



Anti-Synchronization of 3-Cells Cellular Neural Network Attractors via Adaptive Control Method

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Abstract: In this research work, we first discuss the properties of the 3-cells cellular neural network (CNN) attractor discovered by Arena et al. (1998). Recent research has shown the importance of biological control in many biological systems appearing in nature. In computer science, machine learning and biology, cellular neural networks (CNN) are a parallel computing paradigm, similar to neural networks with the difference that communication is allowed between neighbouring units only. CNN has wide applications and recently, CNN is found to have many applications in biology and applied areas of biology. Chua and Yang introduced the cellular neural network (CNN) in 1988 as a nonlinear dynamical system composed by an array of elementary and locally interacting nonlinear subsystems, which are called cells. We also derive new results for the biological chaos anti-synchronization of the identical 3-cells CNN attractors via adaptive control method. All the main results are proved using Lyapunov stability theory. Also, numerical simulations have been plotted using MATLAB to illustrate the main results for the 3-cells cellular neural network (CNN) attractor.

Keywords: Chaos, chaotic systems, biology, cellular neural networks, CNN attractor, anti-synchronization, etc.

1. Introduction

Chaos theory describes the qualitative study of deterministic chaotic dynamical systems, and a chaotic system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal of anti-synchronization is to devise a feedback mechanism so that the trajectories of the master and slave systems are asymptotically equal in magnitude but opposite in sign.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-107], biology [108-125], memristors [126-128], electrical circuits [129], etc.

Recent research has shown the importance of biological control in many biological systems appearing in nature. In computer science, machine learning and biology, cellular neural networks (CNN) are a parallel computing paradigm, similar to neural networks with the difference that communication is allowed between neighbouring units only. CNN has wide applications and recently, CNN is found to have many applications in biology and applied areas of biology.

In 1988, Chua and Yang introduced the cellular neural network (CNN) as a nonlinear dynamical system composed by an array of elementary and locally interacting nonlinear subsystems, which are called cells [130]. In this research work, we first analyze the properties of the 3-cells CNN attractor discovered by Arena et al. [131].

We also derive new results for the biological chaos anti-synchronization of the identical 3-cells CNN attractors with unknown parameters via adaptive control method. All the main results are proved using Lyapunov stability theory [132]. Also, numerical simulations using MATLAB have been shown to illustrate all the main results for the 3-cells cellular neural network (CNN) attractor.

2. 3-Cells Cellular Neural Network Attractor

Arena *et al.* (1998, [131]) derived a 3-cells cellular neural network (CNN) attractor, which is described by the 3-D system of differential equations

$$\begin{cases} \dot{x}_1 = -x_1 + \alpha f(x_1) - \beta f(x_2) - \beta f(x_3) \\ \dot{x}_2 = -x_2 - \beta f(x_1) + \beta f(x_2) - \alpha f(x_3) \\ \dot{x}_3 = -x_3 - \beta f(x_1) + \alpha f(x_2) + f(x_3) \end{cases} \quad (1)$$

where x_1, x_2, x_3 are the states, $\alpha, \beta, \alpha, \beta$ are positive constants and the function $f(z)$ is defined by

$$f(z) = 0.5 (|z+1| - |z-1|) \text{ where } z \in R \quad (2)$$

In [119], it was shown that the 3-cells CNN system (1) is chaotic when we take the parameter values as $\alpha = 1.24$, $\beta = 1.1$, $\alpha = 4.4$ and $b = 3.21$

For numerical simulations, we take the initial conditions as $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $x_3(0) = 0.1$.

The 3-D phase portrait of the 3-cells CNN attractor (1) is depicted in Figure 1. The 2-D projections of the 3-cells CNN attractor (1) on the coordinate planes are depicted in Figures 2-3.

