



Anti-synchronization of the Generalized Lotka-Volterra Three-Species Biological Systems via Adaptive Control

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Abstract: Since the recent research has shown the importance of biological control in many biological systems appearing in nature, this research paper investigates research in the dynamic and chaotic analysis of the generalized Lotka-Volterra three-species biological system, which was studied by Samardzija and Greller (1988). The generalized Lotka-Volterra biological system consists of two predator and one prey populations. This paper depicts the phase portraits of the 3-D generalized Lotka-Volterra system when the system undergoes chaotic behaviour. The anti-synchronization of *master* and *slave* chaotic systems deals with driving the sums of the respective states of the two systems to zero asymptotically with time. Next, this paper derives adaptive biological control law for achieving global and exponential anti-synchronization of the states of the generalized Lotka-Volterra three-species biological systems with unknown parameters. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the three-species generalized Lotka-Volterra biological system and its adaptive anti-synchronization.

Keywords: Chaos, chaotic systems, anti-synchronization, biology, biological system, Lotka-Volterra system, etc.

1. Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The classical chaotic systems are due to Lorenz, who discovered chaos while studying a 3-D weather model in 1963 [3], and Rossler, who discovered chaos, while he was studying chemical reactions in 1976 [4]. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

One of the famous examples of simple biological models is the two-species predator-prey model developed by Lotka and Volterra [130]. Lotka-Volterra system describes the interaction of a two-species predator-prey model and it consists of a system of two nonlinear ordinary differential equations. This is a very popular model and it has many applications of interacting two-species systems. However, this model also has limitations such as it ignores many important factors such as interactions between another species of the same ecosystem, interactions with the environment etc. Thus, three-species models of biological species have more importance. Arneodo et al. [143] have shown that one can obtain chaotic behaviour for three species in an ecosystem. Three species predator-prey models typically consist of one-prey and two predators, and in this research work, we investigate such a three-species biological generalized Lotka-Volterra system investigated by Samardzija and Greller [134].

An agricultural ecosystem comprises a dynamic web of biological relationships among crop plants or trees, herbivores, predators, preys, disease organisms, etc. Organisms in an ecosystem interact in many ways through competition. These organisms constantly evolve and depend on each other and thereby they create a diverse, complex and dynamic environment.

This paper depicts the phase portraits of the 3-D generalized Lotka-Volterra system [145] when the system undergoes chaotic behaviour. The synchronization of *master* and *slave* chaotic systems deals with synchronizing the respective states of the two systems asymptotically with time. Next, this paper derives adaptive biological control law for achieving global and exponential anti-synchronization of the states of the generalized Lotka-Volterra three-species biological systems with unknown parameters. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the three-species generalized Lotka-Volterra biological system and its adaptive anti-synchronization.

2. Generalized Lotka-Volterra Three-Species Biological System

Samardzija and Greller (1988, [145]) derived a generalized Lotka-Volterra three-species biological system, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = x_1 - x_1x_2 + cx_1^2 - ax_1^2x_3 \\ \dot{x}_2 = -x_2 + x_1x_2 \\ \dot{x}_3 = -bx_3 + ax_1^2x_3 \end{cases} \quad (1)$$

In (1), x_1 is the prey population, x_2, x_3 are predator populations and a, b, c are positive constants.

In [145], it was shown that the three-species biological system (1) is chaotic when we take $a = 2.9851, b = 3, c = 2$ (2)

For numerical simulations, we take the initial conditions as $x_1(0) = 1.2, x_2(0) = 1.2$ and $x_3(0) = 1.2$.

The 3-D phase portrait of the generalized Lotka-Volterra system(1) is depicted in Figure 1. The 2-D projections of the generalized Lotka-Volterra systems (2) on the coordinate planes are depicted in Figures 2-3.

