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# Sliding Controller Design for the Global Chaos Synchronization of Forced Van der Pol Chaotic Oscillators

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**Abstract:** Chaos theory has a lot of applications in science and engineering. This paper first details the qualitative properties of the forced Van der Pol chaotic oscillator, which has important applications. Since its introduction in the 1920's, the Van der Pol equation has been a prototype model for systems with self-excited limit cycle oscillations. The Van der Pol equation has been studied over wide parameter regimes, from perturbations of harmonic motion to relaxation oscillations. It has been used by scientists to model a variety of physical and biological phenomena. Next, we derive new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators via sliding mode control (SMC) method. MATLAB plots have been shown to illustrate the phase portraits of the forced Van der Pol chaotic oscillator and the sliding mode control based synchronization of the forced Van der Pol chaotic oscillators.
**Keywords:** Chaos, forced Van der Pol system, oscillators, synchronization, sliding mode control, etc.

## 1. Introduction

Chaos theory is a modern research field which discusses the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

The first famous chaotic system was discovered by Lorenz, when he was developing a 3-D weather model for atmospheric convection in 1963 [3], and subsequently, Rössler discovered a 3-D chaotic system in 1976 [4], which was constructed during the study of a chemical reaction.

Recently, many 3-D chaotic systems have been announced in the literature such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-79]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [80-86], sliding mode control method [87-98], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [99-107], biology [108-125], memristors [126-128], electrical circuits [129], etc.

This paper investigates first the qualitative properties of the forced Van der Pol chaotic oscillator, which was discovered by Van der Pol and Van der Mark ([130], 1927). In [131], it was reported that at certain drive frequencies an irregular noise was heard. This irregular noise was always heard near the natural entrainment frequencies. This was one of the first discovered instances of deterministic chaos. The Van der Pol oscillator has a long history of being used in both the physical and biological sciences. For instance, in biology, Fitzhugh [132] and Nagumo [133] extended the Van der Pol equation in a planar field as a model for action potentials of neurons. A detailed study on forced Van der Pol equation is found in [134].

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system, and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In this paper, we derive new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators [131].

This paper is organized as follows. Section 2 details the dynamics and properties of the forced Van der Pol chaotic oscillator. Section 3 details the global chaos synchronization of the forced Van der Pol chaotic oscillator via sliding mode control method. Section 4 details the numerical simulations illustrating the main sliding mode control result derived in this research paper. Section 5 contains the main conclusions of this work.

## 2. Forced Van der Pol Chaotic Oscillator

The forced Van der Pol chaotic oscillator [131] is described by the second order differential equation

$$\ddot{x} = -x - a(x^2 - 1)\dot{x} + b\cos(\omega t) \qquad (1)$$

In this work, we express the forced Van der Pol equation (1) in system form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - a(x_1^2 - 1)x_2 + b\cos(\omega t) \end{cases} \qquad (2)$$

In Eq. (2), $x_1, x_2$ are the states and $a, b$ are constant, positive, parameters.

It is known in the literature [134] that the system (2) is *chaotic*, when the parameter values are taken as

$$a = 5, \quad b = 5, \quad \omega = 2.467 \qquad (3)$$

For numerical simulations, we take $x_1(0) = 0.1$ and $x_2(0) = 0.1$.

Figure 1 shows the $x_1 -$ waveform of the Van der Pol system (2), while Figure 2 shows the $x_2 -$ waveform of the Van der Pol system (2). Figure 3 shows the chaotic phase portrait of the Van der Pol system (2).

