

## Secure communication by using Chua's model

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**Abstract :** In this work, the secure communication has been studied by using two Chua's system (transmittance and receiver system), which was programed by using Berkley Madonna software. The parameter value ( $a$ ) in Chua's system has been changed to get exactly chaotic system (typical value) where was 15. Full synchronization between two chaotic systems has been done when the coupling factor ( $k$ ) was 20. Sinusoidal wave has been used to test the secure communication in this system.

**Key words :** Chaotic system, Chua's system.

### Introduction

The irregular oscillation for time evolutions in nonlinear dynamical systems are appeared clearly in their output as deterministic manner and it's different from random processes. These oscillations are called dynamical chaos. Chaos may it indicated to any state of disorder or confusion [1]. Dynamic chaos is considered as a very interesting nonlinear phenomenon which has been intensively studied during the last four decade [2]. Nonlinear system has a rich variety of phenomena such as self-sustained oscillation, pattern formation and chaos [3]. Since 1970's the application research was an active area within the chaotic dynamic [4]. Many applications including secure communities, chaotic ranging and ultra-wide-band (UWB) sensor networks are excited by the chaotic dynamic due to their noise-like broadband intensity waveform [5]. Moreover, full synchronization chaos signal has been done by using 2x2 optocouplers network [6-7].

There are many systems have been used in secure communication like Lorenz and Rossler system [8-10]. Chua system (also called Chua circuit) was a simple oscillator exhibiting a variety of bifurcation and chaotic phenomena [11].

### Chua's chaotic system

Chua's oscillator is a third order autonomous system, which can easily realize in electronic from and exhibits a wide variety of nonlinear and chaotic phenomena. Chua's equations can be described by four differential equations, which in dimensionless form [12]:

$$\dot{x} = \alpha(y - x - g(x))$$

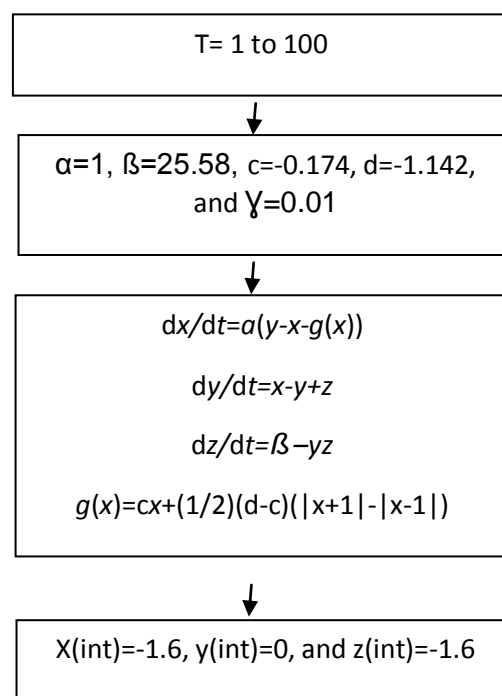
$$\dot{y} = x - y + z$$

$$\dot{z} = -\beta y - \gamma z \quad (1)$$

$$g(x) = cx + \left(\frac{1}{2}\right)(d - c)(|x + 1| - |x - 1|)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are bifurcation parameters, and  $g(x)$  is the nonlinear function. In fact, the chaotic dynamical system described by above equation is dynamics of an electronic circuit invented by Leon Chua in 1983. The main component of Chua's circuit is a nonlinear resistor having electric characteristic described by function  $g(x)$  (Sometimes called Chua's diode). Because the Chua's signal is typically broadband, noise like, and difficult to predict, they can be used in various application one of them can be used in secure communication. Chua's system has important properties, such as: high sensitivity to the initial condition [13], the property of pseudo-randomness, no periodicity, and the dependency of the system parameters. These properties are related to Shannon's requirements for permutation and diffusion in cryptosystem building [14].

The four Chua equations can be solving it numerically by using a fourth order Runge – Kutta integration algorithm of Berkley Madonna, as shown in Figure (1). The first block from this flow chart (Figure 1) represents the time scale of chaotic signal that generated in system and it is changeable in any range of time. Sequentially the second block represents the parameters values of Chua's system that exhibits chaotic dynamic, where it limited values. The Chua's system equations were written in third block. Finally, the initial condition values of the system, these values are very important to change chaotic system behavior.