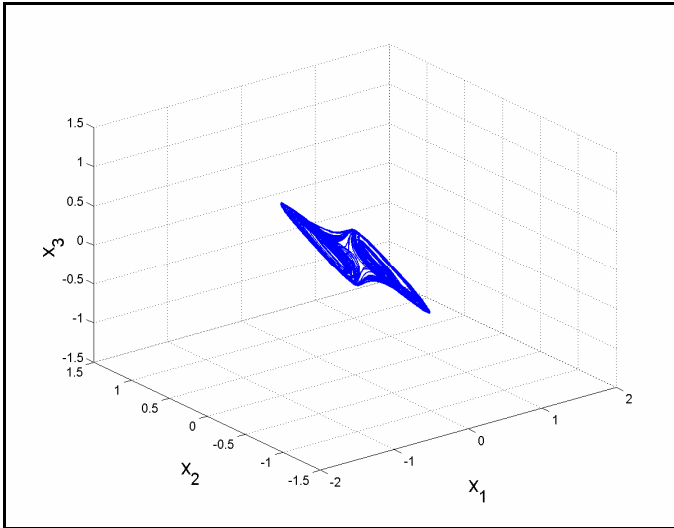


Figure 1. The 3-D phase portrait of the 3-cells CNN attractor

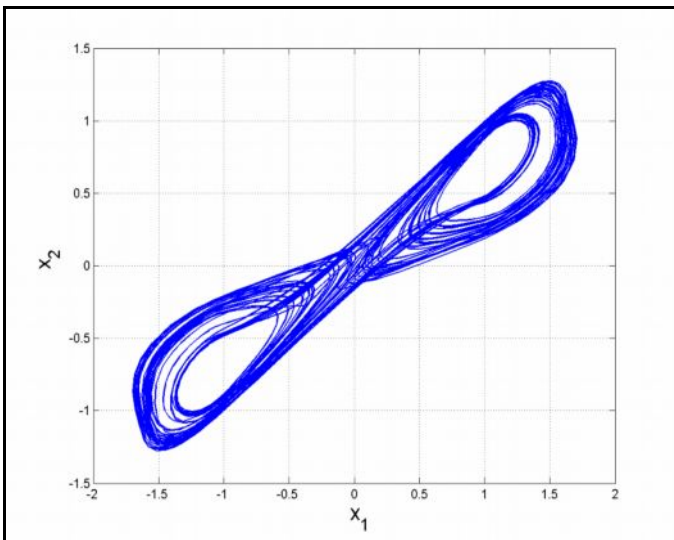


Figure 2. The 2-D projection of the 3-cells CNN attractor on (x_1, x_2) plane

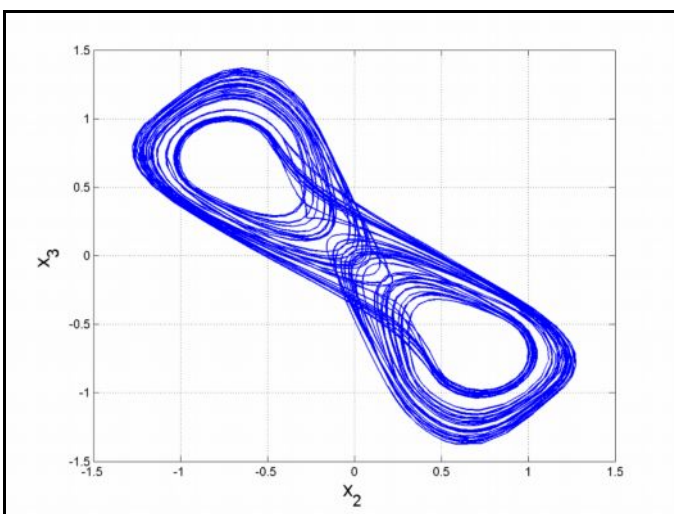


Figure 3. The 2-D projection of the 3-cells CNN attractor on (x_2, x_3) plane

3. Anti-synchronization of the 3-Cells Cellular Neural Network (CNN) Attractors

The chaotic behaviour of the 3-cells cellular neural network (CNN) attractor [131] is a well-known example of a chaotic CNN system. In this section, we consider the anti-synchronization of the identical 3-cells CNN attractors.

As the master system, we consider the 3-cells CNN attractor given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = -x_1 + \alpha f(x_1) - bf(x_2) - bf(x_3) \\ \dot{x}_2 = -x_2 - bf(x_1) + \beta f(x_2) - af(x_3) \\ \dot{x}_3 = -x_3 - bf(x_1) + af(x_2) + f(x_3) \end{cases} \quad (4)$$

In (4), x_1, x_2, x_3 are the states and α, β, a, b are unknown system parameters. Also, the function $f(z), z \in R$ is defined by the equation (2).

