International Journal of ChemTech Research
CODEN (USA): IJCRGG ISSN: 0974-4290
Vol.8, No.9 pp 406-417, 2015

Global Chaos Control of Mathieu-Van der Pol System via Adaptive Control Method

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we describe the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system (2009). We also derive new results for the global chaos control of the Mathieu-Van der Pol chaotic system with unknown parameters. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos control of the Mathieu-Van der Pol chaotic system with unknown system parameters via adaptive control method.
Keywords: Chaos, chaotic systems, chaos control, Mathieu equation, Van der Pol equation, adaptive control.

1. Introduction

A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2]. Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [11, 12], Vaidyanathan systems [13-43], Pehlivan system [44], Pham system [45], etc.

Recently, there is significant result in the chaos literature in the control, regulation and synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to device a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. In the control and regulation of chaotic systems, state feedback control laws are devised so as to regulate the state trajectories of the system to track the reference input signals.

In control theory, active control method is used when the parameters are available for measurement [46-65]. Adaptive control is a popular control technique used for stabilizing systems when the system parameters are unknown [66-80]. There are also other popular methods available for control and synchronization of systems such as backstepping control method [81-87], sliding mode control method [88-100], etc.

Recently, chaos theory is found to have important applications in several areas such as chemistry [101-

114], biology [115-138], memristors [129-141], electrical circuits [142], etc.

Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we describe the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system obtained by Zheng-Ming Ge and Shih-Yu Li [143].

We also derive new results for the global chaos control of the Mathieu-Van der Pol chaotic system with unknown parameters. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos control of the Mathieu-Van der Pol chaotic system with unknown system parameters via adaptive control method.

2. Mathieu-Van der Pol Chaotic System

Mathieu equation and Van der Pol equation are famous non-autonomous systems described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\left[a + b\sin(\omega t)\right]x_1 - \left[a + b\sin(\omega t)\right]x_1^3 - cx_2 + d\sin(\omega t) \end{cases} \quad (1)$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + q\sin(\omega t) \end{cases} \quad (2)$$

Exchanging $\sin(\omega t)$ in Eq. (1) with x_3 and $\sin(\omega t)$ in Eq. (2) with x_1, Zheng-Ming Ge and Shih-Yu Li obtained the Mathieu-Van der Pol system [143], which is a 4-D autonomous system described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -(a + bx_3)x_1 - (a + bx_3)x_1^3 - cx_2 + dx_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + qx_1 \end{cases} \quad (3)$$

In Eq. (3), x_1, x_2, x_3, x_4 are the states and a, b, c, d, p, q are constant, positive, parameters.

In [130], it was shown that the Mathieu-Van der Pol system (3) is *chaotic* when the system parameters are chosen as

$$a = 10, \ b = 3, \ c = 0.4, \ d = 70, \ p = 5, \ q = 0.1 \quad (4)$$

For numerical simulations, we take the initial conditions

$$x_1(0) = 1.5, \ x_2(0) = 1.5, \ x_3(0) = 1.5, \ x_4(0) = 1.5 \quad (5)$$

Figures 1-4 show the 3-D projections of the Mathieu-Van der Pol 4-D chaotic system (3) on the (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

