



A Novel Coupled Van der Pol Conservative Chaotic System and its Adaptive Control

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech University, Avadi, Chennai, Tamil Nadu, India

Abstract: Chaos theory has a lot of applications in science and engineering. This paper first details the qualitative properties of the forced Van der Pol chaotic oscillator, which has important applications. Since its introduction in the 1920's, the Van der Pol equation has been a prototype model for systems with self-excited limit cycle oscillations. The Van der Pol equation has been studied over wide parameter regimes, from perturbations of harmonic motion to relaxation oscillations. It has been used by scientists to model a variety of physical and biological phenomena. In this paper, we announce a novel 4-D coupled Van der Pol conservative chaotic system and discuss its qualitative properties. We show that the Lyapunov exponents of the novel 4-D Van der Pol conservative chaotic system are $L_1 = 14.6$, $L_2 = 0$, $L_3 = -0.46$ and $L_4 = -14.14$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel conservative chaotic system is obtained as $L_1 = 14.6$, which is very large. This shows that the novel Van der Pol conservative chaotic system is highly chaotic. The Kaplan-Yorke dimension of the novel 4-D Van der Pol conservative chaotic system is determined as $D_{KY} = 4$. This shows the high level complexity of the novel 4-D Van der Pol conservative chaotic system. We also derive new results for the adaptive control of the coupled Van der Pol highly chaotic system. The main results are proved using Lyapunov stability theory. MATLAB plots are shown to illustrate the phase portraits of the novel 4-D conservative chaotic system and all the main results.

Keywords: Chaos, chaotic systems, Van der Pol oscillator, coupled oscillator, highly chaotic system, adaptive control, stability.

1. Introduction

Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

In [143], Van der Pol reported that at certain drive frequencies an irregular noise was heard. This irregular noise was always heard near the natural entrainment frequencies. This was one of the first discovered instances of deterministic chaos.

The Van der Pol oscillator has a long history of being used in both the physical and biological sciences. For instance, in biology, Fitzhugh [144] and Nagumo [145] extended the Van der Pol equation in a planar field as a model for action potentials of neurons. A detailed study on forced Van der Pol equation is found in [146].

In this paper, we announce a novel 4-D coupled Van der Pol highly chaotic system and discuss its qualitative properties. We also derive new results for the adaptive control of the coupled Van der Pol highly chaotic system. The main results are established using Lyapunov stability theory. MATLAB plots are shown to illustrate all the main results.

2. A Novel 4-D Coupled Van der Pol Conservative Chaotic System

The forced Van der Pol chaotic oscillator [147] is described by the second order differential equation

$$\ddot{x} = -x + a(1 - x^2)\dot{x} + b \cos(\omega t) \quad (1)$$

The Van der Pol equation (1) is one of the most intensely studied systems in non-linear dynamics. Many efforts have been made to approximate the solutions of the Van der Pol equation or to construct simple maps that qualitatively describe important features of the Van der Pol equation.

In this work, we express the forced Van der Pol equation (1) in system form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + a(1 - x_1^2)x_2 + b \cos(\omega t) \end{cases} \quad (2)$$

In Eq. (2), x_1, x_2 are the states and a, b are constant, positive, parameters.

We also consider a second forced Van der Pol equation written in system form as follows:

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + c(1 - x_3^2)x_4 + d \cos(\omega t) \end{cases} \quad (3)$$

In Eq. (3), x_3, x_4 are the states and c, d are constant, positive, parameters.

In this work, we couple the forced Van der Pol systems (2) and (3) and express it as a 4-D autonomous system as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + a(1 - x_1^2)x_4 + bx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + c(1 - x_3^2)x_2 + dx_1 \end{cases} \quad (4)$$

In the coupled nonlinear system (4), x_1, x_2, x_3, x_4 are the states and a, b, c, d are constant, positive, parameters.

