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Lotka-Volterra Two Species Competitive Biology Models and their Ecological Monitoring

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Abstract: Lotka-Volterra population biology models are important models that describe the interaction between various biological species. This work describes a Lotka-Volterra 2-species competitive population biology model. We show that for this biological model, under an assumption, the two competing species have stable coexistence. Then we shall propose ecological monitoring of the Lotka-Volterra 2-species competitive population biology models by constructing nonlinear observers for them about their stable equilibrium points. The nonlinear observer design for the population biology model is constructed by applying Sundarapandian's theorem (2002) and using only the dynamics of the Lotka-Volterra population biology model and the population size of any of the competitive species as the output function. Numerical example and MATLAB simulations are given to illustrate the ecological monitoring or the nonlinear observer design for the Lotka-Volterra two-species competitive biology model with stable coexistence.
Keywords: Population biology, Lotka-Volterra model, competitive model, two-species model, ecological monitoring, observer design, etc.

1. Introduction

Lotka-Volterra population biology models are important models that describe the interaction between various biological species considered as predator-prey system [1-2]. This work describes a Lotka-Volterra two species competitive biology model [3]. We show that for this biological model, under an assumption, the two competing species have stable co-existence. After discussion on the Lotka-Volterra population biology models, we propose ecological monitoring of the Lotka-Volterra two species competitive biology models by explicitly constructing local exponential observers for the population biology models.

In control systems engineering, a state observer is a system that provides an estimate of the internal state of a given real system, from measurements of the input and output of the real system. It is typically computer-implemented, and provides the basis of many practical applications.

The problem of designing observers for linear control systems was first proposed and fully solved by Luenberger [4]. The problem of designing observers for nonlinear control systems was proposed by Thau [5]. Over the past three decades, significant attention has been paid in the control systems literature on the construction of observers for nonlinear control systems [6].

A characterization of local exponential observers for nonlinear control systems was first obtained by Sundarapandian [7]. In [7], necessary and sufficient conditions were obtained for exponential observers for Lyapunov stable continuous-time nonlinear systems. In [7], an exponential observer design was provided by Sundarapandian for nonlinear control systems, which generalizes the linear observer design of Luenberger [3] for linear control systems. In [8], Sundarapandian obtained necessary and sufficient conditions for exponential observers for Lyapunov stable discrete-time nonlinear systems and also provided a formula for designing

exponential observers for Lyapunov stable discrete-time nonlinear systems. In [9], Sundarapandian derived new results for the global observer design for nonlinear control systems.

The concept of nonlinear observers for nonlinear control systems was extended in many ways. In [10-11], Sundarapandian derived new results for the characterization of local exponential observers for nonlinear bifurcating systems. In [12-15], Sundarapandian derived new results for the exponential observer design for a general class of nonlinear systems with real parametric uncertainty. In [16-19], Sundarapandian derived new results for the general observers for nonlinear systems. In [20], Sundarapandian derived new results for observers around equilibria. In [21-22], Sundarapandian derived new results for the periodic orbits of nonlinear control systems.

This work discusses the Lotka-Volterra two species competitive biology models. Section 2 reviews the definition and results of local exponential observers for nonlinear systems. Section 3 describes the Lotka-Volterra two species competitive biology models. Section 4 details the ecological monitoring of the Lotka-Volterra two species competitive biology model discussed in Section 3 by giving a design of nonlinear exponential observer that estimates the states of the Lotka-Volterra two species competitive biology model. Section 5 contains the conclusions of this work.

2. Review of Nonlinear Observer Design for Nonlinear Systems

An observer for a nonlinear system is a state estimator, and the states of the observer converge to the states of the plant dynamics asymptotically or exponentially as time tends to infinity.

We consider the nonlinear system described by

$$\dot{x} = f(x) \tag{1a}$$
$$y = h(x) \tag{1b}$$

where $x \in R^n$ is the *state* and $y \in R^p$ is the *output*.