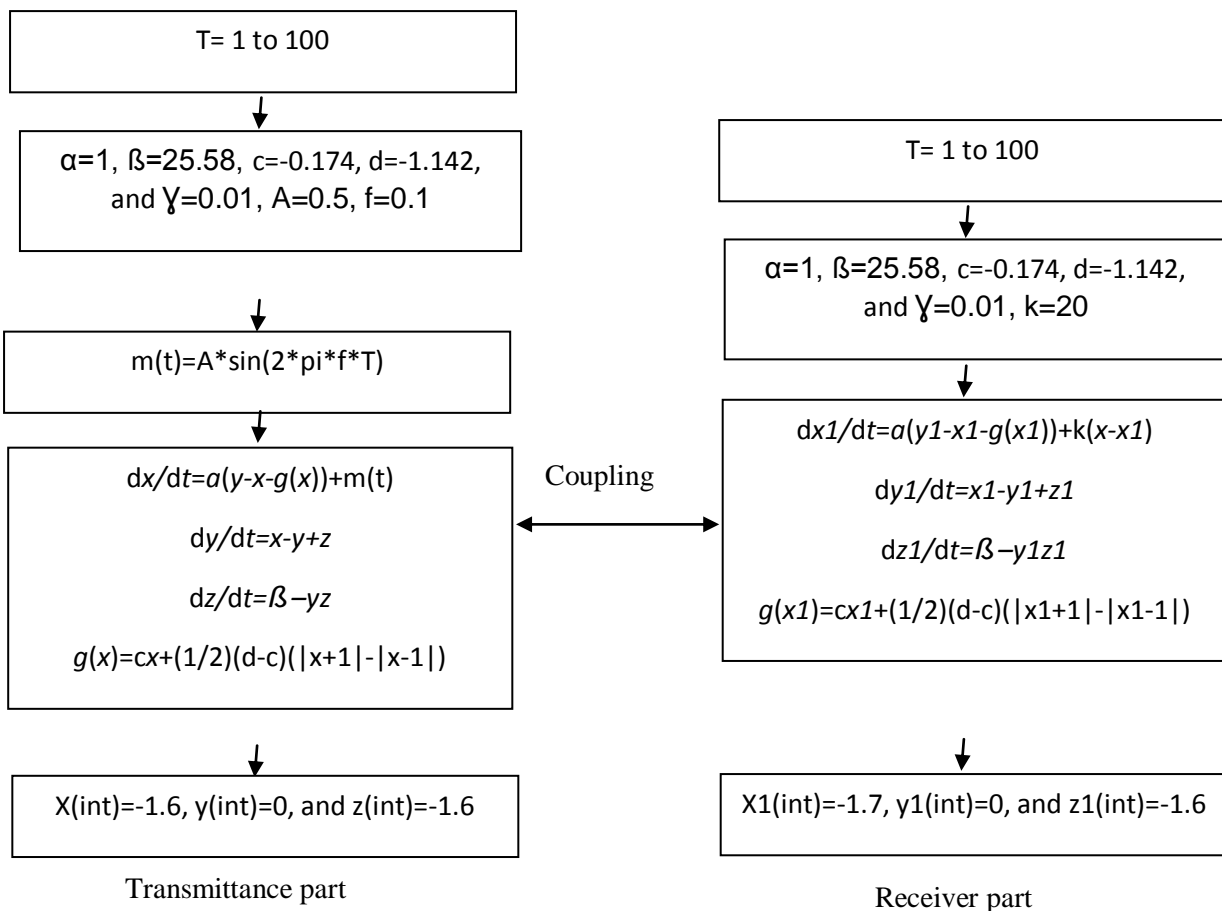


**Fig 1: Flow Chart of Chua's model**

To Study the synchronization in Chua model must take two Chua models, as shown in the flow chart (2) Figure (2) that called (transmitter and receiver part) and the term  $[k(x-x_1)]$  add to first equation in  $x$  scale for receiver part, as shown in Figure (2) where  $k$  called the coupling factor. The receiver part different the transmitter part in initial condition in  $(x_1)$  scale.

To study the secure communications the observably program apply, after adding message on transmitter part in  $y$  scale for Chua equation.

The decoding of the message is achieved by the synchronization between the chaotic oscillator of transmitter and the receiver.



**Fig 2: Flow Chart of synchronization and secure communication by using Chua's model**

### Result of Chua's system

Chua's oscillator is third order autonomous system, which can be easily realized in electronic form and exhibit a wide variety of nonlinear and chaotic phenomena. The Chua's equation were simulated using a flow chart (1), where the total simulation time chosen depends strongly on the magnitude of the temporal scale defined by five parameter  $\alpha$ ,  $\beta$ ,  $c$ ,  $d$  and  $\gamma$  depending on initial conditions  $x_{int}$ ,  $y_{int}$ , and  $z_{int}$ . The system shows a non-chaotic behavior (dc system) for parameters values,  $\alpha=1$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$  and  $\gamma=0.01$ ; initial conditions  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$ . Figure (3) show the time series and Figure (4) show the Fast Fourier Transformation (FFT). By fixed all pervious parameters values and changing the ( $\alpha$ ) value, where at  $\alpha=11$  the system convert to periodic system as shown in Figure (5). The FFT for  $x$  dynamic shows that the system has a singlet frequency as shown in Figure (6). By increasing ( $\alpha$ ) value to 12 the system become period-doubling system as shown in Figures (7) where the FFT shown two frequencies as shown in figure (8). Finally to get chaotic phenomena the ( $\alpha$ ) value was 15, where the Figures (9), (11), and (12) shown time series in ( $x$ ), ( $y$ ), and ( $z$ ) dimensions respectively. The FFT of this system in ( $x$ ) dimension shown in Figures (10), where the system exhibit exponential-decay behavior that indicates of chaotic system, on the contrary of noise system that have Gaussian behavior. The chaotic attractor is presented in Figure (13), where plotted between ( $y$ ) and ( $x$ ) dimensions, and it exhibits a double scroll attractor. The same behavior can see it when plot between ( $z$ ) and ( $x$ ), as shown Figure (14). The double scroll indicating that the system may not be sensitive for some choice of system parametrs, but for some other it exhibits much more sensitbilty. These attractors different of Lorenz (butterfly attractor) and Rossler system so can recognize the system type from shape of attractor [8-10].

