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Global Chaos Synchronization of 3-Cells Cellular Neural Network Attractors via Integral Sliding Mode Control

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Abstract: In this research work, we first discuss the properties of the 3-cells Cellular Neural Network (CNN) attractor discovered by Arena et al. (1998). Recent research has shown the importance of biological control in many biological systems appearing in nature. In computer science, machine learning and biology, cellular neural networks (CNN) are a parallel computing paradigm, similar to neural networks with the difference that communication is allowed between neighbouring units only. CNN has wide applications and recently, CNN is found to have many applications in biology and applied areas of biology. Chua and Yang introduced the Cellular Neural Network (CNN) in 1988 as a nonlinear dynamical system composed by an array of elementary and locally interacting nonlinear subsystems, which are called cells. We also derive new results for the global chaos synchronization of the 3-cells cellular neural networks (CNN) via integral sliding mode control. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the 3-cells cellular neural network (CNN) attractor.
Keywords: Chaos, chaotic systems, biology, cellular neural networks, CNN attractor, integral sliding mode control, chaos synchronization, stability theory, etc.

1. Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The classical chaotic systems are due to Lorenz, who discovered chaos while studying a 3-D weather model in 1963 [3], and Rossler, who discovered chaos, while he was studying chemical reactions in 1976 [4]. These classical systems were followed by the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

Recent research has shown the importance of biological control in many biological systems appearing in nature. In computer science, machine learning and biology, cellular neural networks (CNN) are a parallel computing paradigm, similar to neural networks with the difference that communication is allowed between neighbouring units only. CNN has wide applications and recently, CNN is found to have many applications in biology and applied areas of biology.

In 1988, Chua and Yang introduced the cellular neural network (CNN) as a nonlinear dynamical system composed by an array of elementary and locally interacting nonlinear subsystems, which are called cells [143]. In this research work, we first analyze the properties of the 3-cells CNN attractor discovered by Arena et al. [144].

We also derive new results for the global chaos synchronization of the 3-cells cellular neural network (CNN) using integral sliding mode control. Also, numerical simulations have been plotted using MATLAB to illustrate all the main results derived in this work for the 3-cells cellular neural network (CNN) attractor.

2. 3-Cells CNN Attractor

Arena *et al.* (1998, [144]) derived a 3-cells cellular neural network (CNN) attractor, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = -x_1 + \alpha f(x_1) - bf(x_2) - bf(x_3) \\ \dot{x}_2 = -x_2 - bf(x_1) + \beta f(x_2) - af(x_3) \\ \dot{x}_3 = -x_3 - bf(x_1) + af(x_2) + f(x_3) \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the states, a, b, α, β are positive constants and the function $f(z)$ is defined by

$$f(z) = 0.5\ (|z+1| - |z-1|) \text{ where } z \in R \tag{2}$$

In [144], it was shown that the 3-cells CNN system (1) is chaotic when we take the parameter values as

$$\alpha = 1.24,\ \beta = 1.1,\ a = 4.4 \text{ and } b = 3.21. \tag{3}$$

For numerical simulations, we take the initial conditions as $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $x_3(0) = 0.1$.

The 3-D phase portrait of the 3-cells CNN attractor (1) is depicted in Figure 1. The 2-D projections of the 3-cells CNN attractor (1) on the coordinate planes are depicted in Figures 2 and 3.