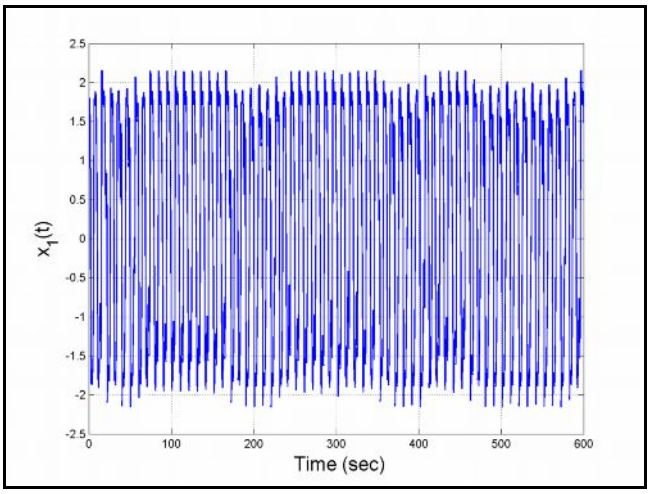


**Figure 1.** $x_1$ **– waveform of the Van der Pol system**

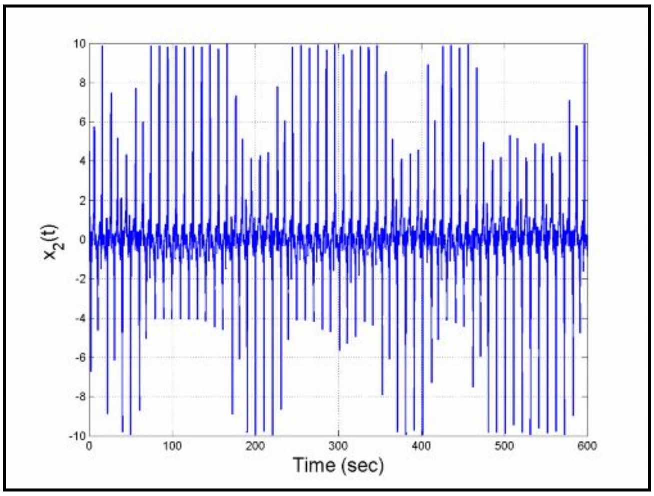


**Figure 2.** $x_2$ **– waveform of the Van der Pol system**

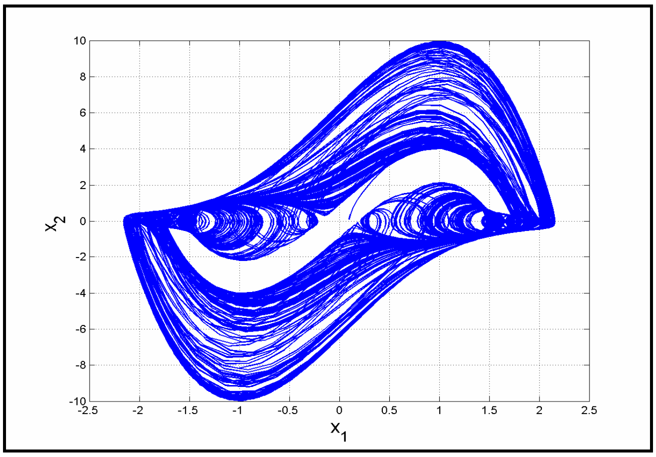


**Figure 3. Chaotic phase portrait of the Forced Van der Pol system**

## 3. Sliding Controller Design for the Global Chaos Synchronization of the Forced Van der Pol Chaotic Oscillators

In this section, we design a sliding mode controller (SMC) for globally synchronizing the forced Van der Pol chaotic oscillators. Our sliding control design is based on the Vaidyanathan's novel sliding mode control method [97] for globally synchronizing identical chaotic systems.

As the master system, we consider the Van der Pol chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - a(x_1^2 - 1)x_2 + b\cos(\omega t) \end{cases} \tag{4}$$

In (4), $x_1, y_1$ are the states and $a, b, \omega$ are constant, positive, parameters.

As the slave system, we consider the controlled Van der Pol chaotic system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -y_1 - a(y_1^2 - 1)y_2 + b\cos(\omega t) + u \end{cases} \tag{5}$$

In (5), $x_2, y_2$ are the states and $u$ is the sliding mode controller to be determined.