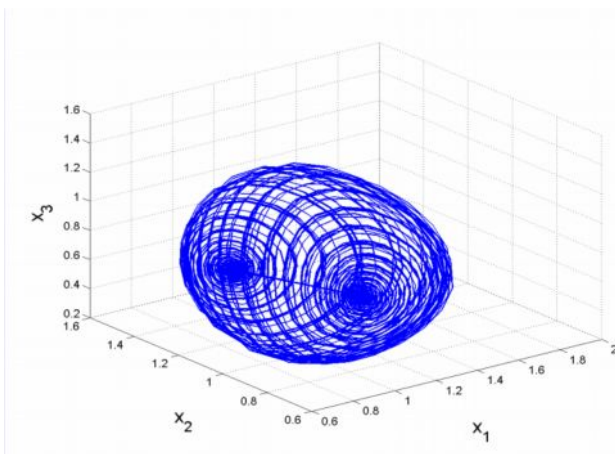


Figure1. The 3-D phase portrait of the generalized Lotka-Volterra chaotic system

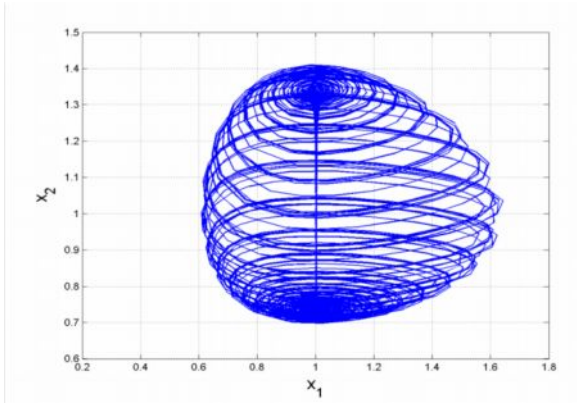


Figure2. The 2-D projection of the generalized Lotka-Volterra system on (x_1, x_2) plane

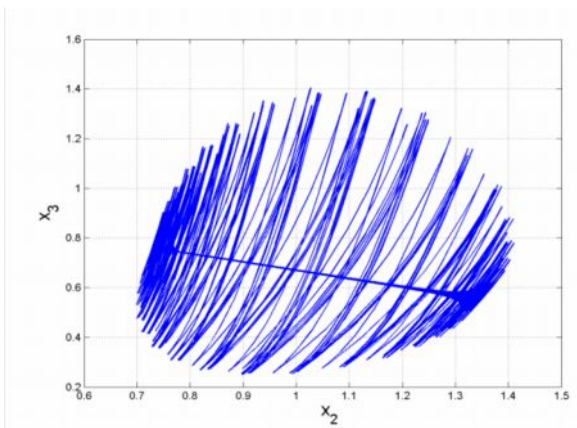


Figure3. The 2-D projection of the generalized Lotka-Volterra system on (x_2, x_3) plane

3. Adaptive Anti-Synchronization of the Generalized Lotka-Volterra Three-Species Biological Systems

The chaotic behaviour of the generalized Lotka-Volterra three-species biological system [45] is an example of the explosive route to chaos and it is attributed to the non-transversal saddle connection type bifurcation. Also, it is observed that the chaotic solution of the generalized Lotka-Volterra biological system (1) portrays a fractal torus in the 3-D phase space.

In this section, we use adaptive control method for achieving global and exponential anti-synchronization of the states of the generalized Lotka-Volterra three-species biological systems.

As the master system, we consider the generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_1 - x_1x_2 + cx_1^2 - ax_1^2x_3 \\ \dot{x}_2 = -x_2 + x_1x_2 \\ \dot{x}_3 = -bx_3 + ax_1^2x_3 \end{cases} \quad (3)$$

In (3), x_1, x_2, x_3 are the states and a, b, c are unknown parameters of the system.

As the slave system, we consider the generalized Lotka-Volterra system given by the 3-D dynamics

$$\begin{cases} \dot{y}_1 = y_1 - y_1y_2 + cy_1^2 - ay_1^2y_3 + u_1 \\ \dot{y}_2 = -y_2 + y_1y_2 + u_2 \\ \dot{y}_3 = -by_3 + ay_1^2y_3 + u_3 \end{cases} \quad (4)$$

In (4), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined.

We define the anti-synchronization error between the systems (3) and (4) as

$$\begin{cases} e_1 = y_1 + x_1 \\ e_2 = y_2 + x_2 \\ e_3 = y_3 + x_3 \end{cases} \quad (5)$$

The anti-synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_1 - y_1 y_2 - x_1 x_2 + c(y_1^2 + x_1^2) - a(y_1^2 y_3 + x_1^2 x_3) + u_1 \\ \dot{e}_2 = -e_2 + y_1 y_2 + x_1 x_2 + u_2 \\ \dot{e}_3 = -b e_3 + a(y_1^2 y_3 + x_1^2 x_3) + u_3 \end{cases} \quad (6)$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = -e_1 + y_1 y_2 + x_1 x_2 - \hat{c}(t)(y_1^2 + x_1^2) + \hat{a}(t)(y_1^2 y_3 + x_1^2 x_3) - k_1 e_1 \\ u_2 = e_2 - y_1 y_2 - x_1 x_2 - k_2 e_2 \\ u_3 = \hat{b}(t) e_3 - \hat{a}(t)(y_1^2 y_3 + x_1^2 x_3) - k_3 e_3 \end{cases} \quad (7)$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (7) into (6), we get the closed-loop control system given by

