Adaptive Synchronization of Chemical Chaotic Reactors

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Abstract: Chaos theory has a manifold variety of applications in science and engineering. This paper investigates the properties of a chemical chaotic attractor discovered by Huang (2005). This paper gives a summary description of the chemical reactor dynamics and the chaos dynamic analysis. Next, an adaptive synchronizer is designed using control theory for the global chaos synchronization of identical chemical chaotic attractors with unknown parameters. The main results for adaptive synchronization of chemical reactors are established using Lyapunov stability theory. MATLAB plots have been shown to illustrate the phase portraits of the chemical chaotic attractor and the adaptive synchronization of identical chemical chaotic attractors.

Keywords: Chaos, chaotic systems, chemical reactor, adaptive synchronization, stability.

Introduction

Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1-2].

Lorenz [3] discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade Rössler [4] discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of many 3-D chaotic systems such as Arneodo system [5], Sprott systems [6], Chen system [7], Lü-Chen system [8], Cai system [9], Tigan system [10], etc. Many new chaotic systems have been also discovered in the recent years such as Sundarapandian systems [11, 12], Vaidyanathan systems [13-20], Pehlivan system [21], Jafari system [22], Pham system [23], etc.

Chaos and control theory have a manifold variety of applications in many fields of science and engineering such as oscillators [24], lasers [25-26], biology [27], chemical reactions [28-30], neural networks [31-32], robotics [33-34], electrical circuits [35-36], etc.

Recently, there is significant result in the chaos literature in the synchronization of physical and chemical systems. A pair of systems called master and slave systems are considered for the synchronization process and the design goal is to device a feedback mechanism so that the trajectories of the slave system asymptotically track the trajectories of the master system. Various methods have been designed for the synchronization of chaotic systems such as active control [37-45], adaptive control [46-60], sliding mode control [61-68], backstepping control [69-73], etc.

This paper investigates the analysis and adaptive synchronization of the chemical chaotic reactor model discovered by Huang in 2005 [74]. Huang derived the chemical reactor model by considering reactor dynamics with five reversible steps. This paper also derives new results of adaptive synchronizer design for the identical chemical chaotic attractors using Lyapunov stability theory [75] and MATLAB plots are shown to illustrate the main results.
Chemical Chaotic Reactor

The well-stirred chemical reactor dynamics [74] consist of the following five reversible steps given below.

\[
A_1 + X \xrightarrow{k_1} 2X, \quad X + Y \xrightarrow{k_2} 2Y, \quad A_3 + Y \xrightarrow{k_3} A_2, \quad X + Z \xrightarrow{k_4} A_3, \quad A_3 + Z \xrightarrow{k_5} 2Z
\]

In (1), \(A_1, A_4, A_2\) are initiators and \(A_2, A_3\) are products. The intermediates whose dynamics are followed are \(X, Y\) and \(Z\). The corresponding non-dimensionalized dynamical evolution equations read as

\[
\begin{align*}
\dot{x} &= (a_1 - k_{-1}x - y - z)x \\
\dot{y} &= (x - a_2)y \\
\dot{z} &= (a_4 - x - k_{-2}z)z
\end{align*}
\]

In (2), \(x, y, z\) are positive mole functions and \(a_1, a_2, a_4, k_{-1}, k_{-2}\) are positive parameters.

To simplify the notations, we rename the constants and express the system (2) as

\[
\begin{align*}
\dot{x} &= (a - px - y - z)x \\
\dot{y} &= (x - c)y \\
\dot{z} &= (b - x - qz)z
\end{align*}
\]

The system (3) is chaotic when the system parameters are chosen as

\[
a = 30, \quad b = 16.5, \quad c = 10, \quad p = 0.5, \quad q = 0.5
\]

For numerical simulations, we take the initial conditions

\[
x(0) = 1.8, \quad y(0) = 2.5, \quad z(0) = 0.6
\]

The 3-D phase portrait of the chemical chaotic reactor is depicted in Fig. 1.

The 2-D projections of the chemical chaotic reactor on the \((x, y)\), \((y, z)\) and \((x, z)\) planes are depicted in Figs. 2-4.
Computational Analysis of the Chemical Chaotic Attractor

The Lyapunov exponents of the chemical chaotic attractor (3) have been obtained in MATLAB as

\[ L_1 = 0.4001, \ L_2 = 0, \ L_3 = -11.8762. \] (6)

Thus, the Lyapunov dimension of the chemical chaotic attractor (3) is deduced as

\[ D_L = 2 + \frac{L_1^2 + L_2^2}{L_3^2} = 2.0337 \] (7)

The chemical chaotic attractor has an equilibrium at \((x, y, z) = (0, 0, 0)\).

The eigenvalues of the linearized system matrix of the attractor (3) at the origin are:

\[ \lambda_1 = 16.5, \ \lambda_2 = 30, \ \lambda_3 = -10. \] (8)

Since there are two positive eigenvalues in the set (7), the origin is an unstable equilibrium of the chemical chaotic attractor (3).