We assume that $f : R^n \to R^n$, $h : R^n \to R^p$ are C^1 mappings and for some $x^* \in R^n$, the following hold:

$$f(x^*) = 0, \quad h(x^*) = 0 \tag{2}$$

Remark 1. The solutions x^* of $f(x) = 0$ are called the *equilibrium points* of the system dynamics (1a). Also, the assumption $h(x^*) = 0$ holds without any loss of generality. Indeed, if $h(x^*) \neq 0$, then we can define a new output function as

$$\psi(x) = h(x) - h(x^*) \tag{3}$$

and it is easy to see that $\psi(x^*) = 0$. ■

The linearization of the nonlinear system (1a)-(1b) at $x = x^*$ is given by

$$\dot{x} = Ax \tag{4a}$$
$$y = Cx \tag{4b}$$

where

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=x^*} \quad \text{and} \quad C = \left[\frac{\partial h}{\partial x}\right]_{x=x^*} \tag{5}$$

Definition 1. [20] A C^1 dynamical system defined by

$$\dot{z} = g(z, y), \quad (z \in R^n) \tag{6}$$

is called a **local asymptotic** (respectively, **local exponential**) observer for the nonlinear system (1a)-(1b) if the following two requirements are satisfied:

(O1) If $z(0) = x(0)$, then $z(t) = x(t)$, for all $t \geq 0$.

(O2) There exists a neighbourhood V of the equilibrium $x^* \in R^n$ such that for all $z(0), x(0) \in V$, the estimation error

$$e(t) = z(t) - x(t) \tag{7}$$

decays asymptotically (respectively, exponentially) to zero as $t \to \infty$. ■

Theorem 1. (Sundarapandian, [20]) Suppose that the nonlinear system dynamics (1a) is Lyapunov stable at the equilibrium $x = x^*$ and that there exists a matrix K such that $A - KC$ is Hurwitz. Then the dynamical system defined by

$$\dot{z} = f(z) + K[y - h(z)] \tag{8}$$

is a local exponential observer for the nonlinear system (1a)-(1b). ■

Remark 2. The estimation error is governed by the error dynamics

$$\dot{e} = f(x+e) - f(x) - K[h(x+e) - h(x)] \tag{9}$$

Linearizing the error dynamics (9) at $x = x^*$, we get the linear system

$$\dot{e} = Ee, \qquad \text{where} \quad E = A - KC \tag{10}$$

If (C, A) is observable, then the eigenvalues of the error matrix $E = A - KC$ can be arbitrarily placed in the complex plane. Thus, when (C, A) is observable, a local exponential observer of the form (8) can be always found such that the transient response of the error decays quickly with any desired speed of convergence. ■

3. Lotka-Volterra Two Species Competitive Biology Models

In this section, we consider a Lotka-Volterra two species competitive biology model, which is modeled by the system of differential equations

$$\begin{cases} \dot{x}_1 = r_1 x_1 \left(1 - \dfrac{x_1}{K_1} - b_{12}\dfrac{x_2}{K_1}\right) \\ \dot{x}_2 = r_2 x_2 \left(1 - \dfrac{x_2}{K_2} - b_{21}\dfrac{x_1}{K_2}\right) \end{cases} \tag{11}$$

In (11), $r_1 > 0, r_2 > 0$ are the rate constants for the two competing species with population densities represented by x_1 and x_2 respectively. Also, K_1 and K_2 are the carrying capacities of the two species. Each b_{ij} is a positive parameter measuring the relative impact of one species j individual on the growth of species i, relative to the impact of one species i individual.

The possible outcomes in the two species competing model (11) are the persistence of both species or persistence of one while the other goes to extinction.

The equilibrium points of the system (11) are obtained by solving the system of equations

$$\begin{cases} r_1 x_1 \left(1 - \dfrac{x_1}{K_1} - b_{12}\dfrac{x_2}{K_1}\right) = 0 \\ r_2 x_2 \left(1 - \dfrac{x_2}{K_2} - b_{21}\dfrac{x_1}{K_2}\right) = 0 \end{cases} \tag{12}$$

In this paper, we suppose that the competing species (11) satisfies the following assumption:

$$\boxed{\textbf{(H)}\ \ \frac{1}{K_1} > \frac{b_{21}}{K_2} \ \text{ and } \ \frac{1}{K_2} > \frac{b_{12}}{K_1}}$$

From (H), it follows that

$$b_{12} < \frac{K_1}{K_2} \quad \text{and} \quad b_{21} < \frac{K_2}{K_1} \tag{13}$$

