(\lambda_1+\lambda_2+\lambda_3))}{c^2[1-g^*(\lambda_1+\lambda_2+\lambda_3)]^2} \right] \lambda_3$$

$$E(T) = -\frac{1}{c(1-g^*(\lambda_1))} - \frac{1}{c(1-g^*(\lambda_2))} - \frac{1}{c(1-g^*(\lambda_3))} + \frac{1}{c(1-g^*(\lambda_1+\lambda_2))}$$
$$+ \frac{1}{c(1-g^*(\lambda_2+\lambda_3))} + \frac{1}{c(1-g^*(\lambda_1+\lambda_3))} - \frac{1}{c(1-g^*(\lambda_1+\lambda_2+\lambda_3))}$$
$$+ \left[-\frac{1}{c(1-g^*(\lambda_3))} - \frac{1}{c(1-g^*(\lambda_2-\lambda_3))} - \frac{1}{c(1-g^*(\lambda_1-\lambda_3))} \right.$$
$$\left. + \frac{1}{c(1-g^*(\lambda_1+\lambda_2-\lambda_3))} \right] \lambda_3$$

$$g^*(.) \sim exp(\mu); \quad g^*(\lambda_1) = \frac{\mu}{\mu+\lambda_1}; \quad g^*(\lambda_2) = \frac{\mu}{\mu+\lambda_2}; \quad g^*(\lambda_3) = \frac{\mu}{\mu+\lambda_3}$$

$$= -\frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_2}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_3}\right]} + \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1+\lambda_2}\right]} + \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_2+\lambda_3}\right]}$$
$$+ \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1+\lambda_3}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1+\lambda_2+\lambda_3}\right]}$$
$$+ \left[-\frac{1}{c\left[1-\frac{\mu}{\mu-\lambda_3}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_2-\lambda_3}\right]} - \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1-\lambda_3}\right]} \right.$$
$$\left. + \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda_1+\lambda_2+\lambda_3}\right]} \right] \lambda_3$$

$$E(T)=\frac{1}{c}\Bigg[\frac{\mu+\lambda_1}{\lambda_1}+\frac{\mu+\lambda_2}{\lambda_2}+\frac{\mu+\lambda_3}{\lambda_3}-\frac{\mu+\lambda_1+\lambda_2}{\lambda_1+\lambda_2}-\frac{\mu+\lambda_2+\lambda_3}{\lambda_2+\lambda_3}-\frac{\mu+\lambda_1+\lambda_3}{\lambda_1+\lambda_3}+\frac{\mu+\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3}-\left[\frac{\mu-\lambda_3}{\lambda_3}+\frac{\mu+\lambda_2-\lambda_3}{\lambda_2-\lambda_3}+\frac{\mu+\lambda_1-\lambda_3}{\lambda_1-\lambda_3}-\frac{\mu+\lambda_1+\lambda_2-\lambda_3}{\lambda_1+\lambda_2-\lambda_3}\right]\lambda_3\Bigg]$$

$$E(T^2)=\frac{d^2}{ds^2}L^*(S)\ /s=0$$

$$=-\frac{2c^2[1-g^*(\lambda_1)]^2}{[(c-g^*(\lambda_1)c)^2]^2}-\frac{2c^2[1-g^*(\lambda_2)]^2}{[(c-g^*(\lambda_2)c)^2]^2}-\frac{2c^2[1-g^*(\lambda_3)]^2}{[(c-g^*(\lambda_3)c)^2]^2}+\frac{2c^2\left(1-g^*(\lambda_1+\lambda_2)\right)}{[(c-g^*(\lambda_1+\lambda_2)c)^2]^2}+\frac{2c^2\left(1-g^*(\lambda_2+\lambda_3)\right)}{[(c-g^*(\lambda_2+\lambda_3)c)^2]^2}+\frac{2c^2\left(1-g^*(\lambda_1+\lambda_3)\right)}{[(c-g^*(\lambda_1+\lambda_3)c)^2]^2}-\frac{2c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)}{[(c-g^*(\lambda_1+\lambda_2+\lambda_3)c)^2]^2}-\left[-\frac{2c^2\left(1-g^*(\lambda_3)\right)}{[(c-g^*(\lambda_3)c)^2]^2}-\frac{2c^2\left(1-g^*(\lambda_2-\lambda_3)\right)}{[(c-g^*(\lambda_2-\lambda_3)c)^2]^2}-\frac{2c^2\left(1-g^*(\lambda_1-\lambda_3)\right)}{[(c-g^*(\lambda_1-\lambda_3)c)^2]^2}+\frac{2c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)}{[(c-g^*(\lambda_1+\lambda_2+\lambda_3)c)^2]^2}\right]\lambda_3$$