Adaptive Control Design of the Chemical Chaotic Attractor

In this section, we use adaptive control method to design an adaptive feedback synchronizer for globally synchronizing the trajectories of identical chemical chaotic reactors with unknown parameters.

Thus, we consider the master system as the chemical chaotic attractor given by the dynamics

\[
\begin{align*}
\dot{x}_1 &= (a - px_1 - y_1 - z_1)x_1 \\
\dot{y}_1 &= (x_1 - c)y_1 \\
\dot{z}_1 &= (b - x_1 - qz_1)z_1
\end{align*}
\] (9)

In (9), \(x_1, y_1, z_1\) are the states of the master system.

Also, we consider the slave system as the chemical chaotic attractor given by the dynamics

\[
\begin{align*}
\dot{x}_2 &= (a - px_2 - y_2 - z_2)x_2 + u_x \\
\dot{y}_2 &= (x_2 - c)y_2 + u_y \\
\dot{z}_2 &= (b - x_2 - qz_2)z_2 + u_z
\end{align*}
\] (10)

Figure 3. The 2-D projection of the chemical chaotic attractor on the \((y, z)\) plane

Figure 4. The 2-D projection of the chemical chaotic attractor on the \((x, z)\) plane
In (10), \(x_1, y_1, z_1\) are the states of the slave system and \(u_x, u_y, u_z\) are adaptive controls to be determined using estimates \(\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t), \hat{\epsilon}(t)\) of the unknown parameters \(\alpha, \beta, \gamma, \delta, \epsilon\), respectively.

The global chaos synchronization error is defined by

\[
\begin{align*}
\epsilon_x &= x_2 - x_3 \\
\epsilon_y &= y_2 - y_1 \\
\epsilon_z &= z_2 - z_1
\end{align*}
\]  

(11)

The error dynamics is obtained as

\[
\begin{align*}
\dot{\epsilon}_x &= a\epsilon_x - p(x_2^2 - x_1^2) - x_2y_1 - x_2z_1 + x_1y_1 + x_1z_1 + u_x \\
\dot{\epsilon}_y &= -c\epsilon_y + x_1y_2 - x_1y_1 + u_y \\
\dot{\epsilon}_z &= b\epsilon_z - q(x_2^2 - z_2^2) - x_2z_2 + x_1z_1 + u_z
\end{align*}
\]  

(12)

We consider the adaptive control law defined by

\[
\begin{align*}
\dot{u}_x &= -\hat{\alpha}(t)\epsilon_x + \hat{\beta}(t)(x_1^2 - x_2^2) + x_2y_1 + x_2z_1 - x_1y_1 - x_1z_1 - k_x\epsilon_x \\
\dot{u}_y &= -\hat{\gamma}(t)\epsilon_y - x_2y_2 + x_1y_1 - k_y\epsilon_y \\
\dot{u}_z &= -\hat{\delta}(t)\epsilon_z - q(\epsilon_2^2 - \epsilon_1^2) + x_2z_2 - x_1z_1 - k_z\epsilon_z
\end{align*}
\]  

(13)

In (13), \(\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t), \hat{\delta}(t), \hat{\epsilon}(t)\) are estimates of the unknown parameters \(\alpha, \beta, \gamma, \delta, \epsilon\), respectively, and \(k_x, k_y, k_z\) are positive gain constants.

Substituting (13) into (12), we obtain the closed-loop error dynamical system

\[
\begin{align*}
\dot{\epsilon}_x &= \left[a - \hat{\alpha}(t)\right]\epsilon_x - \left[p - \hat{\beta}(t)\right](x_1^2 - x_2^2) - k_x\epsilon_x \\
\dot{\epsilon}_y &= -\left[c - \hat{\gamma}(t)\right]\epsilon_y - k_y\epsilon_y \\
\dot{\epsilon}_z &= \left[b - \hat{\delta}(t)\right]\epsilon_z - \left[q - \hat{\epsilon}(t)\right](z_1^2 - z_2^2) - k_z\epsilon_z
\end{align*}
\]  

(14)

Now, we define the parameter estimation errors as

\[
\begin{align*}
\hat{\epsilon}_a &= a - \hat{\alpha}(t) \\
\hat{\epsilon}_b &= b - \hat{\beta}(t) \\
\hat{\epsilon}_c &= c - \hat{\gamma}(t) \\
\hat{\epsilon}_p &= p - \hat{\beta}(t) \\
\hat{\epsilon}_q &= q - \hat{\epsilon}(t)
\end{align*}
\]  

(15)

Using (15), we can simplify the error dynamics (14) as

\[
\begin{align*}
\dot{\hat{\epsilon}}_a &= -\hat{\alpha}(t) \\
\dot{\hat{\epsilon}}_b &= -\hat{\beta}(t) \\
\dot{\hat{\epsilon}}_c &= -\hat{\gamma}(t) \\
\dot{\hat{\epsilon}}_p &= -\hat{\delta}(t) \\
\dot{\hat{\epsilon}}_q &= -\hat{\epsilon}(t)
\end{align*}
\]  

(16)