As the slave system, we consider the 3-cells CNN attractor given by the 3-D dynamics

$$\begin{cases} \dot{y}_1 = -y_1 + \alpha f(y_1) - bf(y_2) - bf(y_3) + u_1 \\ \dot{y}_2 = -y_2 - bf(y_1) + \beta f(y_2) - af(y_3) + u_2 \\ \dot{y}_3 = -y_3 - bf(y_1) + af(y_2) + f(y_3) + u_3 \end{cases} \quad (5)$$

In (5), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined.

To simplify the notation, we define a new function $G(u, v)$ as follows:

$$G(u, v) = f(v) + f(u), \text{ where } u, v \in R. \quad (6)$$

We define the anti-synchronization error between the CNN systems (4) and (5) as

$$\begin{cases} e_1 = y_1 + x_1 \\ e_2 = y_2 + x_2 \\ e_3 = y_3 + x_3 \end{cases} \quad (7)$$

Using (4), (5) and (6), the synchronization error dynamics is obtained as follows:

$$\begin{cases} \dot{e}_1 = -e_1 + \alpha G(x_1, y_1) - bG(x_2, y_2) - bG(x_3, y_3) + u_1 \\ \dot{e}_2 = -e_2 - bG(x_1, y_1) + \beta G(x_2, y_2) - aG(x_3, y_3) + u_2 \\ \dot{e}_3 = -e_3 - bG(x_1, y_1) + aG(x_2, y_2) + G(x_3, y_3) + u_3 \end{cases} \quad (8)$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = e_1 - \hat{\alpha}(t)G(x_1, y_1) + \hat{b}(t)G(x_2, y_2) + \hat{b}(t)G(x_3, y_3) - k_1 e_1 \\ u_2 = e_2 + \hat{b}(t)G(x_1, y_1) - \hat{\beta}(t)G(x_2, y_2) + \hat{a}(t)G(x_3, y_3) - k_2 e_2 \\ u_3 = e_3 + \hat{b}(t)G(x_1, y_1) - \hat{a}(t)G(x_2, y_2) - G(x_3, y_3) - k_3 e_3 \end{cases} \quad (9)$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (9) into (8), we get the closed-loop error dynamics given by

$$\begin{cases} \dot{e}_1 = [\alpha - \hat{\alpha}(t)]G(x_1, y_1) - [b - \hat{b}(t)]G(x_2, y_2) - [b - \hat{b}(t)]G(x_3, y_3) - k_1 e_1 \\ \dot{e}_2 = -[b - \hat{b}(t)]G(x_1, y_1) + [\beta - \hat{\beta}(t)]G(x_2, y_2) - [a - \hat{a}(t)]G(x_3, y_3) - k_2 e_2 \\ \dot{e}_3 = -[b - \hat{b}(t)]G(x_1, y_1) + [a - \hat{a}(t)]G(x_2, y_2) - k_3 e_3 \end{cases} \quad (10)$$

We define parameter estimation errors as follows:

$$\begin{cases} e_\alpha = \alpha - \hat{\alpha}(t) \\ e_\beta = \beta - \hat{\beta}(t) \\ e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \end{cases} \quad (11)$$

Using (11), we can simplify the closed-loop plant dynamics (6) as follows.

$$\begin{cases} \dot{e}_1 = e_\alpha G(x_1, y_1) - e_b G(x_2, y_2) - e_b G(x_3, y_3) - k_1 e_1 \\ \dot{e}_2 = -e_b G(x_1, y_1) + e_\beta G(x_2, y_2) - e_a G(x_3, y_3) - k_2 e_2 \\ \dot{e}_3 = -e_b G(x_1, y_1) + e_a G(x_2, y_2) - k_3 e_3 \end{cases} \quad (12)$$

Differentiating the parameter estimation errors (8) with respect to time, we get

$$\begin{cases} \dot{\hat{\alpha}} = -\hat{\alpha} \\ \dot{\hat{\beta}} = -\hat{\beta} \\ \dot{\hat{a}} = -\hat{a} \\ \dot{\hat{b}} = -\hat{b} \end{cases} \quad (13)$$

Next, we consider the candidate Lyapunov function given by

$$V(e_1, e_2, e_3, e_\alpha, e_\beta, e_a, e_b) = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_a^2 + e_b^2) \quad (14)$$