The another diagram that shows the scenario of nonlinear dynamic behavior of system is bifurcation diagram, as shown in Figure (15), a system transition from one type of behavior to another depending on the values for important parameter like ( $\alpha$ ) value in Chua equation. If ( $\alpha$ ) value is changing from (0) to maximum value (15) the amplitude of  $x$  scale in time series in four parts. The first one is curve line where ( $\alpha$ ) value is

starting from (0) to (11) that called steady state. The second region is regular oscillation (parodic behavior) where  $(\alpha)$  value starting from (11) to (12). The next region stating from (12) to (14), that called period doubling behavior. Finally, the chaos behavior region is starting after  $(\alpha)$  equal (14), where the optimum value to get chaotic behavior at  $\alpha$  equal (15).

To exhibit the sensitive dependence of system on initial condition, the simulation is performed for two different initial values. Two identical systems (transmitter and receiver) are taken with same parameters but starting from different initial condition (nearly however identical). The different between the two system is in the initial condition  $x_0=-1.6$  and  $x_1=-1.7$ , Figure (16) depicts the time series of variable  $x_0$  and  $x_1$  for two Chua system (transmitter and receiver). After some period, the two variables quickly diverge from each other even they started from identical initial conditions. To check the identical chaotic signal between these two systems we must plot between amplitude of  $x$  scale versus amplitude of  $x_1$  scale, as shown in Figure (17). It shows diffusion distribution in  $(x, x_1)$  plane with great accumulate point in  $(0,0)$  point. Synchronization in chaotic systems occurs by using synchronization term  $k(x - x_1)$ . By using synchronization term, the transmitted and received signals are connected to each other, Figure (18) show time series at coupling factor  $k=20$ , from this figure can conclude at this value from coupling factor we can get full synchronization between the transmitter and receiver system. By drawing  $x$  versus  $x_1$  for two similar Chau's system at full synchronization can find linear relation between them, as shown in Figure (19) where the slope of curve was 1.

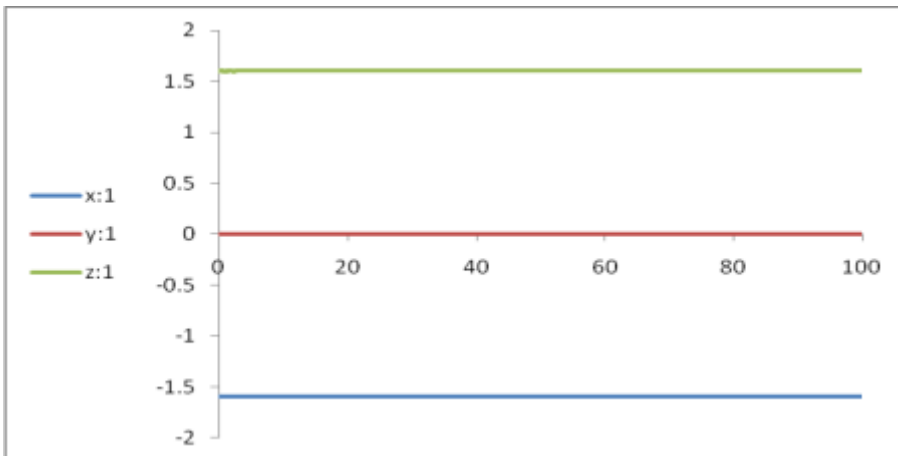


Fig (3): The intensity of x dynamical state vs. time at  $\alpha=1, \beta=25.58, c=-0.714, d=-1.142, \gamma=0.01, x_{int}=-1.6, y_{int}=0, \text{ and } z_{int}=1.6$

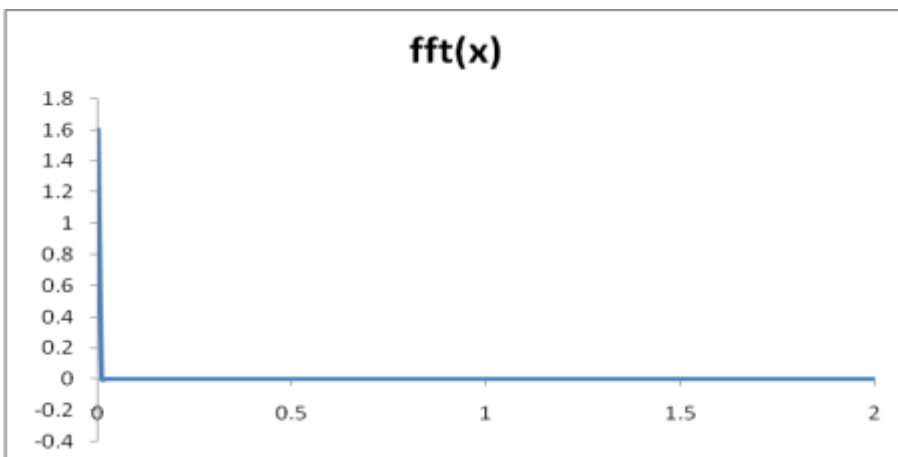


Fig (4): The frequency spectrum of x dynamic.