$$\begin{cases} \dot{e}_1 = [c - \hat{c}(t)](y_1^2 + x_1^2) - [a - \hat{a}(t)](y_1^2 y_3 + x_1^2 x_3) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -[b - \hat{b}(t)] e_3 + [a - \hat{a}(t)](y_1^2 y_3 + x_1^2 x_3) - k_3 e_3 \end{cases} \quad (8)$$

We define parameter estimation errors as follows:

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \\ e_c = c - \hat{c}(t) \end{cases} \quad (9)$$

Using (9), we can simplify the error dynamics (8) as follows.

$$\begin{cases} \dot{e}_1 = e_c (y_1^2 + x_1^2) - e_a (y_1^2 y_3 + x_1^2 x_3) - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -e_b e_3 + e_a (y_1^2 y_3 + x_1^2 x_3) - k_3 e_3 \end{cases} \quad (10)$$

Differentiating the parameter estimation errors (9) with respect to time, we get

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \\ \dot{e}_c = -\dot{\hat{c}}(t) \end{cases} \quad (11)$$

Next, we consider the candidate Lyapunov function given by

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2), \quad (12)$$

which is a positive definite function on R^6 .

Differentiating V along the trajectories of (10) and (11), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a [(y_1^2 y_3 + x_1^2 x_3)(e_3 - e_1) - \dot{\hat{a}}] + e_b [-e_3 - \dot{\hat{b}}] + e_c [e_1 (y_1^2 + x_1^2) - \dot{\hat{c}}] \quad (13)$$

In view of (13), we take the parameter estimates as follows:

$$\begin{cases} \dot{\hat{a}} = (y_1^2 y_3 + x_1^2 x_3)(e_3 - e_1) \\ \dot{\hat{b}} = -e_3^2 \\ \dot{\hat{c}} = e_1 (y_1^2 + x_1^2) \end{cases} \quad (14)$$

Theorem 1. *The generalized Lotka-Volterra three-species biological systems (3) and (4) are globally and exponentially anti-synchronized for all initial states $x(0), y(0) \in R^3$ by the adaptive biological control law (7) and the parameter update law (14), where k_1, k_2, k_3 are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by Eq. (12) is a positive definite function on R^6 .

Substituting the parameter update law (14) into (13), the time-derivative of V is obtained as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \quad (15)$$

which is a negative semi-definite function on R^6 .

Thus, by Lyapunov stability theory [133], we conclude that the anti-synchronization error $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (3), (4) and the parameter update law (14).

We take the gain constants as

$$k_1 = 5, \quad k_2 = 5, \quad k_3 = 5 \quad (16)$$

The parameter values of the systems (3) and (4) are taken as in the chaotic case, i.e.

$$a = 2.9851, \quad b = 3, \quad c = 2 \quad (17)$$

We take the initial conditions of the master system (3) as

$$x_1(0) = 3.2, \quad x_2(0) = 4.7, \quad x_3(0) = 21.6 \quad (18)$$

We take the initial conditions of the slave system (4) as

$$y_1(0) = 6.5, \quad y_2(0) = 2.4, \quad y_3(0) = 10.3 \quad (19)$$

Also, we take the initial conditions of the parameter estimates as

$$\hat{a}(0) = 2.1, \quad \hat{b}(0) = 3.6, \quad \hat{c}(0) = 8.4 \quad (20)$$

Figures 4-6 show the anti-synchronization of the states of the generalized Lotka-Volterra 3-species biological systems (3) and (4). Figure 7 shows the time-history of the anti-synchronization errors e_1, e_2, e_3 .