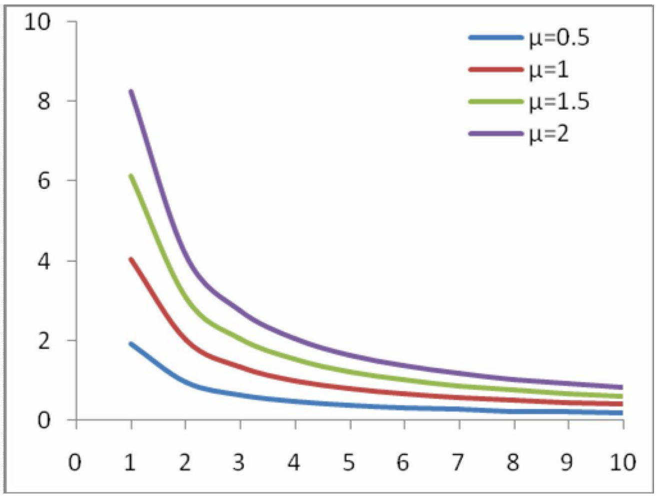
$$E(T^2)=\frac{2}{c^2[1-g^*(\lambda_1)]^2}+\frac{2}{c^2[1-g^*(\lambda_2)]^2}+\frac{2}{c^2[1-g^*(\lambda_3)]^2}-\frac{2}{c^2\left(1-g^*(\lambda_1+\lambda_2)\right)^2}-\frac{2}{c^2\left(1-g^*(\lambda_2+\lambda_3)\right)^2}-\frac{2}{c^2\left(1-g^*(\lambda_1+\lambda_3)\right)^2}+\frac{2}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)^2}-\left[+\frac{2}{c^2\left(1-g^*(\lambda_3)\right)^2}+\frac{2}{c^2\left(1-g^*(\lambda_2-\lambda_3)\right)}+\frac{2}{c^2\left(1-g^*(\lambda_1-\lambda_3)\right)}-\frac{2}{c^2\left(1-g^*(\lambda_1+\lambda_2+\lambda_3)\right)^2}\right]\lambda_3$$

$$=\frac{2}{c^2\left[1-\frac{\mu}{\mu+\lambda_1}\right]^2}+\frac{2}{c^2\left[1-\frac{\mu}{\mu+\lambda_2}\right]^2}+\frac{2}{c^2\left[1-\frac{\mu}{\mu+\lambda_3}\right]^2}-\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_1+\lambda_2}\right)^2}-\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_2+\lambda_3}\right)^2}-\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_1+\lambda_3}\right)^2}+\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_1+\lambda_2+\lambda_3}\right)^2}-\left[+\frac{2}{c^2\left(1-\frac{\mu}{\mu-\lambda_3}\right)^2}+\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_2-\lambda_3}\right)}+\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_1-\lambda_3}\right)}-\frac{2}{c^2\left(1-\frac{\mu}{\mu+\lambda_1+\lambda_2-\lambda_3}\right)^2}\right]\lambda_3$$