Now, we define the chaos synchronization errors as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \end{cases} \tag{6}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -e_1 - ae_2 - a\left(y_1^2 y_2 - x_1^2 x_2\right) + u \end{cases} \tag{7}$$

Next, we arrange the error system (8) in matrix form as

$$\dot{e} = Ae + \psi(X,Y) + Bu \tag{8}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \ A = \begin{bmatrix} 0 & 1 \\ -1 & -a \end{bmatrix}, \ X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ X_2 = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{9}$$

and

$$\psi\left(X,Y\right) = \begin{bmatrix} 0 \\ \varphi(X,Y) \end{bmatrix} \tag{10}$$

with

$$\varphi(X,Y) = -a\left(y_1^2 y_2 - x_1^2 x_2\right) \tag{11}$$

We define the nonlinear control $u$ as

$$u = -\varphi(X,Y) + v \tag{12}$$

Then the nonlinear error dynamics (9) reduces to the linear error dynamics

$$\dot{e} = Ae + Bv \tag{13}$$

where $v(t)$ is a sliding mode controller to be determined using Vaidyanathan's novel sliding control method [97].

First, we shall verify that $(A,B)$ is completely controllable.

We take the parameter values as in the chaotic case, i.e.

$$a = 5, \ \ b = 5, \ \ \omega = 2.467 \tag{14}$$

The controllability matrix for the linear pair $(A,B)$ is easily obtained as

$$Q = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -a \end{bmatrix} \tag{15}$$

We find that $\det(Q) = -1 \neq 0.$ Thus, the controllability matrix $Q$ has full rank.

Hence, by Kalman's rank condition for controllability [135], the pair $(A,B)$ is completely controllable.

We select the sliding variable as

$$s = Ce = \begin{bmatrix} 20 & 1 \end{bmatrix} e = 20e_1 + e_2 \tag{16}$$

With the choice of $C = \begin{bmatrix} 20 & 1 \end{bmatrix}$, the eigenvalues of the matrix $E = \left[ I - B(CB)^{-1} C \right] A$ are given by

$$\text{eig}(E) = \{-20, 0\}. \tag{17}$$

This shows that the dynamics along the sliding manifold is globally asymptotically stable.

Next, we take the sliding constants as $k = 6$ and $q = 0.2.$

Then the sliding mode control $v$ is obtained by the Vaidyanathan's theorem [97] as

$$v(t) = -(CB)^{-1} \left[ C(kI + A)e + qs^2 \,\text{sgn}(s) \right] \tag{18}$$

A simple calculation gives

$$v(t) = -119e_1 - 21e_2 - 0.2s^2 \,\text{sgn}(s) \tag{19}$$

As an application of Vaidyanathan's theorem [97], we obtain the following result.

**Theorem 1.** The forced Van der Pol chaotic systems (4) and (5) are globally and asymptotically synchronized for all initial conditions by the sliding mode control $u$ given by (12), where $\varphi(X,Y)$ is defined by (11) and $v$ is defined by (19). ■

## 4. Numerical Simulations

We use classical fourth-order Runge-Kutta method in MATLAB with step-size $h = 10^{-8}$ for solving the systems of differential equations given by (4) and (5), when the sliding mode control law (12) is applied.

We take the sliding constants as $k = 6$ and $q = 0.2$.
We take the initial conditions of the master system (5) as

$$x_1(0) = 3.5, \quad x_2(0) = 6.7 \tag{20}$$

We take the initial conditions of the slave system (6) as

$$y_1(0) = 1.8, \quad y_2(0) = 3.2 \tag{21}$$

The parameter values are taken as in (3) for the chaotic case, viz.

$$a = 5, \quad b = 5, \quad \omega = 2.467 \tag{22}$$

Figures 4-5 show the global chaos synchronization of the forced Van der Pol chaotic systems (4) and (5).

Figure 6 shows the time-history of the chaos synchronization errors $e_1(t), e_2(t)$.

From Figure 6, it is clear that the synchronization errors converge to zero in just one second.