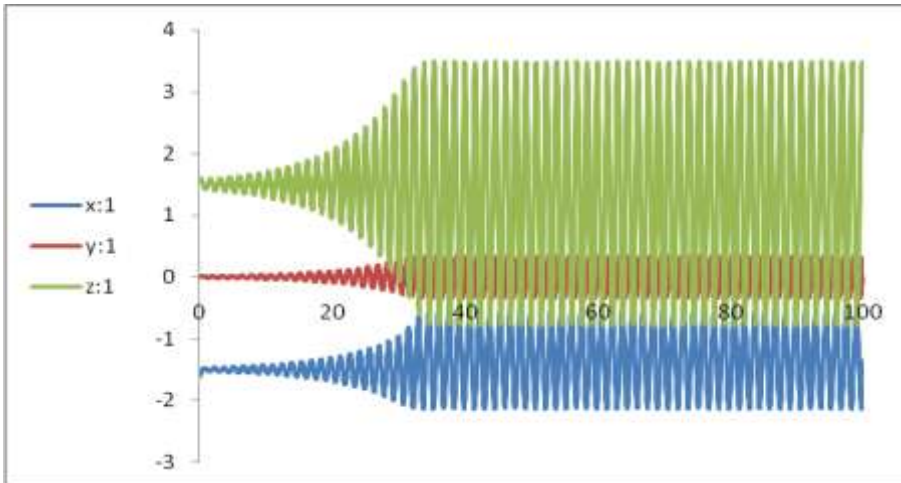


Fig (5): The intensity of x,y, and z dynamical state vs. time at  $\alpha=11$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$ ,  $\gamma=0.01$ ,  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$ .(Periodic system).

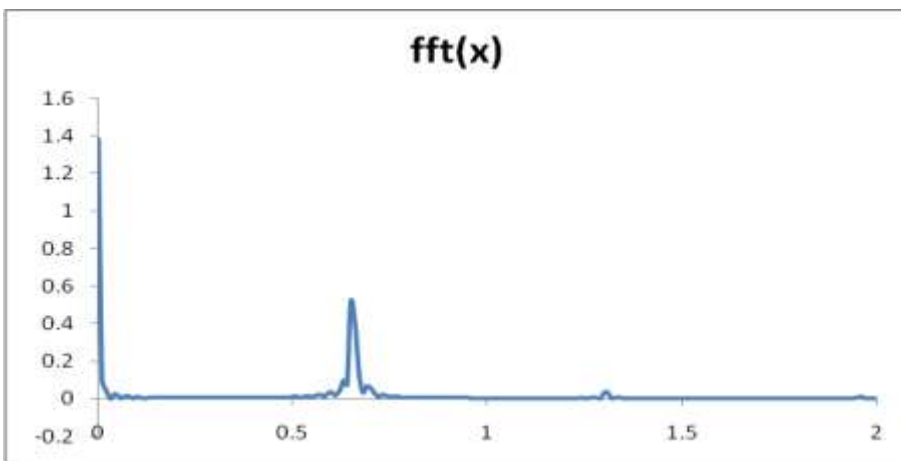


Fig (6): The frequency spectrum of x dynamic (Periodic system).

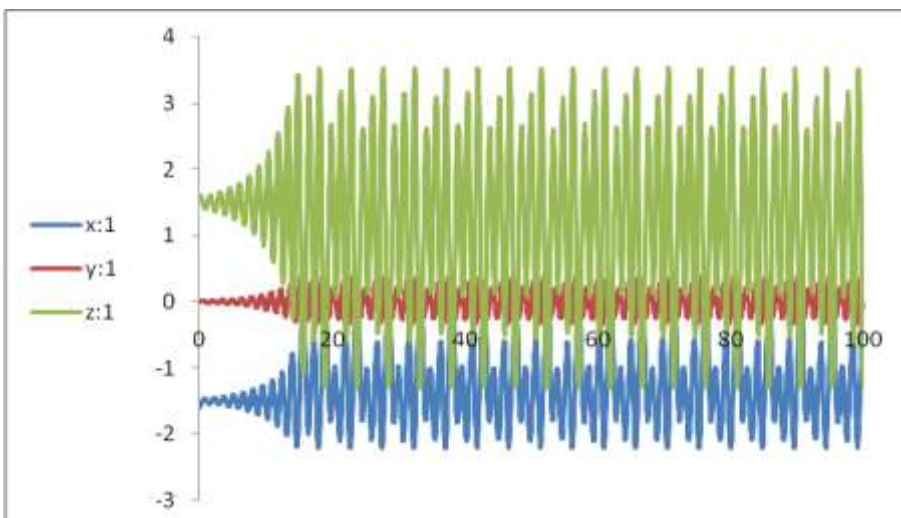


Fig (7): The intensity of x,y, and z dynamical state vs. time at  $\alpha=12$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$ ,  $\gamma=0.01$ ,  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$  (period-doubling system).

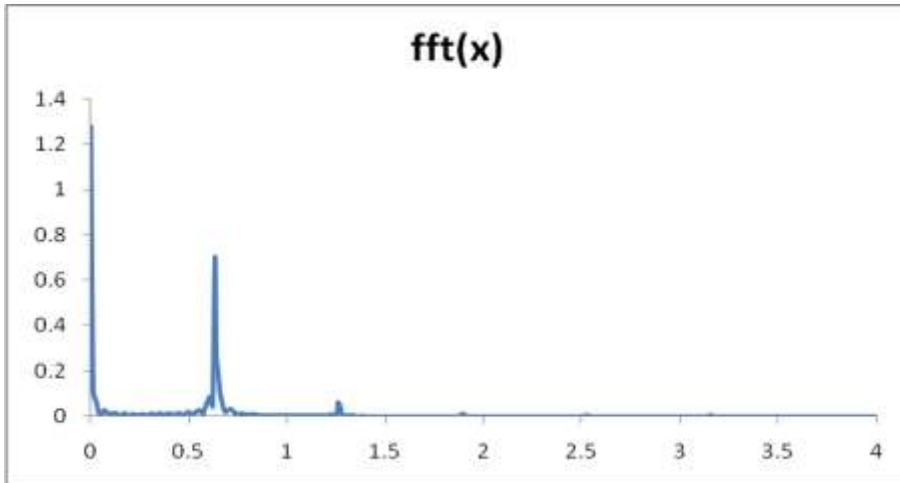


Fig (8): The frequency spectrum of x dynamic (period-doubling system).