In this work, we show that the novel coupled Van der Pol autonomous system (4) is *highly chaotic* and *conservative* when we take the parameter values as

$$a = 8.5, \quad b = 0.5, \quad c = 8.5, \quad d = 0.5 \tag{5}$$

For numerical simulations, we take the initial conditions of the novel 4-D chaotic system (4) as

$$x_1(0) = 0.4, \quad x_2(0) = 0.4, \quad x_3(0) = 0.4, \quad x_4(0) = 0.4 \tag{6}$$

The Lyapunov exponents of the novel coupled Van der Pol autonomous system (4) for the parameter values (5) and the initial conditions (6) are numerically obtained using MATLAB as

$$L_1 = 14.6, \quad L_2 = 0, \quad L_3 = -0.46, \quad L_4 = -14.14 \tag{7}$$

From (7), it is clear that the Maximal Lyapunov Exponent of the system (4) is $L_1 = 14.6$, which is very large. This shows that the novel coupled Van der Pol autonomous system (4) is highly chaotic.

Since the sum of the Lyapunov exponents of the system (4) is zero, the system (4) is conservative.

The Kaplan-Yorke dimension of the novel coupled Van der Pol autonomous system (4) is obtained as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3 + \frac{14.14}{14.14} = 3 + 1 = 4 \tag{8}$$

Figures 1-4 show the 3-D projections of the novel 4-D coupled Van der Pol conservative chaotic system (4) on the (x_1, x_2, x_3) , (x_1, x_2, x_4) , (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

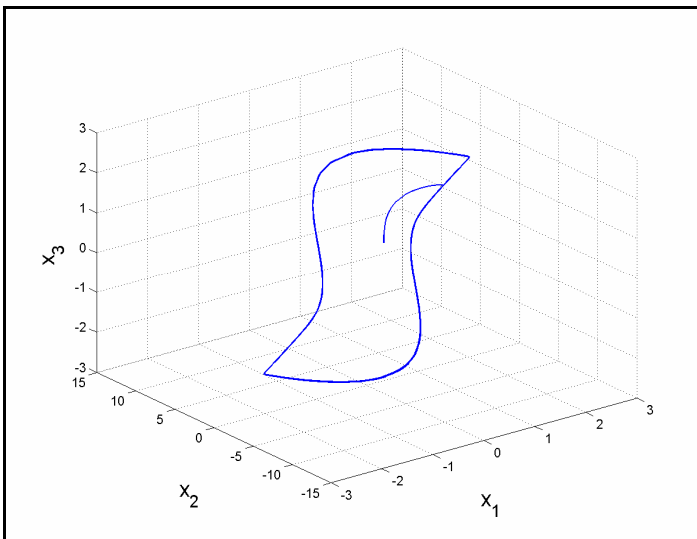


Figure 1. 3-D Projection of the Conservative Chaotic System on the (x_1, x_2, x_3) space

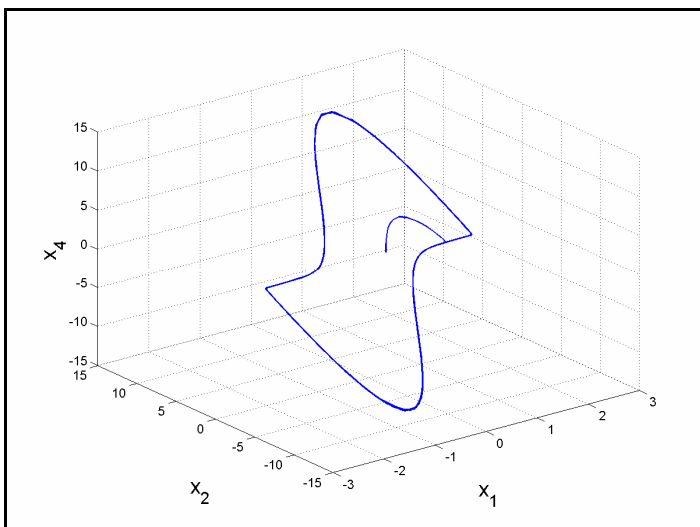


Figure 2. 3-D Projection of the Conservative Chaotic System on the (x_1, x_2, x_4) space