From (13), it is immediate that

$$b_{12}b_{21} < 1 \tag{14}$$

Under the assumption (H), the Lotka-Volterra two species competition population biology system (11) has four equilibria given by

$$E_1(0,0),\ E_2\left(K_1, 0\right),\ E_3\left(0, K_2\right),\ E_4\left(x_1^*, x_2^*\right) \tag{15}$$

where the positive equilibrium E_4 has the coordinates

$$x_1^* = \frac{K_1 - b_{12}K_2}{1 - b_{12}b_{21}},\quad x_2^* = \frac{K_2 - b_{21}K_1}{1 - b_{12}b_{21}} \tag{16}$$

The Jacobian or community matrix corresponding to $E_4(x_1^*, x_2^*)$ is obtained as

$$A = \begin{pmatrix} -\dfrac{r_1 x_1^*}{K_1} & -\dfrac{r_1 b_{12} x_1^*}{K_1} \\ -\dfrac{r_1 b_{21} x_2^*}{K_2} & -\dfrac{r_2 x_2^*}{K_2} \end{pmatrix} \tag{17}$$

Next, we find the characteristic equation of the community matrix A as

$$\lambda^2 - \text{Trace}(A)\lambda + \det(A) = 0 \tag{18}$$

We note that

$$\text{Trace}(A) = -\frac{r_1 x_1^*}{K_1} - \frac{r_2 x_2^*}{K_2} < 0 \tag{19}$$

$$\det(A) = \frac{r_1 r_2}{K_1 K_2} x_1^* x_2^* (1 - b_{12}b_{21}) > 0 \tag{20}$$

Since all the coefficients of the quadratic equation (18) are positive, it is immediate from Hurwitz criterion that all the eigenvalues of the community matrix A are stable.

Thus, A is a Hurwitz matrix.

Thus, from Lyapunov stability theory [23], it is immediate that the positive equilibrium $E_4\left(x_1^*, x_2^*\right)$ is locally asymptotically stable. Hence, we have proved the following theorem.

Theorem 2. Suppose that the Lotka-Volterra two-species competitive biology system (11) satisfies the assumption (H) stated as follows.

$$\boxed{\textbf{(H)}\ \frac{1}{K_1} > \frac{b_{21}}{K_2} \text{ and } \frac{1}{K_2} > \frac{b_{12}}{K_1}}$$

Then $E_4\left(x_1^*, x_2^*\right)$ is a positive equilibrium of the system. Then the unique positive equilibrium $E_4\left(x_1^*, x_2^*\right)$ of the Lotka-Volterra population biology system (11) is locally asymptotically stable. ■

Remark 3. By Theorem 1, it follows that the Lotka-Volterra two-species competitive biology system (11) has stable coexistence of both competing species when assumption (H) is satisfied, *i.e.* when

$$\textbf{(H)}\ \frac{1}{K_1} > \frac{b_{21}}{K_2} \text{ and } \frac{1}{K_2} > \frac{b_{12}}{K_1}$$

Thus, we can that stable coexistence for the two-species competitive model (11) occurs when within-species effects of density-the impact of one individual on the per-capita growth rate of its own species – is larger than the between-species effect. For our study, we assume that the assumption (H) for stable coexistence of the competing species is satisfied. ■

4. Ecological Monitoring for the Lotka-Volterra Two-Species Competitive Biology Systems

In this section, we discuss how to do ecological monitoring of the Lotka-Volterra two-species competitive biology systems by designing a local exponential observer to estimate their states.

4.1 Ecological Monitoring of the Competitive Models with Population Density of Species 1 as Output

We consider the Lotka-Volterra two-species competition biology systems given by

$$\begin{cases} \dot{x}_1 = r_1 x_1 \left(1 - \dfrac{x_1}{K_1} - b_{12} \dfrac{x_2}{K_1} \right) \\ \dot{x}_2 = r_2 x_2 \left(1 - \dfrac{x_2}{K_2} - b_{21} \dfrac{x_1}{K_2} \right) \end{cases} \tag{21}$$

We suppose that the population density of species 1 is given as the system output, *i.e.*

$$y = x_1 \tag{22}$$

We suppose that the assumption (H) holds so that $\left(x_1^*, x_2^*\right)$ is a unique positive equilibrium of the system (21), where

$$x_1^* = \frac{K_1 - b_{12}K_2}{1 - b_{12}b_{21}}, \quad x_2^* = \frac{K_2 - b_{21}K_1}{1 - b_{12}b_{21}} \tag{23}$$

In Section 3, we showed that the community matrix of the system (21) about the unique positive equilibrium $\left(x_1^*, x_2^*\right)$ is given by

$$A = \begin{pmatrix} -\dfrac{r_1 x_1^*}{K_1} & -\dfrac{r_1 b_{12} x_1^*}{K_1} \\ -\dfrac{r_1 b_{21} x_2^*}{K_2} & -\dfrac{r_2 x_2^*}{K_2} \end{pmatrix}, \tag{24}$$

which is a Hurwitz matrix. Thus, the equilibrium $\left(x_1^*, x_2^*\right)$ is locally asymptotically stable.