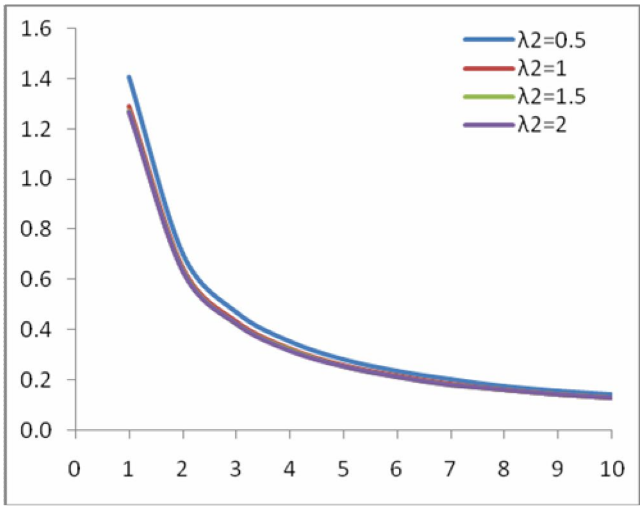
$$V(T) = E(T^2) - [E(T)]^2$$

$$\begin{aligned}
= \frac{2}{c^2}\Bigg[& \left(\frac{\mu+\lambda_1}{\lambda_1}\right)^2 + \left(\frac{\mu+\lambda_2}{\lambda_2}\right)^2 + \left(\frac{\mu+\lambda_3}{\lambda_3}\right)^2 - \left(\frac{\mu+\lambda_1+\lambda_2}{\lambda_1+\lambda_2}\right)^2 - \left(\frac{\mu+\lambda_2+\lambda_3}{\lambda_2+\lambda_3}\right)^2 \\
& - \left(\frac{\mu+\lambda_1+\lambda_3}{\lambda_1+\lambda_3}\right)^2 + \left(\frac{\mu+\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)^2 \\
& - \left\{ -\left(\frac{\mu-\lambda_3}{\lambda_3}\right)^2 + \left(\frac{\mu+\lambda_2-\lambda_3}{\lambda_2-\lambda_3}\right)^2 + \left(\frac{\mu+\lambda_1-\lambda_3}{\lambda_1-\lambda_3}\right)^2 \right. \\
& \left. - \left(\frac{\mu+\lambda_1+\lambda_2-\lambda_3}{\lambda_1+\lambda_2-\lambda_3}\right)^2 \right\} \lambda_3 \Bigg] \\
& - \frac{1}{c}\Bigg[\frac{\mu+\lambda_1}{\lambda_1} + \frac{\mu+\lambda_2}{\lambda_2} + \frac{\mu+\lambda_3}{\lambda_3} - \frac{\mu+\lambda_1+\lambda_2}{\lambda_1+\lambda_2} - \frac{\mu+\lambda_2+\lambda_3}{\lambda_2+\lambda_3} - \frac{\mu+\lambda_1+\lambda_3}{\lambda_1+\lambda_3} \\
& + \frac{\mu+\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+\lambda_3} \\
& - \left\{ + \frac{\mu-\lambda_3}{\lambda_3} + \frac{\mu+\lambda_2-\lambda_3}{\lambda_2-\lambda_3} + \frac{\mu+\lambda_1-\lambda_3}{\lambda_1-\lambda_3} - \frac{\mu+\lambda_1+\lambda_2-\lambda_3}{\lambda_1+\lambda_2-\lambda_3} \right\} \lambda_3 \Bigg]
\end{aligned}$$