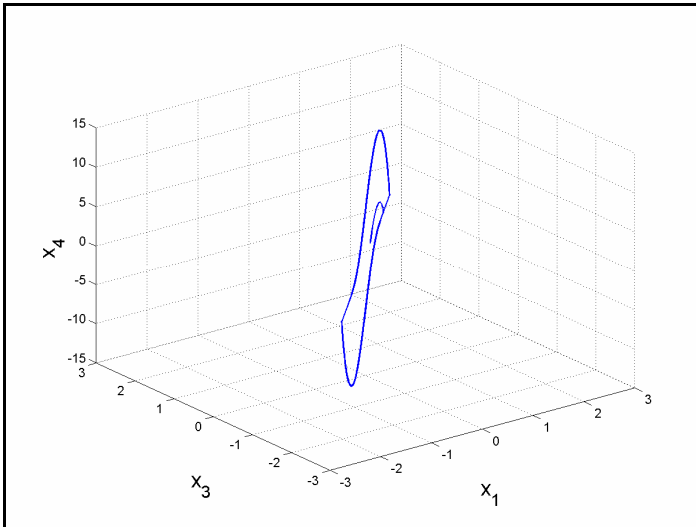


Figure 3. 3-D Projection of the Conservative Chaotic System on the (x_1, x_3, x_4) space

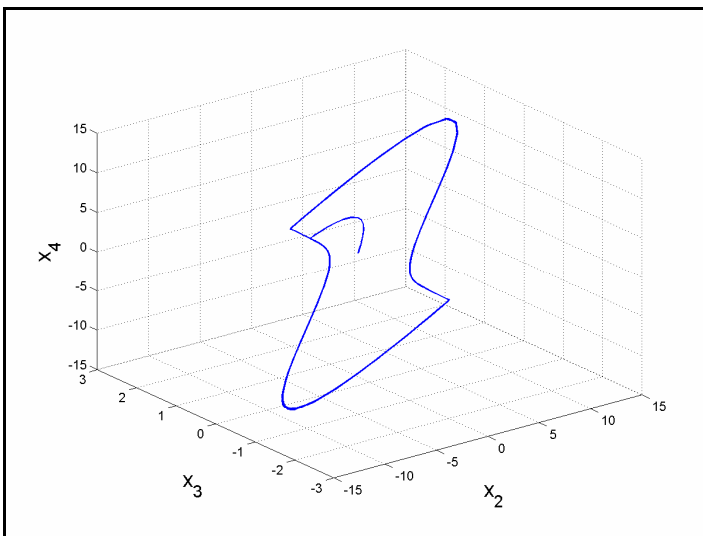


Figure 4. 3-D Projection of the Conservative Chaotic System on the (x_2, x_3, x_4) space

3. Analysis of the Novel 4-D Coupled Van der Pol Conservative Chaotic System

In this section, we discuss the qualitative properties of the novel 4-D coupled Van der Pol conservative chaotic system (4). We take the parameter values as in the chaotic case (5), i.e. $a = 8.5$, $b = 0.5$, $c = 8.5$ and $d = 0.5$.

3.1 Volume conservation of the flow

In vector notation, we can express the 4-D system (4) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4) \\ f_2(x_1, x_2, x_3, x_4) \\ f_3(x_1, x_2, x_3, x_4) \\ f_4(x_1, x_2, x_3, x_4) \end{bmatrix} \tag{9}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3, x_4) = x_2 \\ f_2(x_1, x_2, x_3, x_4) = -x_1 + a(1 - x_1^2)x_4 + bx_3 \\ f_3(x_1, x_2, x_3, x_4) = x_4 \\ f_4(x_1, x_2, x_3, x_4) = -x_3 + c(1 - x_3^2)x_2 + dx_1 \end{cases} \quad (10)$$

Let Ω be any region in R^4 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f . Furthermore, let $V(t)$ denote the hypervolume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int_{\Omega(t)} (\nabla \cdot f) dx_1 dx_2 dx_3 \quad (11)$$

The divergence of the novel 4-D coupled Van der Pol system (4) is easily calculated as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = 0 + 0 + 0 + 0 = 0 \quad (12)$$

Substituting the value of $\nabla \cdot f$ from (12) into (11), we obtain

$$\dot{V}(t) \equiv 0 \quad (13)$$

Integrating (13), we obtain the unique solution as

$$V(t) = V(0) \quad \text{for all } t \geq 0 \quad (14)$$

This shows that the novel 4-D coupled Van der Pol chaotic system (4) is volume-conserving. Hence, the novel 4-D coupled Van der Pol system (4) is a conservative chaotic system.