Moreover, the linearization of the output function (22) about the equilibrium $\left(x_1^*, x_2^*\right)$ is given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{25}$$

Thus, the observability matrix for the system (21)-(22) is given by

$$W = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{r_1 x_1^*}{K_1} & -\dfrac{r_1 b_{12} x_1^*}{K_1} \end{bmatrix} \tag{26}$$

We find that

$$\text{`}\det(W) = -\frac{r_1 b_{12} x_1^*}{K_1} \neq 0 \tag{27}$$

which shows that the observability matrix W has full rank.

Thus, by Kalman's rank test for observability [24], the system (21)-(22) is completely observable.

Hence, by Sundarapandian's theorem (Theorem 1, Section 2), we obtain the following main result, which gives the ecological monitoring of the Lotka-Volterra competitive population biology systems.

Theorem 3. Suppose that the assumption (H) is satisfied. Then the Lotka-Volterra two-species competitive biology system (21) with output (22) has a local exponential observer of the form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} r_1 z_1 \left(1 - \dfrac{z_1}{K_1} - b_{12} \dfrac{z_2}{K_1} \right) \\ r_2 z_2 \left(1 - \dfrac{z_2}{K_2} - b_{21} \dfrac{z_1}{K_2} \right) \end{bmatrix} + K \left[y - z_1 \right] \tag{28}$$

where K is a matrix chosen such that $A - KC$ is Hurwitz. Since (C, A) is observable, an observer gain matrix K can be found such that the error matrix $E = A - KC$ has arbitrarily assigned set of stable eigenvalues. ■

Example 1. We consider a two species Lotka-Volterra two-species competitive biology system given by

$$\begin{cases} \dot{x}_1 = x_1(16 - 2x_1 - x_2) \\ \dot{x}_2 = x_2(12 - x_1 - x_2) \end{cases} \tag{29}$$

with the output function given by the density of competing species 1, *i.e.*

$$y = x_1 \tag{30}$$

We find the positive equilibrium of the system (29) by solving the equations

$$\begin{cases} x_1(16 - 2x_1 - x_2) = 0 \\ x_2(12 - x_1 - x_2) = 0 \end{cases} \tag{31}$$

Since $x_1 \neq 0$ and $x_2 \neq 0$, we obtain

$$\begin{cases} 16 - 2x_1 - x_2 = 0 \\ 12 - x_1 - x_2 = 0 \end{cases} \tag{32}$$

This can be easily arranged in matrix form as

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} \tag{33}$$

By solving the linear system (33), we get the unique positive equilibrium as $\left(x_1^*, x_2^* \right) = (4,\ 8)$.

As shown in Section 3, the Lotka-Volterra population biology system (29) is locally asymptotically stable about the unique positive equilibrium point $\left(x_1^*, x_2^* \right) = (4,\ 8)$.

The linearization of the population biology dynamics (29) at $\left(x_1^*, x_2^* \right) = (4, 8)$ is given by

$$A = \begin{bmatrix} -2x_1^* & -x_1^* \\ -x_2^* & -x_2^* \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ -8 & -8 \end{bmatrix} \tag{34}$$

Also, the linearization of the output function (30) at $\left(x_1^*, x_2^* \right) = (4, 8)$ is given by

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{35}$$

It is easy to check that the observability matrix $W = \begin{bmatrix} C \\ CA \end{bmatrix}$ has full rank. This shows that the given system (29) with output (30) is completely observable near the positive equilibrium point $\left(x_1^*, x_2^* \right) = (4, 8)$.

For numerical simulations, we take $x_1(0) = 3.2$ and $x_2(0) = 5.4$. Figure 1 illustrates that the unique positive equilibrium point $\left(x_1^*, x_2^* \right) = (4,\ 8)$ is locally asymptotically stable.