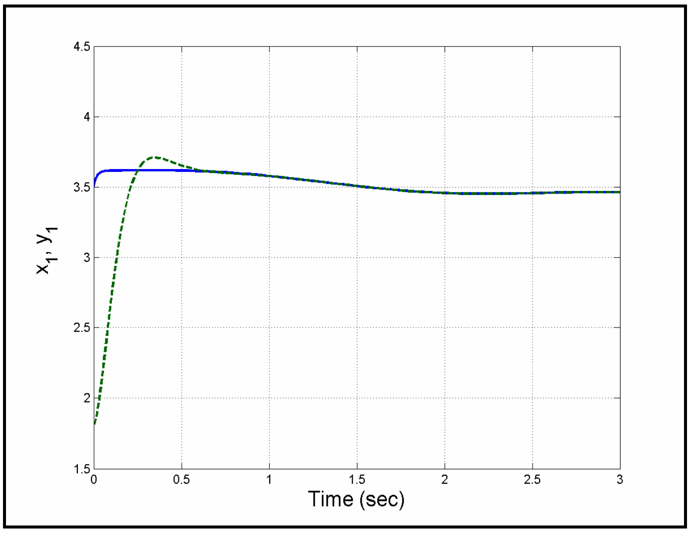


**Figure 4. Synchronization of the states** $x_1(t), y_1(t)$

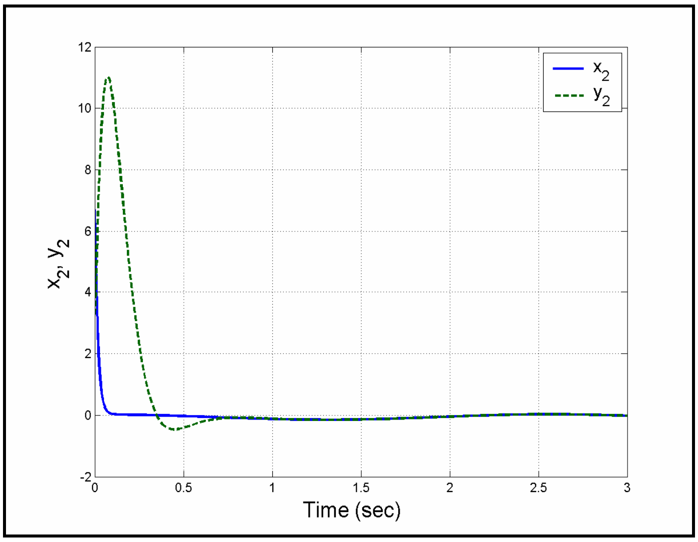


**Figure 5. Synchronization of the states** $x_2(t), y_2(t)$

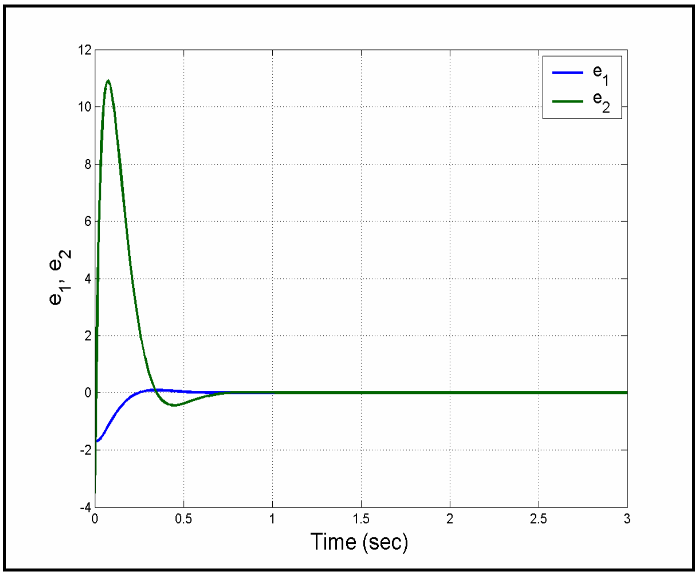


**Figure 6. Time-history of the synchronization errors** $e_1(t), e_2(t)$

## 5. Conclusions

In this paper, we first discussed the qualitative properties of the forced Van der Pol chaotic oscillator, which has important applications. Since its introduction in the 1920's, the Van der Pol equation has been a prototype model for systems with self-excited limit cycle oscillations. The Van der Pol equation has been studied over wide parameter regimes, from perturbations of harmonic motion to relaxation oscillations. It has been used by scientists to model a variety of physical and biological phenomena. Next, we derived new results for the global chaos synchronization of the identical forced Van der Pol chaotic oscillators via sliding mode control (SMC) method. MATLAB plots were depicted to illustrate the phase portraits of the forced Van der Pol chaotic oscillator and the sliding mode control based synchronization of the forced Van der Pol chaotic oscillators.

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