3.2 Symmetry

We see that the novel 4-D coupled Van der Pol chaotic system (4) is invariant under the coordinates transformation

$$(x_1, x_2, x_3, x_4) \mapsto (-x_1, -x_2, -x_3, -x_4) \quad (15)$$

This shows that the novel 4-D coupled Van der Pol chaotic system (4) has point-reflection symmetry about the origin. Hence, any non-trivial trajectory of the novel 4-D coupled Van der Pol chaotic system (4) must have a twin trajectory.

3.3 Equilibrium Points

The equilibrium points of the novel 4-D coupled Van der Pol chaotic system (4) are obtained by solving the system of equations

$$\begin{cases} x_2 = 0 \\ -x_1 + a(1 - x_1^2)x_4 + bx_3 = 0 \\ x_4 = 0 \\ -x_3 + c(1 - x_3^2)x_2 + dx_1 = 0 \end{cases} \quad (16)$$

Solving the system (16), we get the unique solution $(x_1, x_2, x_3, x_4) = (0, 0, 0, 0)$.

This shows that the novel 4-D coupled Van der Pol chaotic system (4) has a unique equilibrium at the origin,

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

The Jacobian of the novel 4-D coupled Van der Pol chaotic system (4) at E_0 is calculated as

$$J_0 = J(E_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0.5 & 8.5 \\ 0 & 0 & 0 & 1 \\ 0.5 & 8.5 & 0 & 0 \end{bmatrix} \quad (18)$$

which has the eigenvalues

$$\lambda_1 = -0.0667, \lambda_2 = -0.0526, \lambda_3 = -8.3809, \lambda_4 = 8.5002 \quad (19)$$

This shows that the equilibrium point E_0 is a saddle-point, which is unstable.

3.4 Lyapunov Exponents and Kaplan-Yorke Dimension

The parameter values of the novel 4-D coupled Van der Pol chaotic system (4) are taken as in the chaotic case (5), *i.e.*

$$a = 8.5, b = 0.5, c = 8.5, d = 0.5 \quad (20)$$

The initial conditions of the novel 4-D coupled Van der Pol chaotic system (4) are taken as

$$x_1(0) = 0.4, x_2(0) = 0.4, x_3(0) = 0.4, x_4(0) = 0.4 \quad (21)$$

Then the Lyapunov exponents of the novel 4-D coupled Van der Pol chaotic system (4) are numerically obtained as

$$L_1 = 14.6, L_2 = 0, L_3 = -0.46, L_4 = -14.14 \quad (22)$$

Since the sum of the Lyapunov exponents in (22) is zero, the novel 4-D coupled Van der Pol chaotic system (4) is conservative.

Also, the Kaplan-Yorke dimension of the novel 4-D coupled Van der Pol chaotic system (4) is calculated as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3 + \frac{14.14}{14.14} = 3 + 1 = 4 \quad (23)$$

The large value of D_{KY} shows the high complexity of the novel 4-D coupled Van der Pol chaotic system (4).

Figure 5 shows the Lyapunov exponents of the novel 4-D coupled Van der Pol chaotic system (4). From this figure, we note that the Maximum Lyapunov Exponent (MLE) of the novel 4-D coupled Van der Pol chaotic system (4) is $L_1 = 14.6$, which is very large. This shows the high complexity of the novel 4-D coupled Van der Pol conservative chaotic system (4).

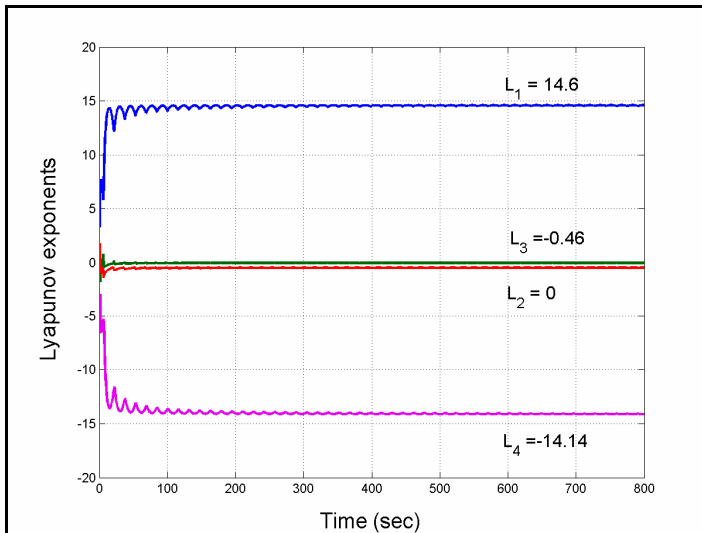


Figure 5. Lyapunov exponents of the Novel 4-D Coupled Van der Pol Conservative Chaotic System