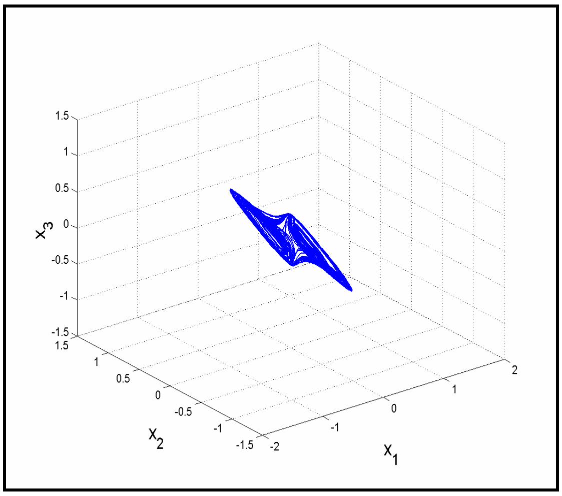

Figure 1. The 3-D phase portrait of the 3-cells CNN attractor

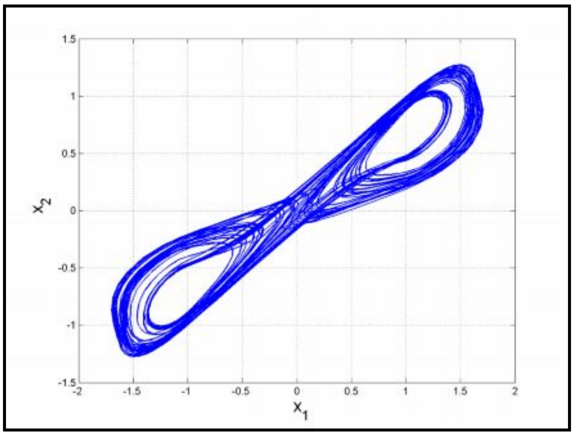

Figure 2. The 2-D projection of the 3-cells CNN attractor on (x_1, x_2) plane

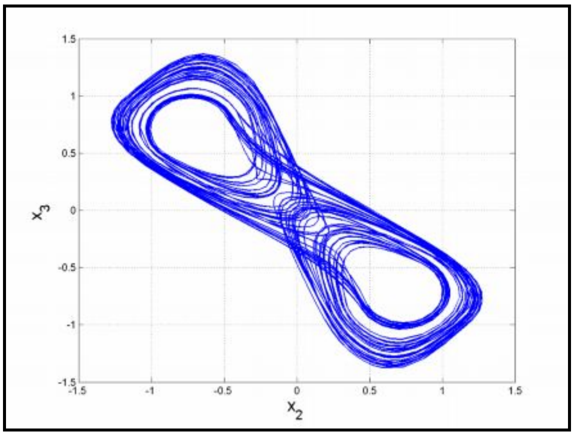

Figure 3. The 2-D projection of the 3-cells CNN attractor on (x_2, x_3) plane

3. Global Chaos Synchronization of the 3-Cells Cellular Neural Network (CNN) Attractors

The chaotic behaviour of the 3-cells cellular neural network (CNN) attractor [144] is a well-known example of a chaotic CNN system. In this section, we derive new results for the global chaos synchronization of the identical CNN attractors using the integral sliding mode control.

As the master system, we consider the 3-cells CNN attractor with controls given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = -x_1 + \alpha f(x_1) - bf(x_2) - bf(x_3) \\ \dot{x}_2 = -x_2 - bf(x_1) + \beta f(x_2) - af(x_3) \\ \dot{x}_3 = -x_3 - bf(x_1) + af(x_2) + f(x_3) \end{cases} \tag{4}$$

In (4), x_1, x_2, x_3 are the states and α, β, a, b are constant, positive, parameters. Also, the function $f(z), z \in R$ is defined by the equation (2).

As the slave system, we consider the controlled 3-cells CNN attractor given by the 3-D dynamics

$$\begin{cases} \dot{y}_1 = -y_1 + \alpha f(y_1) - bf(y_2) - bf(y_3) + u_1 \\ \dot{y}_2 = -y_2 - bf(y_1) + \beta f(y_2) - af(y_3) + u_2 \\ \dot{y}_3 = -y_3 - bf(y_1) + af(y_2) + f(y_3) + u_3 \end{cases} \tag{5}$$

In (5), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the controls to be determined.

The synchronization error between the 3-cells CNN attractors (4) and (5) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{6}$$

To simplify the notation, we define the new function

$$G(u,v) = f(v) - f(u), \qquad \text{for all } u, v \in R \tag{7}$$

Then the synchronization error dynamics is calculated as follows.

$$\begin{cases} \dot{e}_1 = -e_1 + \alpha G(x_1, y_1) - bG(x_2, y_2) - bG(x_3, y_3) + u_1 \\ \dot{e}_2 = -e_2 - bG(x_1, y_1) + \beta G(x_2, y_2) - aG(x_3, y_3) + u_2 \\ \dot{e}_3 = -e_3 - bG(x_1, y_1) + aG(x_2, y_2) + G(x_3, y_3) + u_2 \end{cases} \tag{8}$$

Based on the sliding mode control theory [146], the integral sliding surface of each state variable $x_i, (i = 1, 2, 3)$ is defined as follows:

$$s_i = \left(\frac{d}{dt} + \lambda_i \right) \left(\int_0^t e_i(\tau) d\tau \right) = e_i + \lambda_i \int_0^t e_i(\tau) d\tau, \quad (i = 1, 2, 3) \tag{9}$$

The derivative of each equation in (5) yields:

$$\dot{s}_i = \dot{e}_i + \lambda_i e_i, \quad (i = 1, 2, 3) \tag{10}$$

The Hurwitz condition is satisfied if $\lambda_i > 0$ for $i = 1, 2, 3$.