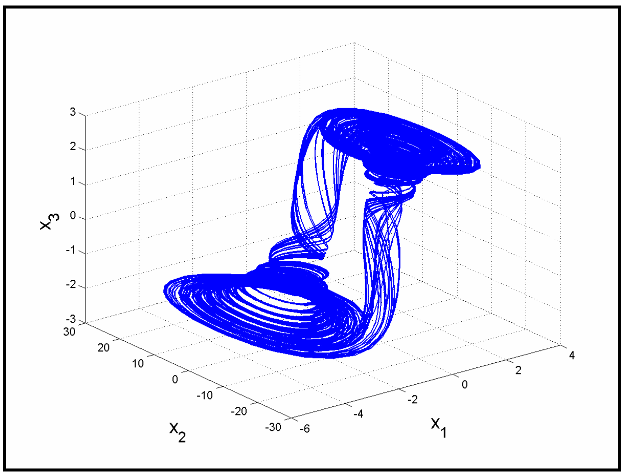

Figure 1. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_2, x_3) space

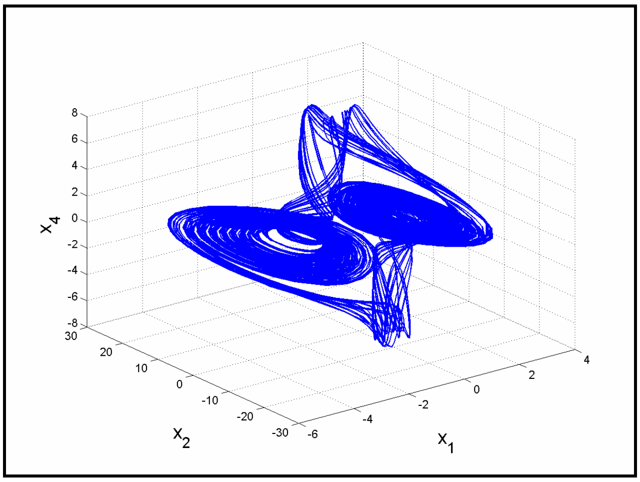

Figure 2. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_2, x_4) **space**

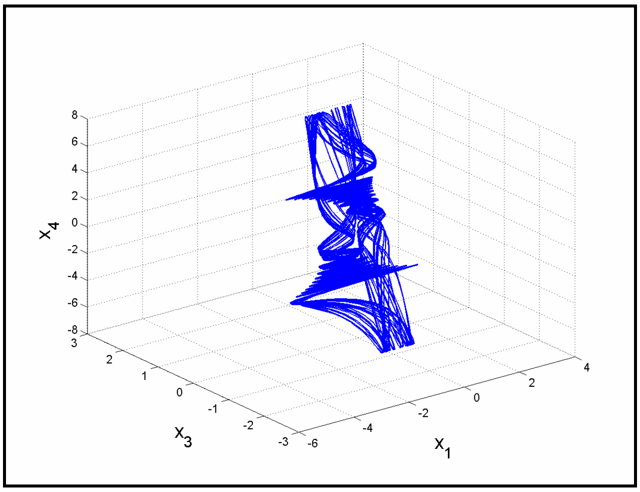

Figure 3. The 3-D projection of the Mathieu-Van der Pol system on the (x_1, x_3, x_4) **space**

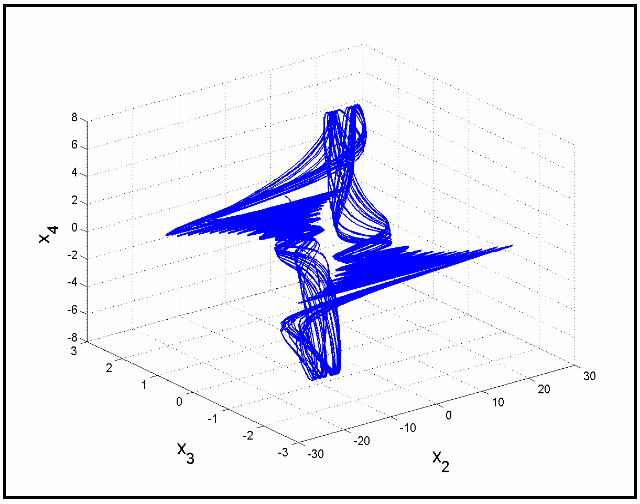

Figure 4. The 3-D projection of the Mathieu-Van der Pol system on the (x_2, x_3, x_4) **space**

The Lyapunov exponents of the Mathieu-Van der Pol system (3) are numerically found as

$$L_1 = 0.05225, \quad L_2 \quad 0, \quad L_3 \quad -0.49938, \quad L_4 \quad -7.27089 \tag{6}$$

From the LE spectrum (6), it is immediate that the Mathieu-Van der Pol system (3) is a chaotic system and the

Maximal Lyapunov Exponent (MLE) of the Mathieu-Van der Pol system (1) is $L_1 = 0.05225$.