4. Adaptive Control Design for the Novel 4-D Van der Pol Conservative Chaotic System

In this section, we derive new results for the global chaos control of the novel 4-D Van der Pol conservative chaotic system with unknown parameters via adaptive control method. The main result is established using Lyapunov stability theory [148].

We consider the novel 4-D Van der Pol conservative chaotic system with controls given by

$$\begin{cases} \dot{x}_1 = x_2 + u_1 \\ \dot{x}_2 = -x_1 + a(1 - x_1^2)x_4 + bx_3 + u_2 \\ \dot{x}_3 = x_4 + u_3 \\ \dot{x}_4 = -x_3 + c(1 - x_3^2)x_2 + dx_1 + u_4 \end{cases} \quad (24)$$

where x_1, x_2, x_3, x_4 are the states, a, b, c, d are constant, unknown, parameters and u_1, u_2, u_3, u_4 are the feedback controls to be determined using estimates of the unknown system parameters.

Next, we consider the adaptive controller defined by

$$\begin{cases} u_1 = -x_2 - k_1x_1 \\ u_2 = x_1 - \hat{a}(t)(1 - x_1^2)x_4 - \hat{b}(t)x_3 - k_2x_2 \\ u_3 = -x_4 - k_3x_3 \\ u_4 = x_3 - \hat{c}(t)(1 - x_3^2)x_2 - \hat{d}(t)x_1 - k_4x_4 \end{cases} \quad (25)$$

where k_1, k_2, k_3, k_4 are positive gain constants and $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{d}(t)$ are estimates of the unknown parameters a, b, c, d , respectively.

Substituting the control law (25) into (24), we obtain the following closed-loop system dynamics.

$$\begin{cases} \dot{x}_1 = -k_1x_1 \\ \dot{x}_2 = [a - \hat{a}(t)](1 - x_1^2)x_4 + [b - \hat{b}(t)]x_3 - k_2x_2 \\ \dot{x}_3 = -k_3x_3 \\ \dot{x}_4 = [c - \hat{c}(t)](1 - x_3^2)x_2 + [d - \hat{d}(t)]x_1 - k_4x_4 \end{cases} \quad (26)$$

Next, we define the parameter estimation errors as follows.

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \\ e_c = c - \hat{c}(t) \\ e_d = d - \hat{d}(t) \end{cases} \quad (27)$$

Using (27), the closed-loop dynamics (26) can be simplified as follows.

$$\begin{cases} \dot{x}_1 = -k_1 x_1 \\ \dot{x}_2 = e_a(1-x_1^2)x_4 + e_b x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \\ \dot{x}_4 = e_c(1-x_3^2)x_2 + e_d x_1 - k_4 x_4 \end{cases} \quad (28)$$

Differentiating the dynamics (27), we obtain the following system.

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \\ \dot{e}_c = -\dot{\hat{c}}(t) \\ \dot{e}_d = -\dot{\hat{d}}(t) \end{cases} \quad (29)$$

Now, we consider the Lyapunov function defined by

$$V(x_1, x_2, x_3, x_4, e_a, e_b, e_c, e_d) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2) + \frac{1}{2}(e_a^2 + e_b^2 + e_c^2 + e_d^2) \quad (30)$$

Differentiating V along the trajectories of (28) and (29), we obtain the following.

$$\begin{aligned} \dot{V} = & -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a \left[(1-x_1^2)x_2 x_4 - \dot{\hat{a}} \right] + e_b \left[x_2 x_3 - \dot{\hat{b}} \right] \\ & + e_c \left[(1-x_3^2)x_2 x_4 - \dot{\hat{c}} \right] + e_d \left[x_1 x_4 - \dot{\hat{d}} \right] \end{aligned} \quad (31)$$

In view of (31), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = (1-x_1^2)x_2 x_4 \\ \dot{\hat{b}} = x_2 x_3 \\ \dot{\hat{c}} = (1-x_3^2)x_2 x_4 \\ \dot{\hat{d}} = x_1 x_4 \end{cases} \quad (32)$$

Next, we state and prove the main result of this section.