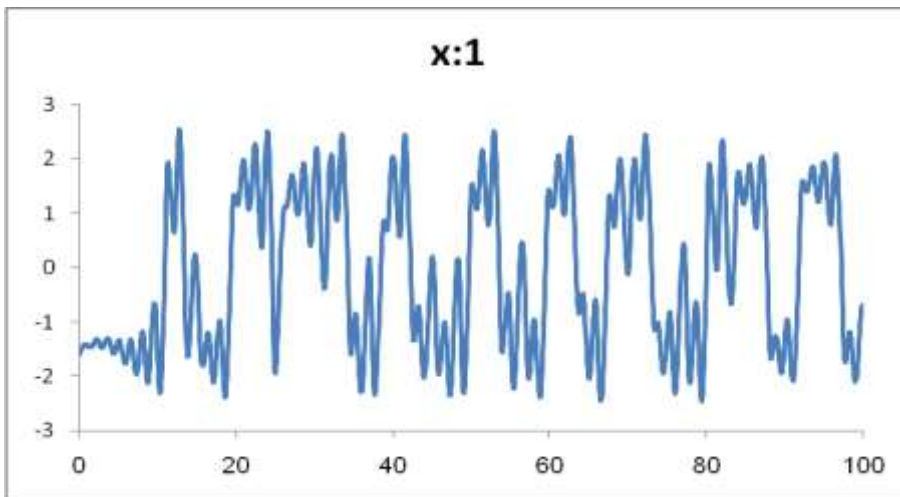


Fig (9): The intensity of x dynamical state vs. time at  $\alpha=15$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$ ,  $\gamma=0.01$ ,  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$ (chaotic system).

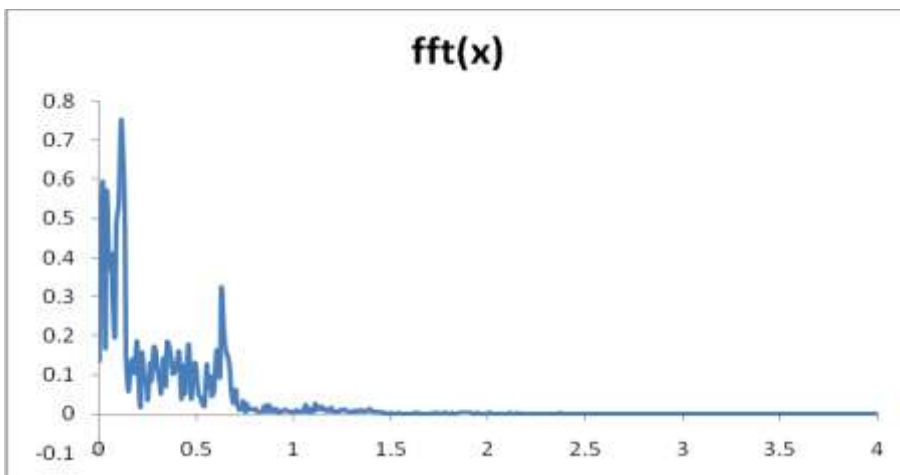


Fig (10): The frequency spectrum of x dynamic (chaotic system).

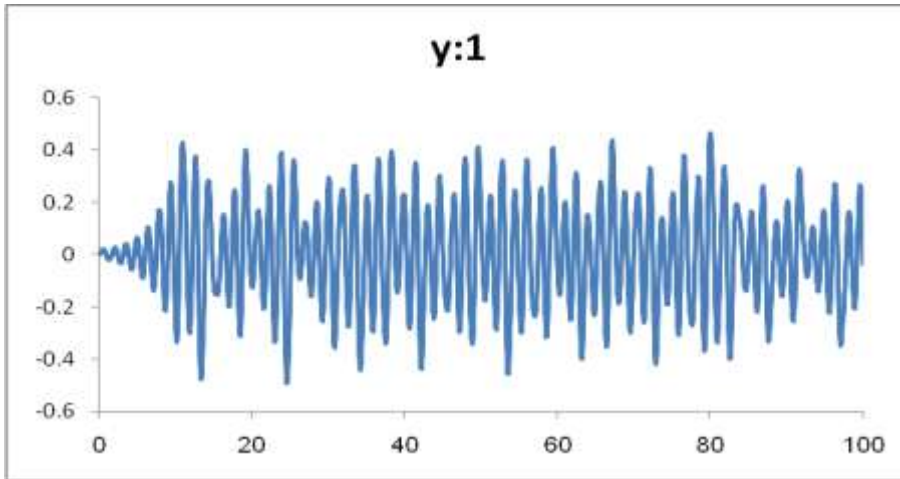


Fig (11): The intensity of y dynamical state vs. time at  $\alpha=15$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$ ,  $\gamma=0.01$ ,  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$ (chaotic system).