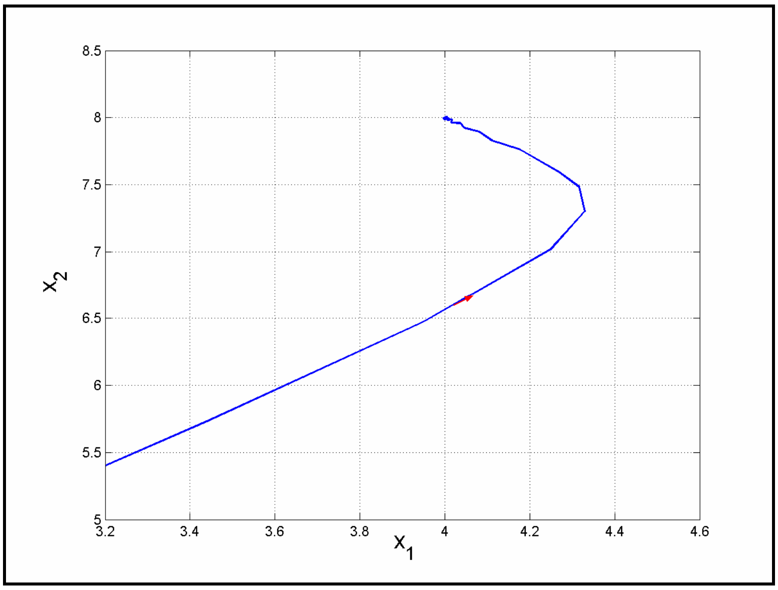

Figure 1. State Orbit of the Lotka-Volterra Competition Biology System (29)

Since (C, A) is observable, the eigenvalues of the error matrix $E = A - KC$ can be placed arbitrarily.

Using the Ackermann's formula [22] for the observer gain matrix, we can choose K so that the error matrix $E = A - KC$ has the stable eigenvalues $\{-8, -8\}$.

A simple calculation using MATLAB gives

$$K = \begin{bmatrix} 0 \\ -8 \end{bmatrix}.$$

By Theorem 3, a local exponential observer for the Lotka-Volterra competitive biology system (29)-(30) around the unique positive equilibrium point $\left(x_1^*, x_2^*\right) = (4, 8)$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_1(16 - 2z_1 - z_2) \\ z_2(12 - z_1 - z_2) \end{bmatrix} + \begin{bmatrix} 0 \\ -8 \end{bmatrix} [y - z_1] \tag{36}$$

For simulations, we choose the initial conditions of the plant dynamics (29) as

$$x_1(0) = 2, \quad x_2(0) = 3 \tag{37}$$

Also, we choose the initial conditions of the observer dynamics (36) as

$$z_1(0) = 10, \quad x_2(0) = 1 \tag{38}$$

Figures 2-3 depict the exponential convergence of the observer states z_1 and z_2 of the system (36) to the states x_1 and x_2 of the Lotka-Volterra competitive biology system (28)-(29).