Theorem 1. The adaptive control law (25) and the parameter update law (32) achieve global and exponential stabilization of the novel 4-D coupled Van der Pol conservative chaotic system (24), where k_1, k_2, k_3, k_4 are positive gain constants.

Proof. This result is a consequence of Lyapunov stability theory [135].

The quadratic Lyapunov function V defined by (30) is positive definite on R^8 . Substituting (32) into (31), we obtain the time-derivative of V as

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 - k_4x_4^2 \quad (33)$$

which is negative semi-definite on R^8 .

Thus, using Barbalat's lemma [148], we conclude that the closed loop system dynamics (28) is globally exponentially stable.

This completes the proof. ■

5. Numerical Simulations

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with step-size $h = 10^{-8}$ to solve the novel 4-D coupled Van der Pol conservative chaotic system (24), when the adaptive control law (25) and the parameter update law (32) are implemented.

We take the parameter values as in the chaotic case, i.e. $a = 8.5$, $b = 0.5$, $c = 8.5$ and $d = 0.5$.

We take the positive gain constants as $k_i = 6$ for $i = 1, 2, 3, 4$.

We take the initial conditions of the novel 4-D coupled Van der Pol conservative chaotic system (24) as $x_1(0) = 12.3$, $x_2(0) = 7.5$, $x_3(0) = 3.8$, $x_4(0) = 16.9$ (34)

We take the initial conditions of the parameter estimates as

$$\hat{a}(0) = 5.4, \hat{b}(0) = 14.2, \hat{c}(0) = 0.5, \hat{d}(0) = 2.1 \quad (35)$$

Figure 6 shows the time-history of the controlled states x_1, x_2, x_3, x_4 .

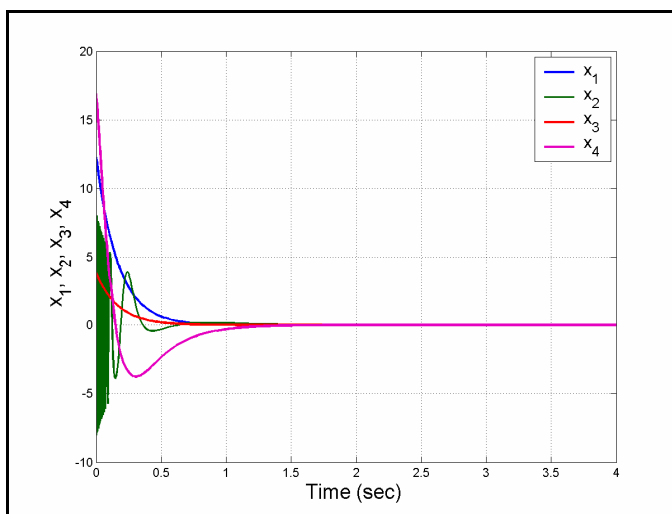


Figure 6. Time-history of the controlled states x_1, x_2, x_3, x_4

6. Conclusions

In this paper, we announced a novel 4-D coupled Van der Pol conservative chaotic system and discussed its qualitative properties. We established that the novel 4-D coupled Van der Pol conservative chaotic system has a unique equilibrium at the origin, which is a saddle-point and unstable. We showed that the Lyapunov exponents of the novel 4-D Van der Pol conservative chaotic system are $L_1 = 14.6$, $L_2 = 0$, $L_3 = -0.46$ and $L_4 = -14.14$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel conservative chaotic system is seen as $L_1 = 14.6$, which is very large. This shows that the novel Van der Pol conservative chaotic system is highly chaotic. The Kaplan-Yorke dimension of the novel 4-D Van der Pol conservative chaotic system is determined as

$D_{KY} = 4$. This shows the high level complexity of the novel 4-D Van der Pol conservative chaotic system. We also derived new results for the adaptive control of the coupled Van der Pol highly chaotic system. The main results have been proved using Lyapunov stability theory. MATLAB plots were shown to illustrate the phase portraits of the novel 4-D conservative chaotic system and all the main results.

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