Differentiating V along the trajectories of (12) and (13), we obtain

$$\begin{aligned} \dot{V} = & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha [e_1 G(x_1, y_1) - \hat{\alpha}] + e_\beta [e_2 G(x_2, y_2) - \hat{\beta}] \\ & + e_a [-e_2 G(x_3, y_3) + e_3 G(x_2, y_2) - \hat{a}] \\ & + e_b [-e_1 [G(x_2, y_2) + G(x_3, y_3)] - (e_2 + e_3) G(x_1, y_1) - \hat{b}] \end{aligned} \quad (15)$$

In view of (11), we take the parameter estimates as follows:

$$\begin{cases} \dot{\hat{\alpha}} = e_1 G(x_1, y_1) \\ \dot{\hat{\beta}} = e_2 G(x_2, y_2) \\ \dot{\hat{a}} = -e_2 G(x_3, y_3) + e_3 G(x_2, y_2) \\ \dot{\hat{b}} = -e_1 [G(x_2, y_2) + G(x_3, y_3)] - (e_2 + e_3) G(x_1, y_1) \end{cases} \quad (16)$$

Theorem 1. *The 3-cells CNN chaotic attractors (4) and (5) are globally and exponentially anti-synchronized by the adaptive control law (9) and the parameter update law (16), where k_1, k_2, k_3 are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by Eq. (14) is a positive definite function on R^7 .

Substituting the parameter update law (12) into (11), the time-derivative of V is obtained as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2, \quad (17)$$

which is a negative semi-definite function on R^7 .

Thus, by the Barbalat's lemma in Lyapunov stability theory [132], we conclude that the error vector $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (4) and (12). We take the gain constants as $k_i = 8$ for $i = 1, 2, 3$.

The parameter values of the 3-cells CNN chaotic attractor (4) are taken as in the chaotic case, viz.

$$\alpha = 1.24, \beta = 1.1, a = 4.4, b = 3.21.$$

We take the initial conditions of the master system (4) as

$$x_1(0) = 5.2, \quad x_2(0) = 4.1, \quad x_3(0) = 10.8$$

We take the initial conditions of the slave system (5) as

$$y_1(0) = 2.6, \quad y_2(0) = 7.8, \quad y_3(0) = 12.4$$

Also, we take the initial conditions of the parameter estimates as

$$\hat{\alpha}(0) = 10.4, \quad \hat{\beta}(0) = 5.1, \quad \hat{a}(0) = 2.3, \quad \hat{b}(0) = 17.8$$

Figures 4-6 show the anti-synchronization of the 3-cells CNN chaotic attractors (4) and (5).

Figure 7 shows the time-history of the anti-synchronization errors e_1, e_2, e_3 .