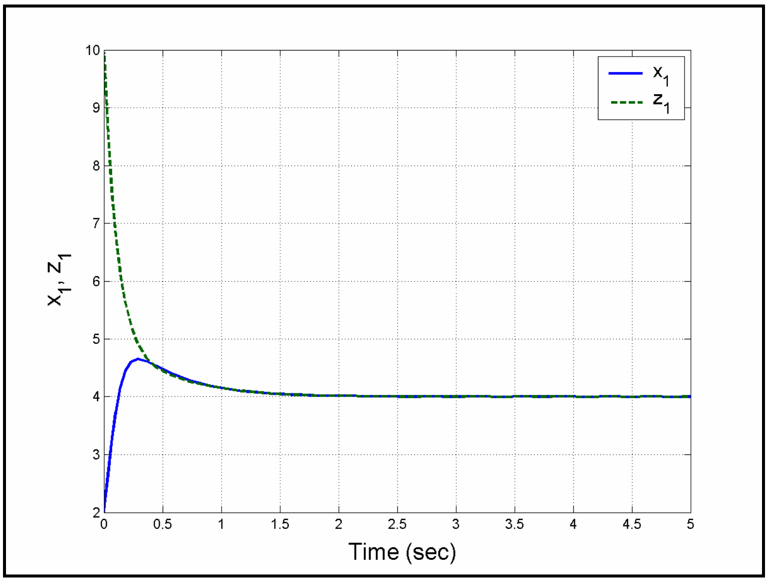

Figure 2. Synchronization of the States x_1 and z_1

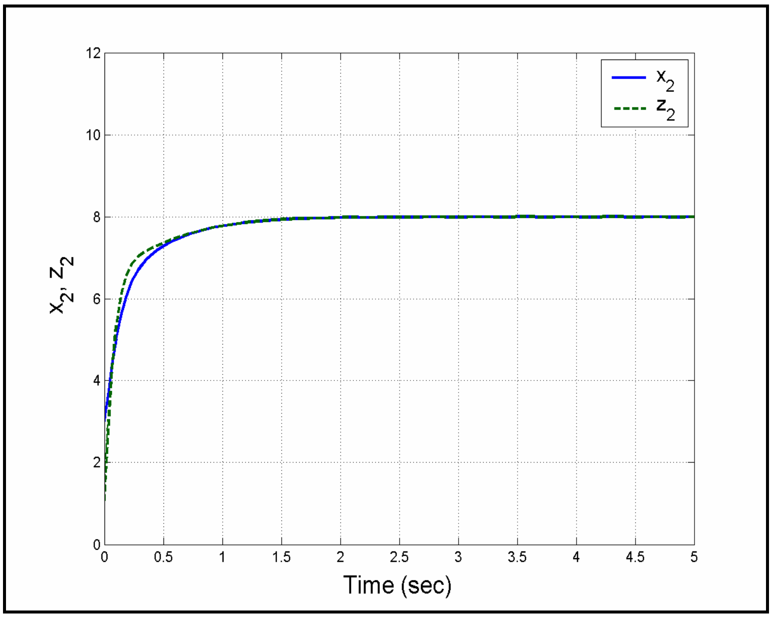

Figure 3. Synchronization of the States x_2 and z_2

4.2 Ecological Monitoring of the Competitive Models with Population Density of Species 2 as Output

Here, we consider the Lotka-Volterra two-species competition biology systems given by

$$\begin{cases} \dot{x}_1 = r_1 x_1 \left(1 - \dfrac{x_1}{K_1} - b_{12} \dfrac{x_2}{K_1} \right) \\ \dot{x}_2 = r_2 x_2 \left(1 - \dfrac{x_2}{K_2} - b_{21} \dfrac{x_1}{K_2} \right) \end{cases} \tag{39}$$

We suppose that the population density of species 2 is given as the system output, *i.e.*

$$y = x_2 \tag{40}$$

We suppose that the assumption (H) holds so that (x_1^*, x_2^*) is a unique positive equilibrium of the system (39), where,

$$x_1^* = \frac{K_1 - b_{12}K_2}{1 - b_{12}b_{21}}, \quad x_2^* = \frac{K_2 - b_{21}K_1}{1 - b_{12}b_{21}} \tag{41}$$

In Section 3, we showed that the community matrix of the system (39) about the unique positive equilibrium (x_1^*, x_2^*) is given by

$$A = \begin{pmatrix} -\frac{r_1 x_1^*}{K_1} & -\frac{r_1 b_{12} x_1^*}{K_1} \\ -\frac{r_1 b_{21} x_2^*}{K_2} & -\frac{r_2 x_2^*}{K_2} \end{pmatrix}, \tag{42}$$

which is a Hurwitz matrix. Thus, the equilibrium (x_1^*, x_2^*) is locally asymptotically stable.

Moreover, the linearization of the output function (40) about the equilibrium (x_1^*, x_2^*) is given by

$$C = [0 \quad 1] \tag{43}$$

Thus, the observability matrix for the system (39)-(40) is given by

$$W = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{r_1 b_{21} x_2^*}{K_2} & -\frac{r_2 x_2^*}{K_2} \end{bmatrix} \tag{44}$$

We find that

$$\text{` } \det(W) = -\frac{r_1 b_{21} x_2^*}{K_2} \neq 0 \tag{45}$$

which shows that the observability matrix W has full rank.

Thus, by Kalman's rank test for observability [22], the system (39)-(40) is completely observable.

Hence, by Sundarapandian's theorem (Theorem 1, Section 2), we obtain the following main result, which gives the ecological monitoring of the Lotka-Volterra competitive population biology systems.