Based on the exponential reaching law [146], we set

$$\dot{s}_i = -\eta_i \operatorname{sgn}(s_i) - k_i s_i, \quad (i = 1, 2, 3) \tag{11}$$

where $\eta_i, k_i, \ (i = 1, 2, 3)$ are positive constants.

Comparing the equations (6) and (7), we get

$$\begin{cases} \dot{e}_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ \dot{e}_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ \dot{e}_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{12}$$

Using Eq. (8), we can rewrite Eq. (12) as follows:

$$\begin{cases} -e_1 + \alpha G(x_1, y_1) - bG(x_2, y_2) - bG(x_3, y_3) + u_1 + \lambda_1 e_1 = -\eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ -e_2 - bG(x_1, y_1) + \beta G(x_2, y_2) - aG(x_3, y_3) + u_2 + \lambda_2 e_2 = -\eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ -e_3 - bG(x_1, y_1) + aG(x_2, y_2) + G(x_3, y_3) + u_3 + \lambda_3 e_3 = -\eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{13}$$

From Eq. (13), the control laws are obtained as follows.

$$\begin{cases} u_1 = (1 - \lambda_1) e_1 - \alpha G(x_1, y_1) + bG(x_2, y_2) + bG(x_3, y_3) - \eta_1 \operatorname{sgn}(s_1) - k_1 s_1 \\ u_2 = (1 - \lambda_2) e_2 + bG(x_1, y_1) - \beta G(x_2, y_2) + aG(x_3, y_3) - \eta_2 \operatorname{sgn}(s_2) - k_2 s_2 \\ u_3 = (1 - \lambda_3) e_3 + bG(x_1, y_1) - aG(x_2, y_2) - G(x_3, y_3) - \eta_3 \operatorname{sgn}(s_3) - k_3 s_3 \end{cases} \tag{14}$$

Next, we state and prove the main result of this section.

Theorem 1. The 3-cells cellular neural network (CNN) attractors (4) and (5) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in R^3$ by the integral sliding mode control law (14), where the constants λ_i, η_i, k_i are positive for $i = 1, 2, 3$.

Proof. We consider the following quadratic Lyapunov function

$$V(s_1, s_2, s_3) = \frac{1}{2} \left(s_1^2 + s_2^2 + s_3^2 \right) \tag{15}$$

where s_1, s_2, s_3 are as defined in Eq. (5). The time-derivative of V is obtained as

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 + s_3 \dot{s}_3 \tag{16}$$

Substituting from Eq. (7) into Eq. (12), we obtain

$$\dot{V} = s_1(-\eta_1 \operatorname{sgn}(s_1) - k_1 s_1) + s_2(-\eta_2 \operatorname{sgn}(s_2) - k_2 s_2) + s_3(-\eta_3 \operatorname{sgn}(s_1) - k_3 s_1) \qquad (17)$$

Simplifying Eq. (13), we obtain

$$\dot{V} = -\eta_1 |s_1| - k_1 s_1^2 - \eta_2 |s_2| - k_2 s_2^2 - \eta_3 |s_3| - k_3 s_3^2 \qquad (18)$$

Since $k_i > 0$ and $\eta_i > 0$ for $i = 1, 2, 3$, it follows from (18) that $\dot{V}$ is a negative definite function.

Thus, by Lyapunov's stability theory [145], it is immediate that the closed-loop error dynamics (12) is globally asymptotically stable.

This completes the proof. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the system of differential equations (4) and (5).

The parameter values of the 3-cells CNN chaotic attractors (4) and (5) are taken as in the chaotic case, *viz.*

$$\alpha = 1.24, \quad \beta = 1.1, \quad a = 4.4, \quad b = 3.21.$$

We take the sliding constants as

$$\eta_i = \lambda_i = 0.1, \quad k_i = 30, \qquad (i = 1, 2, 3)$$

We take the initial conditions of the CNN chaotic attractor (4) as

$$x_1(0) = 5.4, \quad x_2(0) = 2.7, \quad x_3(0) = 20.3$$

We take the initial conditions of the CNN chaotic attractor (5) as

$$y_1(0) = 11.3, \quad y_2(0) = 3.4, \quad y_3(0) = 8.7$$

Figures 4-6 exhibit the complete synchronization of the CNN chaotic attractors (4) and (5).

Figure 7 shows the time-history of the synchronization errors e_1, e_2, e_3.

5. Conclusions

In this paper, the phase portraits and properties of the 3-cells Cellular Neural Network (CNN) chaotic attractor (Arena et al., 1998) were first discussed. Next, an integral sliding mode controller has been designed for the global chaos synchronization of the 3-cells CNN chaotic attractors. Numerical simulations have been illustrated using MATLAB.