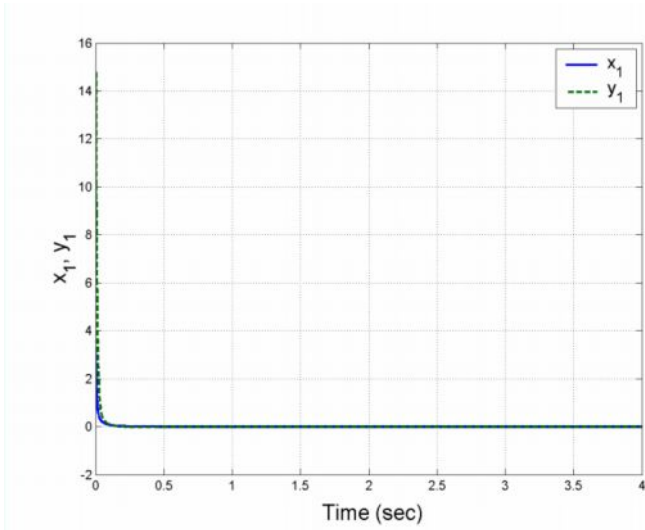


Figure4. Anti-synchronization of the states x_1 and y_1

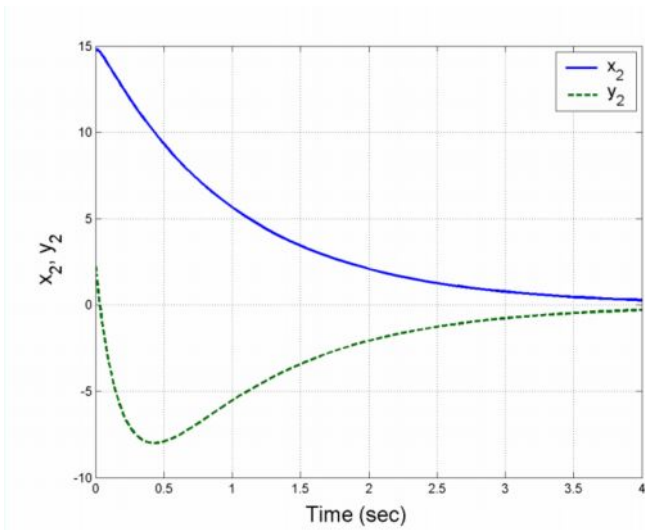


Figure5. Anti-synchronization of the states x_2 and y_2

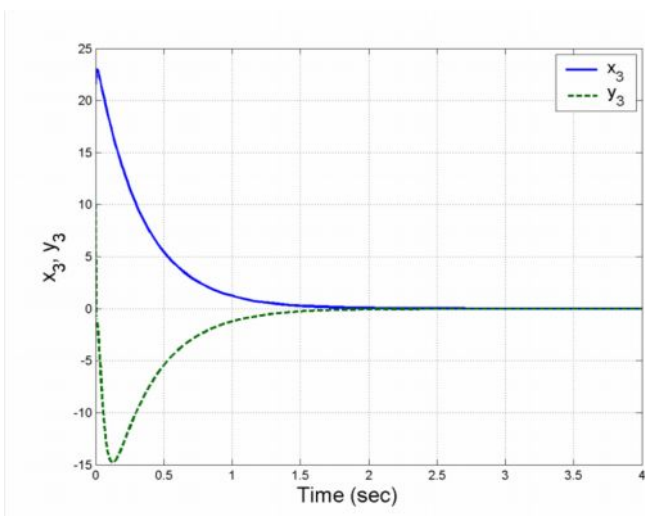


Figure6. Anti-synchronization of the states x_3 and y_3

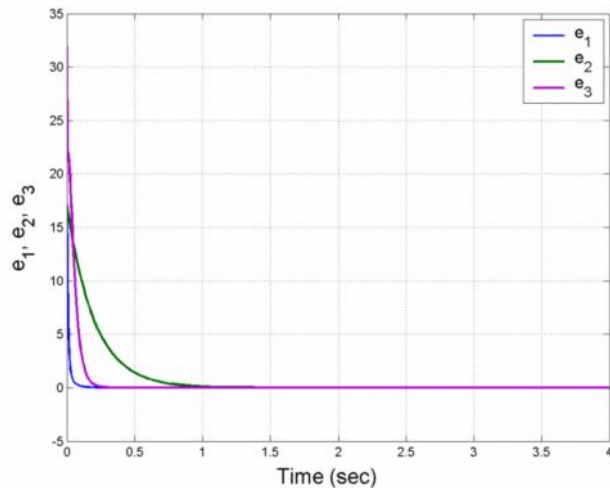


Figure 7. Time-history of the anti-synchronization errors e_1, e_2, e_3

5. Conclusions

In this paper, new results have been derived for the analysis and adaptive anti-synchronization of the three-species generalized Lotka-Volterra biological systems discovered by Samardzija and Greller (1988). After a description and dynamic analysis of the chaotic 3-D three species Samardzija-Greller model, we have designed an adaptive biological feedback controller for the global exponential and anti-synchronization of the states of the three-species generalized Lotka-Volterra biological systems. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

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