Theorem 4. Suppose that the assumption (H) is satisfied. Then the Lotka-Volterra two-species competitive biology system (39) with output (40) has a local exponential observer of the form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} r_1 z_1 \left(1 - \frac{z_1}{K_1} - b_{12}\frac{z_2}{K_1}\right) \\ r_2 z_2 \left(1 - \frac{z_2}{K_2} - b_{21}\frac{z_1}{K_2}\right) \end{bmatrix} + K\left[y - z_2\right] \tag{46}$$

where K is a matrix chosen such that $A - KC$ is Hurwitz. Since (C, A) is observable, an observer gain matrix K can be found such that the error matrix $E = A - KC$ has arbitrarily assigned set of stable eigenvalues. ■

Example 2. We consider a two species Lotka-Volterra two-species competitive biology system given by

$$\begin{cases} \dot{x}_1 = x_1(18 - 3x_1 - x_2) \\ \dot{x}_2 = x_2(10 - x_1 - x_2) \end{cases} \tag{47}$$

with the output function given by the density of competing species 2, *i.e.*

$$y = x_2 \tag{48}$$

We find the positive equilibrium of the system (47) by solving the equations

$$\begin{cases} x_1(18 - 3x_1 - x_2) = 0 \\ x_2(10 - x_1 - x_2) = 0 \end{cases} \tag{49}$$

Since $x_1 \neq 0$ and $x_2 \neq 0$, we obtain

$$\begin{cases} 18-3x_1-x_2=0 \\ 10-x_1-x_2=0 \end{cases} \tag{50}$$

This can be easily arranged in matrix form as

$$\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 18 \\ 10 \end{bmatrix} \tag{51}$$

By solving the linear system (51), we get the unique positive equilibrium as $\left(x_1^*, x_2^*\right)=(4,\ 6)$.

As shown in Section 3, the Lotka-Volterra population biology system (47) is locally asymptotically stable about the unique positive equilibrium point $\left(x_1^*, x_2^*\right)=(4,\ 6)$.

The linearization of the population biology dynamics (47) at $\left(x_1^*, x_2^*\right)=(4,6)$ is given by

$$A=\begin{bmatrix} -2x_1^* & -x_1^* \\ -x_2^* & -x_2^* \end{bmatrix}=\begin{bmatrix} -8 & -4 \\ -6 & -6 \end{bmatrix} \tag{52}$$

Also, the linearization of the output function (48) at $\left(x_1^*, x_2^*\right)=(4,6)$ is given by

$$C=\begin{bmatrix} 0 & 1 \end{bmatrix} \tag{53}$$

It is easy to check that the observability matrix $W=\begin{bmatrix} C \\ CA \end{bmatrix}$ has full rank. This shows that the given system (47) with output (48) is completely observable near the positive equilibrium point $\left(x_1^*, x_2^*\right)=(4,6)$.

For numerical simulations, we take $x_1(0)=1.5$ and $x_2(0)=2.7$. Figure 4 illustrates that the unique positive equilibrium point $\left(x_1^*, x_2^*\right)=(4,\ 6)$ is locally asymptotically stable.

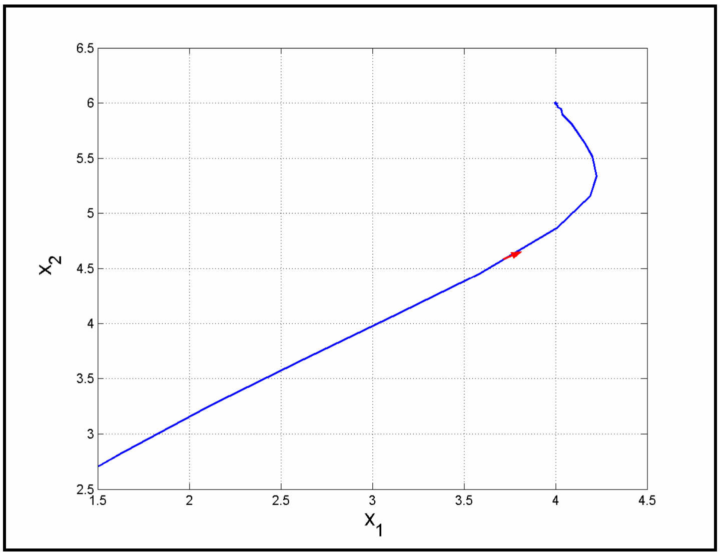

Figure 4. State Orbit of the Lotka-Volterra Competition Biology System (47)

Since (C, A) is observable, the eigenvalues of the error matrix $E=A-KC$ can be placed arbitrarily.

Using the Ackermann's formula [22] for the observer gain matrix, we can choose K so that the error matrix $E=A-KC$ has the stable eigenvalues $\{-8,-8\}$.