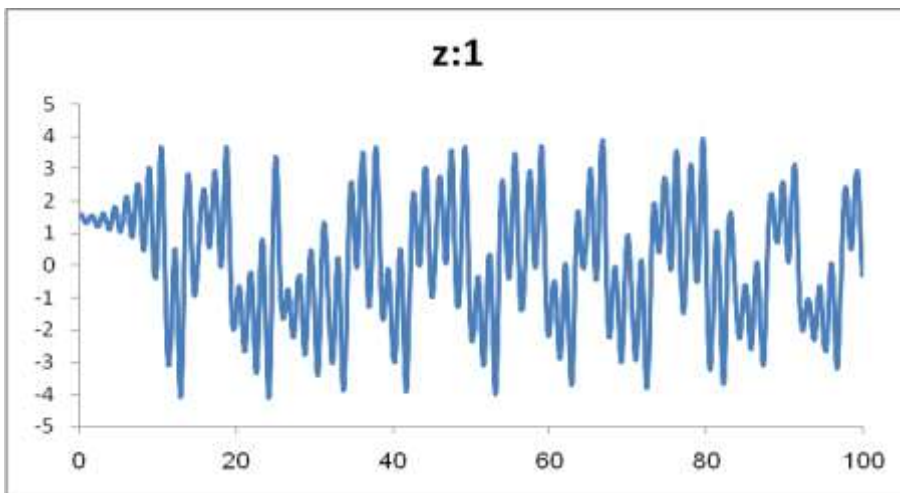


Fig (12): The intensity of z dynamical state vs. time at  $\alpha=15$ ,  $\beta=25.58$ ,  $c=-0.714$ ,  $d=-1.142$ ,  $\gamma=0.01$ ,  $x_{int}=-1.6$ ,  $y_{int}=0$ , and  $z_{int}=1.6$ (chaotic system).

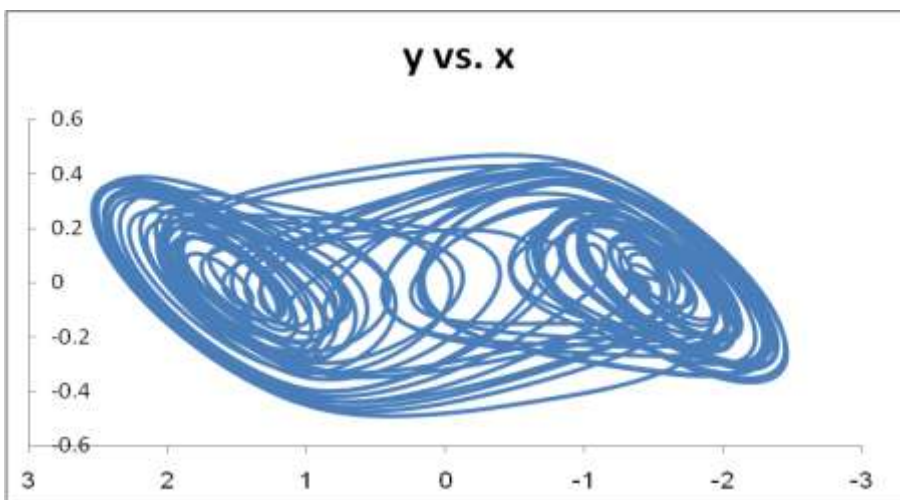


Fig (13): Strange attractor of the Chua system plotting y vs. x dynamics.



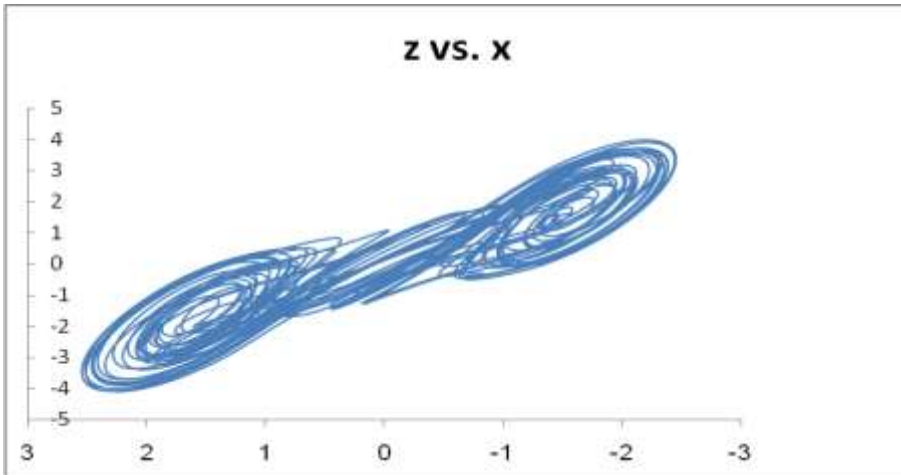


Fig (14): Strange attractor of the Chua system plotting z vs. x dynamics.

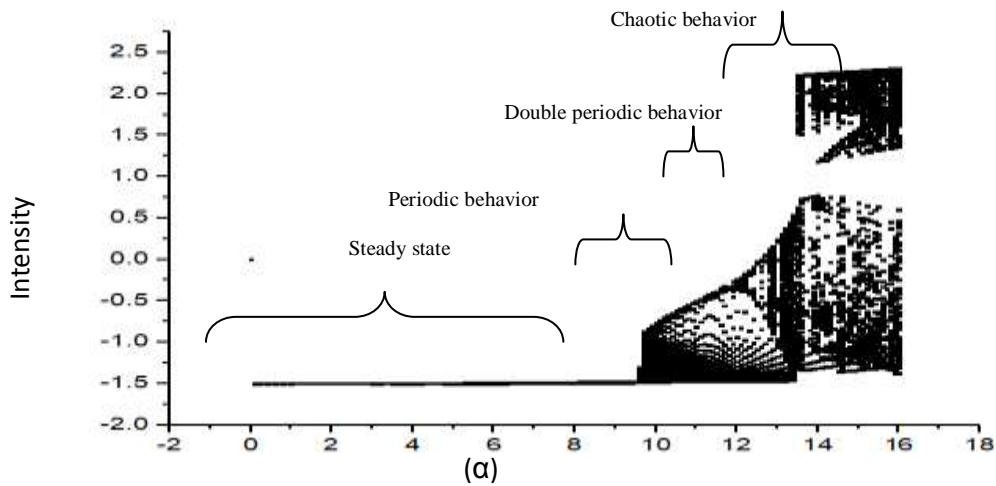


Fig (15): Bifurcation diagram, which show the scenario of nonlinear dynamic behavior of system.