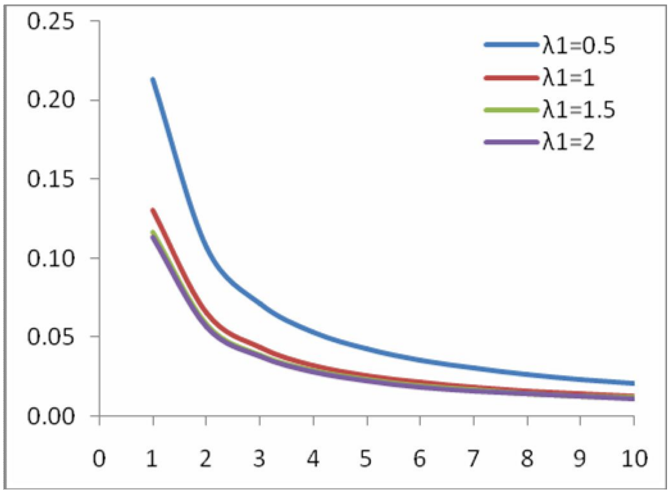
$E(T)$ *Figure* 1.1



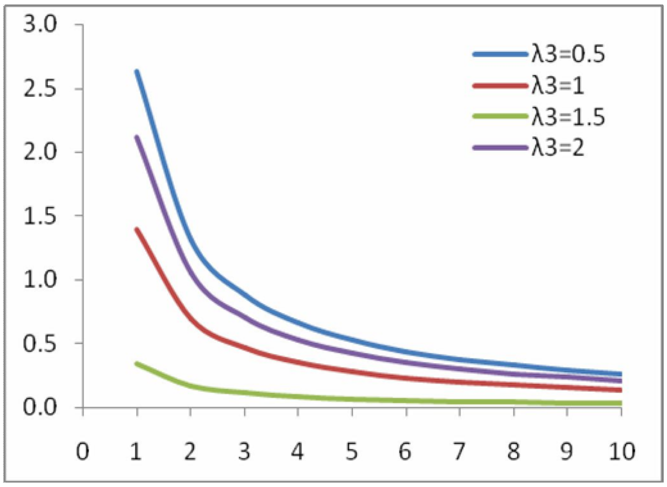
$E(T)$ *Figure* 1.3



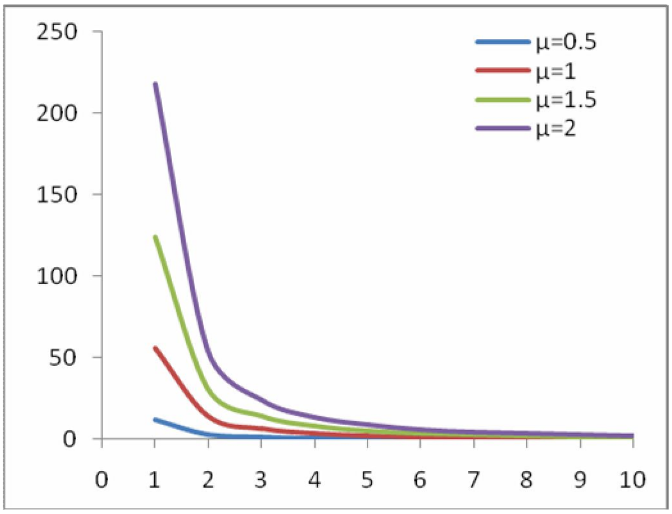
$E(T)$ *Figure* 1.2



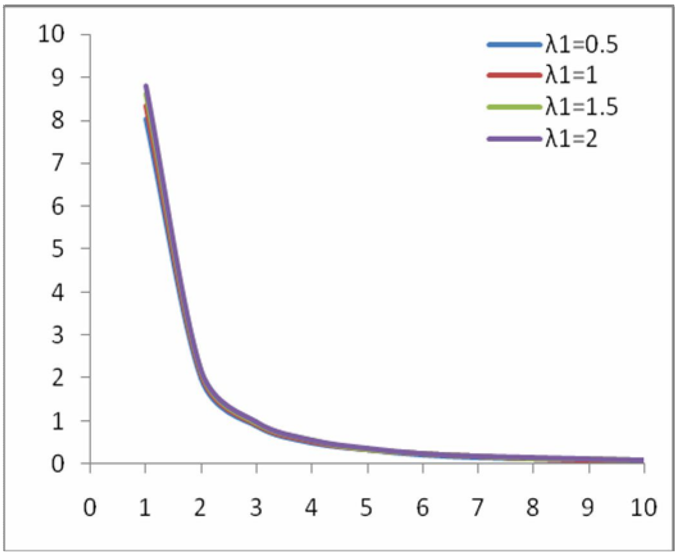
$E(T)$ *Figure* 1.4



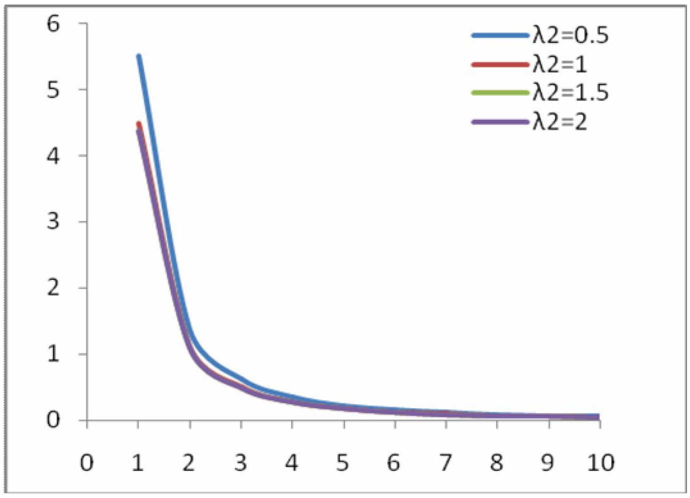
V(T) Figure 2.1



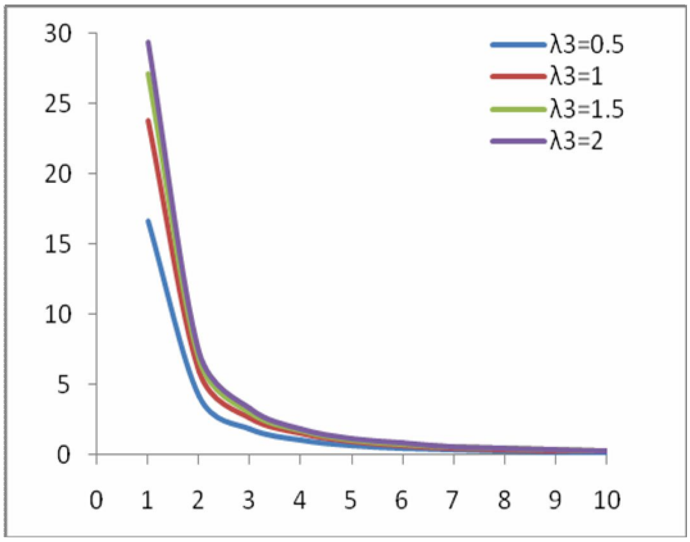
V(T) Figure 2.2



V(T) Figure 2.3



V(T) Figure 2.4



Discussion

To illustrate the method described in this paper, some limited simulation results are discussed. The theory developed was tested using stimulated data in *Mathcad* software. Numerical examples with one parameter varying and keeping the other parameter fixed. Considering four simulated data to the model it is concluded that:

From corresponding Figure 1.1 and 2.1, the Threshold level of the inter-arrival time μ increases the time to cross the threshold level decreases in both Expected time and Variance.

The Sexual contact which follows Exponential distribution with parameter λ_1 is increased for the Mean (Expected) time and Variance. The expected time to cross the threshold level decreases which is observed in Figure 1.2 and 2.2.

Figure 1.3 and 2.3 observes the Needle sharing which follows Exponential distribution with parameter λ_2. The expected time and Variance to cross the threshold level decreases by increasing the parameter of needle Sharing λ_2.

The graphical diagram in Figure 1.4 and 2.4, we observe the infected blood product which follows Erlang Distribution with parameter λ_3. The expected time and Variance to cross the threshold level decreases as the parameter λ_3 increases.

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