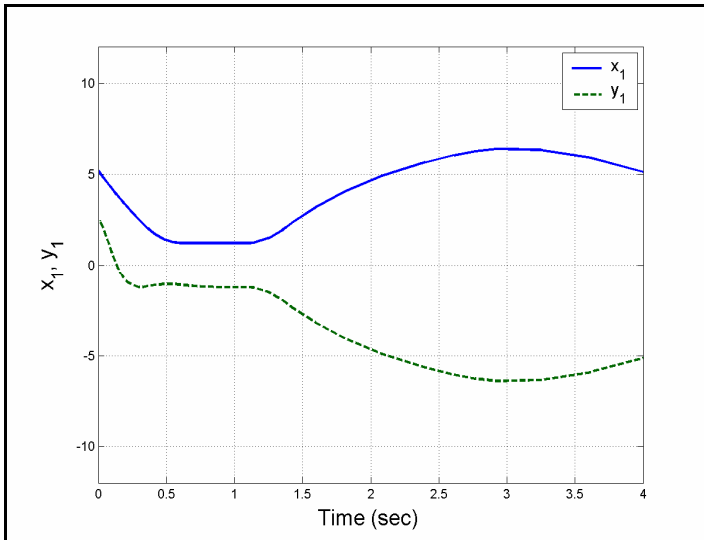


Figure 4. Anti-synchronization of the states x_1 and y_1

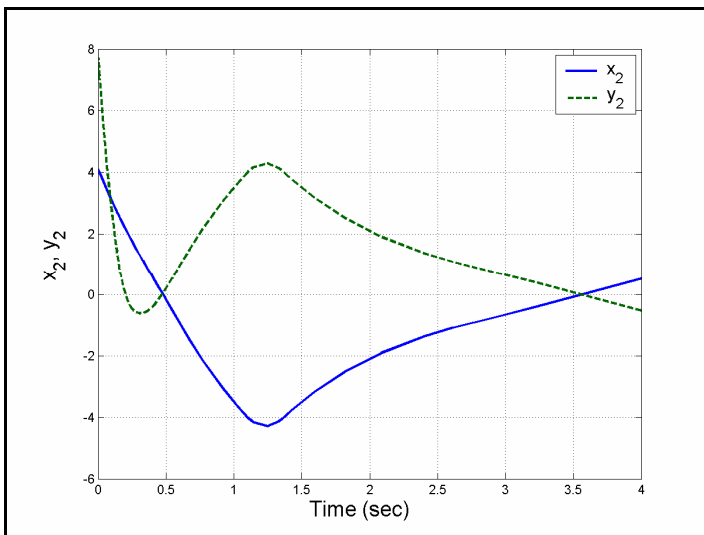


Figure 5. Anti-synchronization of the states x_2 and y_2

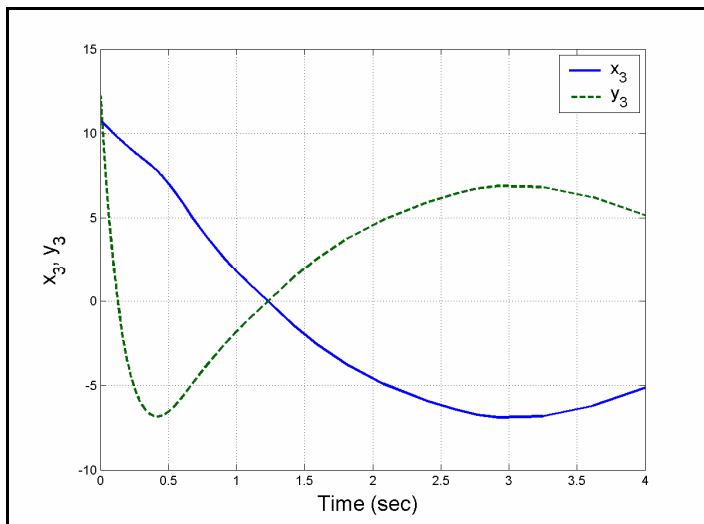


Figure 6. Anti-synchronization of the states x_3 and y_3

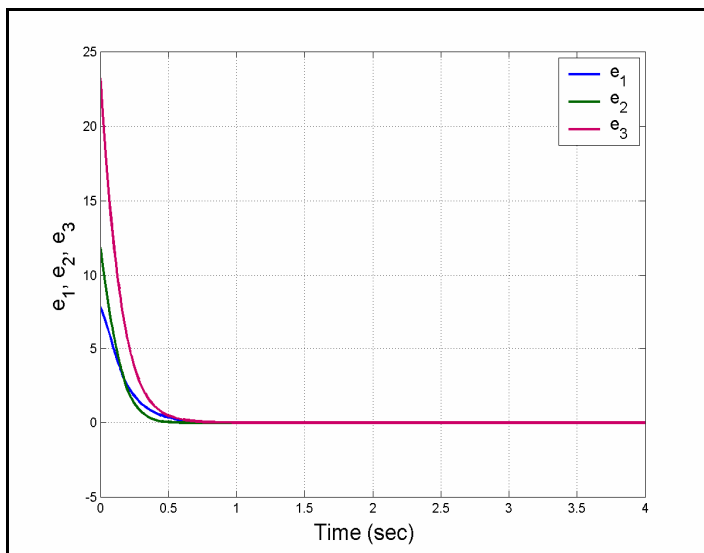


Figure 7. Time-history of the synchronization errors e_1, e_2, e_3

5. Conclusions

In this paper, new results have been derived for the analysis and global chaos anti-synchronization of the 3-cells cellular neural network (CNN) chaotic attractor obtained by Arena *et al.* (1998). After a description and phase portraits of the 3-cells CNN chaotic attractor, we have designed an adaptive feedback controller for the complete and exponential anti-synchronization of the states of the 3-cells CNN chaotic attractors. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.

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