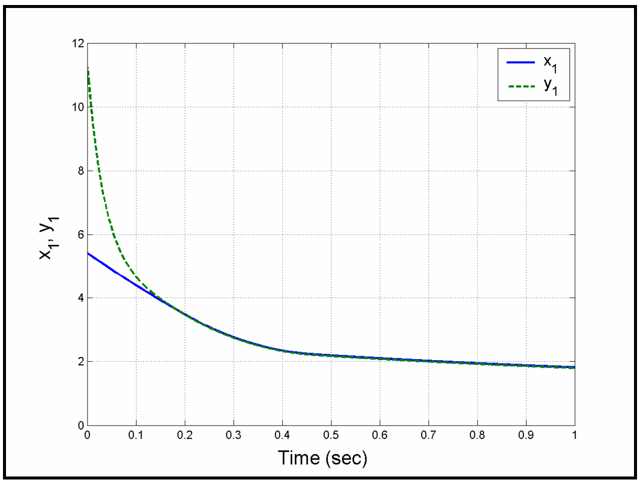

Figure 4. Synchronization of the states x_1 and y_1

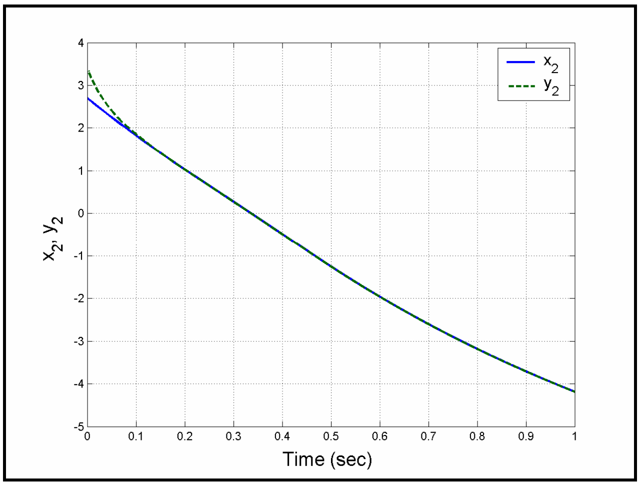

Figure 5. Synchronization of the states x_2 and y_2

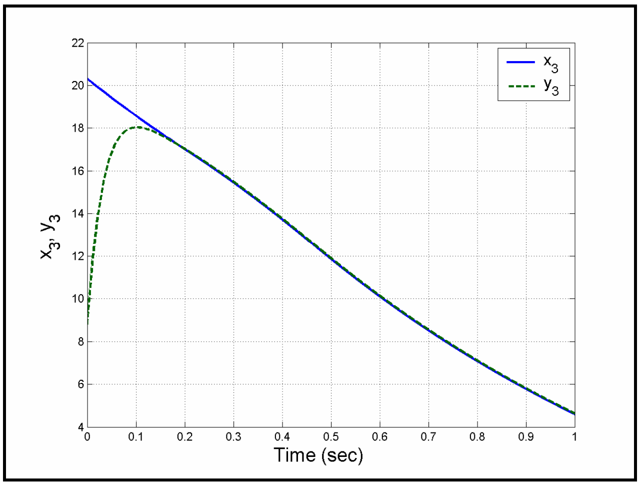

Figure 6. Synchronization of the states x_3 and y_3

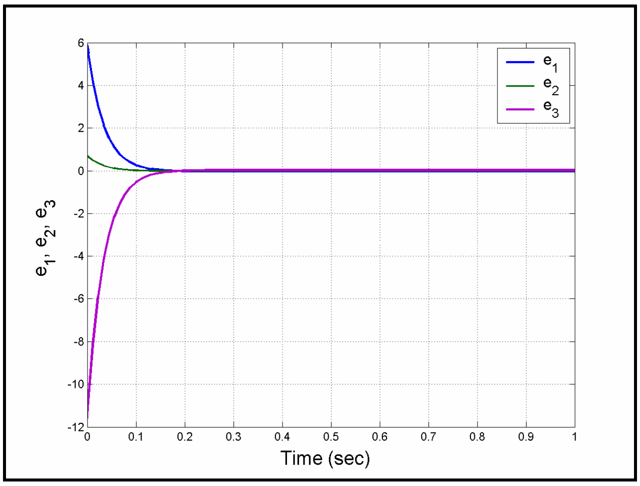

Figure 7. Time-history of the synchronization errors e_1, e_2, e_3

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