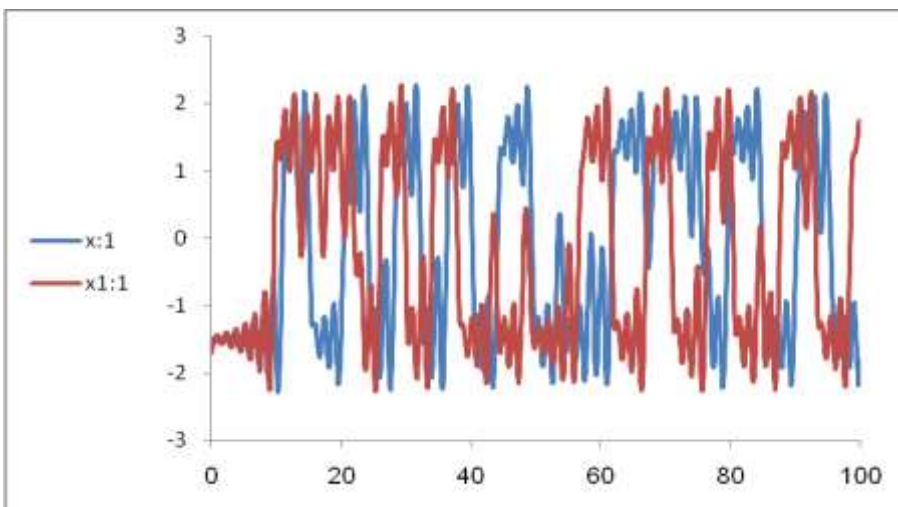


Fig (16): Time series plot of variable x and x1 of two similar Chua system starting from a nearly identical initial condition,  $x_0 = -1.6$  and  $x_1 = -1.7$ , and without synchronization.



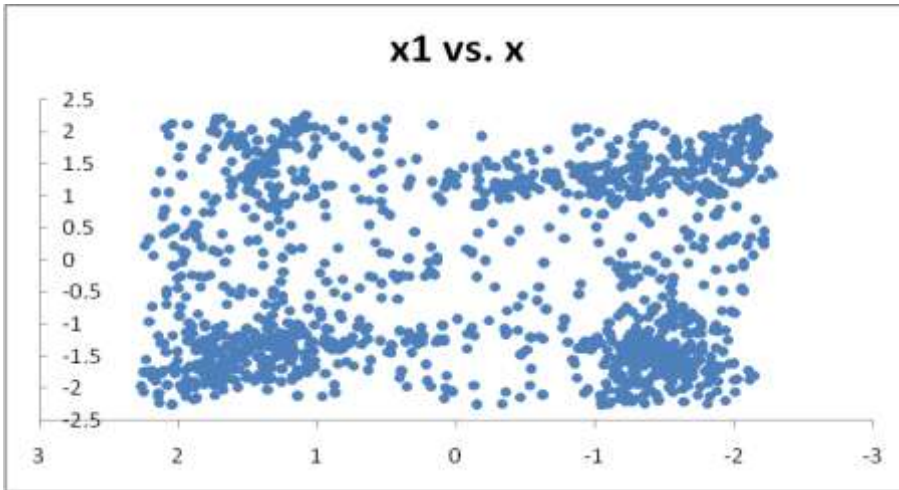


Fig (17):  $x_1$  vs.  $x$  of two similar Chua system at  $x_0 = -1.6$  and  $x_1 = -1.7$  and without synchronization.

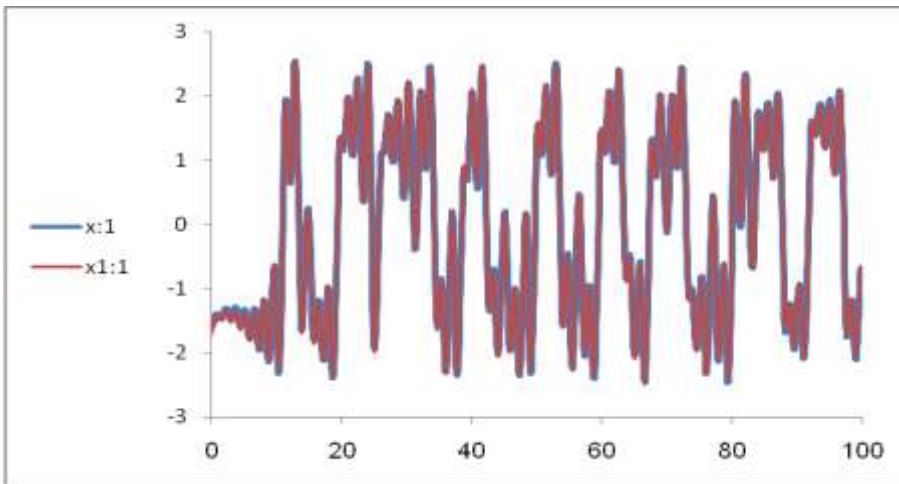


Fig (18): Time series plot of variable  $x$  and  $x_1$  of two similar Chua system starting from a nearly identical initial condition,  $x_0 = -1.6$  and  $x_1 = -1.7$ , and  $k=20$  (full synchronization).