Since the sum of the Lyapunov exponents in (6) is negative, it follows that the Mathieu-Van der Pol system (3) is dissipative.

Also, the Lyapunov dimension of the Mathieu-Van der Pol system (3) is derived as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1046 \tag{7}$$

3. Global Chaos Control of the Mathieu-Van der Pol System via Adaptive Control

In this section, we use adaptive control method to achieve global chaos control of the Mathieu-Van der Pol chaotic system with unknown parameters. We use Lyapunov stability theory [144] to prove the main adaptive control result derived in this section using estimates of the unknown system parameters.

Thus, we consider the Mathieu-Van der Pol system with controls given by

$$\begin{cases} \dot{x}_1 = x_2 + u_1 \\ \dot{x}_2 = -(a + bx_3)(x_1 + x_1^3) - cx_2 + dx_3 + u_2 \\ \dot{x}_3 = x_4 + u_3 \\ \dot{x}_4 = -x_3 + p(1 - x_3^2)x_4 + qx_1 + u_4 \end{cases} \tag{8}$$

In (8), x_1, x_2, x_3, x_4 are the states of the Mathieu-Van der Pol system, a, b, c, d, p, q are unknown parameters and u_1, u_2, u_3, u_4 are adaptive feedback controls.

Now, we consider the adaptive controller defined by

$$\begin{cases} u_1 = -x_2 - k_1 x_1 \\ u_2 = [\hat{a}(t) + \hat{b}(t)x_3](x_1 + x_1^3) + \hat{c}(t)x_2 - \hat{d}(t)x_3 - k_2 x_2 \\ u_3 = -x_4 - k_3 x_3 \\ u_4 = x_3 - \hat{p}(t)(1 - x_3^2)x_4 - \hat{q}(t)x_1 - k_4 x_4 \end{cases} \tag{9}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (9) into (8), we get the closed-loop system dynamics as

$$\begin{cases} \dot{x}_1 = -k_1 x_1 \\ \dot{x}_2 = -[a - \hat{a}(t)](x_1 + x_1^3) - [b - \hat{b}(t)]x_3(x_1 + x_1^3) - [c - \hat{c}(t)]x_2 + [d - \hat{d}(t)]x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \\ \dot{x}_4 = [p - \hat{p}(t)](1 - x_3^2)x_4 + [q - \hat{q}(t)]x_1 - k_4 x_4 \end{cases} \tag{10}$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_d(t) = d - \hat{d}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{11}$$

Using (11), the closed-loop system (10) can be simplified as

$$\begin{cases} \dot{x}_1 = -k_1 x_1 \\ \dot{x}_2 = -e_a(x_1 + x_1^3) - e_b x_3 (x_1 + x_1^3) - e_c x_2 + e_d x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \\ \dot{x}_4 = e_p (1 - x_3^2) x_4 + e_q x_1 - k_4 x_4 \end{cases} \tag{12}$$

Differentiating (11) with respect to time, we get

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_d(t) = -\dot{\hat{d}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{13}$$

Next, we consider the candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b, e_c, e_d, e_p, e_q) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_p^2 + e_q^2\right) \tag{14}$$

Differentiating (14) along the trajectories of (12) and (13), we get the following dynamics

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[-x_2(x_1 + x_1^3) - \dot{\hat{a}}\right] + e_b\left[-x_2 x_3 (x_1 + x_1^3) - \dot{\hat{b}}\right] \\ &+ e_c\left[-x_2^2 - \dot{\hat{c}}\right] + e_d\left[x_2 x_3 - \dot{\hat{d}}\right] + e_p\left[(1 - x_3^2) x_4^2 - \dot{\hat{p}}\right] + e_q\left[x_1 x_4 - \dot{\hat{q}}\right] \end{aligned} \tag{15}$$

In view of (15), we take the following parameter update law:

$$\begin{cases} \dot{\hat{a}}(t) = -x_2\left(x_1 + x_1^3\right) \\ \dot{\hat{b}}(t) = -x_2 x_3\left(x_1 + x_1^3\right) \\ \dot{\hat{c}}(t) = -x_2^2 \\ \dot{\hat{d}}(t) = x_2 x_3 \\ \dot{\hat{p}}(t) = (1 - x_3^2) x_4^2 \\ \dot{\hat{q}}(t) = x_1 x_4 \end{cases} \tag{16}$$

Next, we state and prove the main result of this section.