A simple calculation using MATLAB gives

$$K = \begin{bmatrix} -4 \\ 2 \end{bmatrix}. \tag{54}$$

By Theorem 4, a local exponential observer for the Lotka-Volterra competitive biology system (47)-(48) around the unique positive equilibrium point $\left(x_1^*, x_2^*\right) = (4,6)$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_1(18 - 3z_1 - z_2) \\ z_2(10 - z_1 - z_2) \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix} \left[y - z_2 \right] \tag{55}$$

For simulations, we choose the initial conditions of the plant dynamics (47) as

$$x_1(0) = 1, \quad x_2(0) = 3 \tag{56}$$

Also, we choose the initial conditions of the observer dynamics (55) as

$$z_1(0) = 7, \quad x_2(0) = 18 \tag{57}$$

Figures 5-6 depict the exponential convergence of the observer states z_1 and z_2 of the system (55) to the states x_1 and x_2 of the Lotka-Volterra competitive biology system (47)-(48).

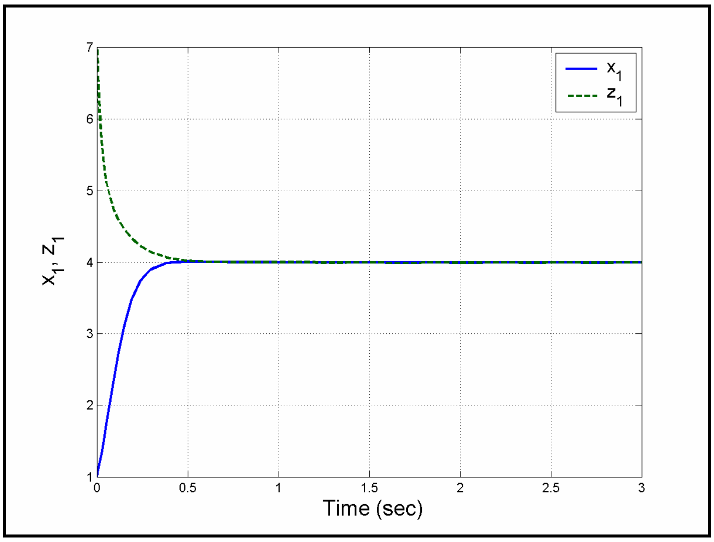

Figure 5. Synchronization of the states x_1 and z_1

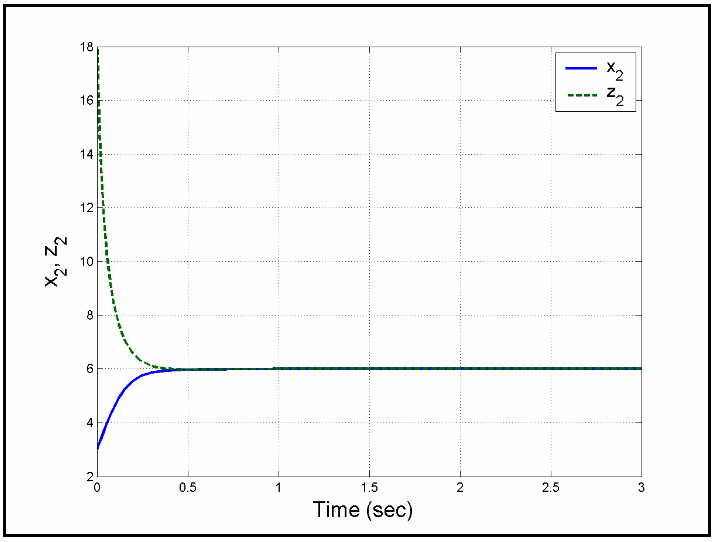

Figure 6. Synchronization of the States x_2 and z_2

5. Conclusions

In this paper, we described Lotka-Volterra two-species competitive biology models. We showed that for this biological model, under an assumption, the two competing species have stable coexistence. Then we achieved ecological monitoring of the population biology model by constructing a nonlinear exponential observer for the competitive biology model under study. The nonlinear observer design for the competitive biology model was constructed by applying Sundarapandian's theorem (2002) and using only the dynamics of the Lotka-Volterra population biology model and any of the density of the two competing species as the output function. Numerical example and MATLAB simulations were shown to illustrate the ecological monitoring or the nonlinear observer design for the two-species Lotka-Volterra competitive biology models.

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