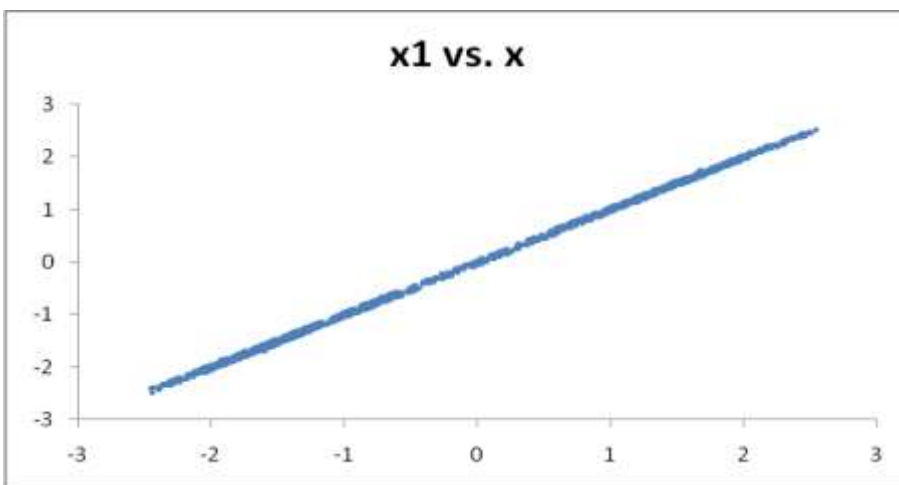
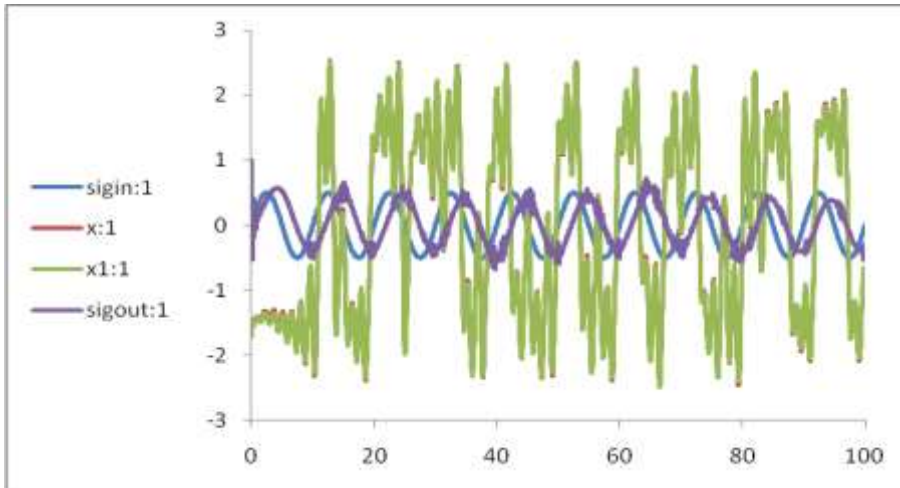


Fig (19):  $x_1$  vs.  $x$  of two similar Chua system at,  $x_0 = -1.6$  and  $x_1 = -1.7$ , and  $k=20$  (Full synchronization).



**Fig (20): Transmitter, receiver, signal in, and signal out in secure communication system by using Chua's model**

### Secure communication in Chua's system

One might feel that since the signals generated from chaotic systems are irregular in nature, therefore, these types of signals cannot have any particle application and should be avoided. However, the properties of the chaotic signal can in fact be used in different field of engineering communications particularly spread spectrum applications to achieve chaotic synchronization, there had to be some sort of coupling present between the two chaotic systems. If we consider unidirectional coupling, then a signal from one chaotic (transmitter system) is being transmitted to another chaotic (receiver system), see Figure (20). This is analogous to communication systems where a carrier signal is modulated by a message prior signal transmission, Therefore the chaotic signals can be used as the carrier signal. By using the chaotic masking method that is one of the realer methods to use chaotic signal for transmitting a message signal. In this scheme, a message signal is added i.e., masked to the output of chaotic oscillator at the transmitter side prior to transmission, as shown in Figure (18). By operating the above system (secure communication) with information message is  $[m(t)=A \sin (2 \pi f)]$ , where  $A$  and  $f$  is a amplitude and frequency respectively of a signal that chosen to be 0.5 and 0.1 Hz. Few thing are worth to be mentioned regarding to the implementation of the chaotic communication system which will be also helpful in our result. One point worth pointing out is, the chaotic signals really broadband, since, the basic idea was to hide the narrow band message spectrum within the wide band of chaotic signals, therefore the chaotic signals being used as the carrier should have a wide spectrum.

### Conclusion

In the work, we present result of chaotic generated by using Chua system. This is programed by Berkley Madonna soft were. This work demonstrates the feasibility of a simple method to synchronize two Chua system with different initial condition and the extend it to the recovery of the information signal, which is originally inserted into a chaotic signal, with satisfactory precision. To keep the communication more secure, the peak of power spectrum of the information signal had better be as undistinguished from these of neighboring frequencies of the masking variable as possible that is almost hidden in the transmitted signal.

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