Theorem 1. The adaptive control law (9) and the parameter update law (16) achieve global and exponential stabilization of the 4-D Mathieu-Van der Pol chaotic system (8), where k_1, k_2, k_3, k_4 are positive gain constants.

Proof. The result is proved using Lyapunov stability theory [144].

The quadratic Lyapunov function V defined by (14) is positive definite on R^{10}.

Substituting the parameter update law (16) into (15), we get the time derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{17}$$

which is negative semi-definite on R^{10}.

Thus, by Barbalat's lemma in Lyapunov stability theory [144], it follows that the closed-loop system dynamics (12) is globally exponentially stable.

This completes the proof. ■

4. Numerical Simulations

We use the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ to solve the system of differential equations (8), when the adaptive control law (9) and parameter update law (16) are implemented.

We take the parameter values of the Mathieu-Van der Pol chaotic system (8) as in the chaotic case, viz.

$$a = 10, \; b = 3, \; c = 0.4, \; d = 70, \; p = 5, \; q = 0.1 \quad (18)$$

We take the gain constants as

$$k_1 = 6, \; k_2 = 6, \; k_3 = 6, \; k_4 = 6 \quad (19)$$

We take the initial values of the Mathieu-Van der Pol system (8) as

$$x_1(0) = 3.2, \; x_2(0) = 12.1, \; x_3(0) = 7.8, \; x_4(0) = 10.5 \quad (20)$$

We take the initial values of the parameter estimates as

$$\hat{a}(0) = 7, \; \hat{b}(0) = 12, \; \hat{c}(0) = 4, \; \hat{d}(0) = 3, \; \hat{p}(0) = 12, \; \hat{q}(0) = 6 \quad (21)$$

Figure 5 shows the time-history of the controlled states x_1, x_2, x_3, x_4.

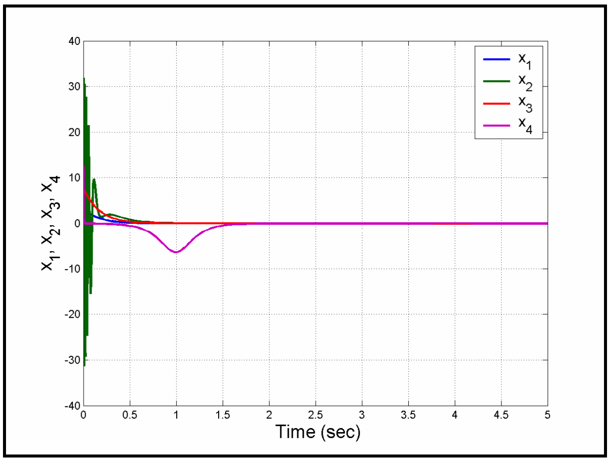

Figure 5. Time-history of the controlled states x_1, x_2, x_3, x_4

5. Conclusions

Mathieu equation and Van der Pol equation are two typical nonlinear non-autonomous systems. Recently, Zheng-Ming Ge and Shih-Yu Li (2009) combined Mathieu equation and Van der Pol equation to obtain a 4-D autonomous chaotic system. In this paper, we described the dynamic equations and qualitative properties of the Mathieu-Van der Pol chaotic system (2009). We also derived new results for the global chaos control of the Mathieu-Van der Pol chaotic system with unknown parameters. MATLAB plots have been depicted to illustrate the phase portraits of the Mathieu-Van der Pol chaotic system (2009) and the global chaos control of the Mathieu-Van der Pol chaotic system with unknown system parameters via adaptive control method.

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