Differentiating (16) with respect to \(t\), we get

\[
\begin{align*}
\hat{\dot{\epsilon}}_a &= -\hat{\dot{\alpha}}(t) \\
\hat{\dot{\epsilon}}_b &= -\hat{\dot{\beta}}(t) \\
\hat{\dot{\epsilon}}_c &= -\hat{\dot{\gamma}}(t) \\
\hat{\dot{\epsilon}}_p &= -\hat{\dot{\delta}}(t) \\
\hat{\dot{\epsilon}}_q &= -\hat{\dot{\epsilon}}(t)
\end{align*}
\]  

(17)

We consider the quadratic Lyapunov function defined by

\[
V = \frac{1}{2}(\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 + \epsilon_p^2 + \epsilon_q^2 + \epsilon_a^2 + \epsilon_b^2 + \epsilon_c^2 + \epsilon_d^2)
\]  

(18)

Clearly, \(V\) is a positive definite function on \(\mathbb{R}^8\).

Differentiating \(V\) along the trajectories of (16) and (17), we obtain

\[
\begin{align*}
\dot{V} &= -k_x\epsilon_x^2 - k_x\epsilon_x^2 - k_x\epsilon_x^2 + e_a[\epsilon_x^2 - \hat{\alpha}(t)] + e_b[\epsilon_y^2 - \hat{\beta}(t)] + e_c[\epsilon_z^2 - \hat{\gamma}(t)] + e_p[\epsilon_p^2 - \hat{\delta}(t)] + e_q[\epsilon_q^2 - \hat{\epsilon}(t)] \\
&\quad + e_a[\epsilon_x^2 - \hat{\epsilon}(t)] + e_b[\epsilon_y^2 - \hat{\epsilon}(t)] + e_c[\epsilon_z^2 - \hat{\epsilon}(t)] + e_p[\epsilon_p^2 - \hat{\epsilon}(t)] + e_q[\epsilon_q^2 - \hat{\epsilon}(t)]
\end{align*}
\]  

(19)

In view of (19), we take the parameter update law as follows.
The identical chemical chaotic attractors (9) and (10) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (13) and the parameter update law (20), where \(k_x, k_y, k_z\) are positive gain constants.

**Proof.** We prove this result by Lyapunov stability theory.

We consider the quadratic Lyapunov function \(V\) defined in (18), which is positive definite on \(R^8\).

Substituting the parameter update law (20) into (19), we obtain

\[
\dot{V} = -k_x x^2 - k_y y^2 - k_z z^2 \tag{21}
\]

By (21), it follows that \(\dot{V}\) is a negative semi-definite function on \(R^8\).

By Barbalat’s lemma in Lyapunov stability theory [75], it follows that the states \(x(t), y(t), z(t)\) exponentially converge to zero as \(t \to \infty\) for all initial conditions.

Hence, identical chemical chaotic attractors (9) and (10) are globally and exponentially synchronized for all initial conditions by the adaptive control law (13) and the parameter update law (20).

This completes the proof. \(\blacksquare\)

**Numerical Simulations**

We use classical fourth-order Runge-Kutta method in MATLAB with step-size \(h = 10^{-8}\) for solving the systems of differential equations given by (9) and (10), when the adaptive control law (13) is applied.

We take the gain constants as \(k_x = 6, k_y = 6, k_z = 6\).

We take the initial conditions of the chemical reactor (9) as \(x_1(0) = 2.3, y_1(0) = 1.7, z_1(0) = 0.8\).

We take the initial conditions of the chemical reactor (10) as \(x_2(0) = 0.7, y_2(0) = 2.9, z_2(0) = 2.5\).

The parameter values of the chemical reactor are taken as in the chaotic case, viz. \(a = 30, b = 16.5, c = 10, p = 0.5, q = 0.5\).

Also, we take \(\dot{a}(0) = 3.1, \dot{b}(0) = 2.3, \dot{c}(0) = 5.1, \dot{p}(0) = 3.6, \dot{q}(0) = 2.8\).

Figs. 5-7 show the complete chaos synchronization of the chemical chaotic reactors (9) and (10). Fig. 8 shows the time-history of the chaos synchronization errors \(e_{x_1}, e_{y_1}, e_{z_1}\).

![Figure 5. Complete synchronization of the states \(x_1(t), x_2(t)\)](image)
Figure 6. Complete synchronization of the states $y_1(t), y_2(t)$

Figure 7. Complete synchronization of the states $z_1(t), z_2(t)$

Figure 8. Time-history of the chaos synchronization errors $e_x(t), e_y(t), e_z(t)$

Conclusions

In this paper, new results have been derived for the analysis and adaptive synchronization of a chemical chaotic attractor discovered by Haung (2005). First, the paper discussed the qualitative properties, Lyapunov exponents, stability of equilibrium point at the origin and phase portraits of the chemical chaotic attractor discovered by Haung. Then this paper derived new results for the adaptive synchronizer for the global chaos synchronization of the states of the identical chemical chaotic reactors. The main results have been proved using Lyapunov stability theory and numerical simulations have been illustrated using MATLAB.
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