



Statistical study using multiple regression model in reverse osmosis

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Abstract: The present study discusses about the influence of operational parameters on response in reverse osmosis membrane separation process. The performance in the design of reverse osmosis depends on various operating parameters like feed pressure, feed flow rate, feed water quality etc. Regression analysis can be applied for correlating independent input variables and the dependent variable response. The most frequently applied statistical methods in regression analysis is multiple regression if the factor for analysis is more than one and significance of the model was justified by analysis of variance (ANOVA). It was found at optimum feed pressure 9 kg/cm², improvement of permeate flux was seen. Salt rejection depends on feed TDS and permeate TDS. Salt rejection was found to be 97.3%. Water recovery ratio was around 67% for feed pressure 11 kg/cm². Multiple regression model was performed for the input parameters like feed pressure, feed flow rate on the response water recovery ratio. Residual error shows deviation of 1% of predicted value with the observed experimental response. Experimental data was fed to statistical tool MINITAB release 17, developed by Minitab Inc., USA. Multiple regression model equation was validated by ANOVA estimates. ANOVA estimates the model to be highly significant confirming by R²= 0.98, for regression feed pressure coefficient- 9.56, P-Value- 0.013, F-Value-10.11, which has highest influence on water recovery ratio.

Keywords: Reverse osmosis, desalination, multiple regression model, response surface, optimization.

1. Introduction

Globally water demand for fresh water is drastically increasing due to global warming, climatic changes, population growth, urbanization and industrial growth. Because of the growing demand for potable water, development of sustainable water purification technology plays an indispensable role in the current scenario of nexus of water and energy. The effective solution for potable water crisis globally is desalination of water from the major water resources which are not consumable. Conventional distillation like multi effect distillation (MED), multi-stage flash evaporation (MSF) and vapour compression (VC) are the traditional technologies used for desalination of sea water and brackish water. Conventional distillation methods are energy intensive process. Desalination of water can also be achieved by membrane separation technology. Membrane separation process are classified as isothermal and non isothermal systems. Membrane distillation is an example for non-isothermal process. In isothermal membrane system transmembrane hydrostatic pressure, concentration, electric or chemical potential are driving forces. Eg.: Reverse osmosis, electrodialysis, microfiltration, ultrafiltration and nanofiltration. Reverse osmosis is defined as the process by which hydraulic

pressure is exerted, greater than the sum of the osmotic pressure difference and the pressure loss of diffusion through the membrane can cause water to diffuse in the opposite direction into more concentrated solution. If greater pressure is applied, then more rapid will be the diffusion. Energy consumption for sea water reverse osmosis (SWRO) plant is around 3- 4 kwhr/m³ for 50% recovery. Brackish groundwater has much lower osmotic pressure than seawater, therefore its desalination requires much less energy¹. The efficiency of reverse osmosis rely on properties of feed water, operating parameters and the membrane. Spiral-wound and thin film cellulose polyamide are commercial membranes in reverse osmosis². The performance of reverse osmosis design was assessed by predicting the product flow rate and salt rejection correlation. In the whole design and control process, statistical model uses regression analysis for feed water conditions like feed flow rate, TDS, pH correlating with permeate TDS, recovery ratio. The polynomial expression arrived out of regression assessment used for sensitivity analysis and design of the system.³ Decline of permeate flux in the sea water desalination experiments was due to change in osmotic pressure and membrane compaction.⁴ Multiple regression is a powerful and versatile method applicable to situations in which research goal is to explain or predict a single dependent on the basis of multiple independent variables.⁵ Regression model replaced the hydrodynamic model in optimizing the design of sea water RO brine outfalls. Regression model was developed to relate input and output parameters of the simulation model. Design study minimizes the cost and requirements for environmental constraints.⁶

1.1. Reverse Osmosis

RO membrane should have the following characteristics^{1, 7},

- They should be hydrophilic and have high water flux i.e highly permeable to water and less susceptible to fouling.
- Membrane should be easily manufactured with good salt rejection i.e. slightly permeable to salt.
- The membrane should be chemically, physically and thermally stable in saline waters.

Advantages

- Higher productivity of water and high water efficiency.
- Highly commercialised for large scale desalination plant.

Disadvantages

- Higher power requirement of pressure pump.
- Membrane fouling is high because of high pressure liquid entry and requires membrane replacement once in 2 years .
- Pretreatment of water is must for free of pathogens and suspended particles for feed water.
- More space for auxiliary requirement necessary and requires higher maintenance.
- Brine disposal causes environmental pollution.

2. Experimental set up

Reverse osmosis membrane model used for this work is BW30-365 purchased from Lenn Tech membranes. Reverse osmosis set up was designed by Crystal water technologies, Chennai. Photographs of Reverse osmosis is shown in figure.1. Reverse osmosis set up for brackish water is automated with proper instrumentation like product flow indicator, reject flow indicator, feed pressure, membrane pressure, total dissolved solids (TDS) indicator etc. RO membrane specifications is mentioned in the below table.1

Table: 1 Reverse osmosis membrane specifications

Membrane model	BW30-365 (Thin film composite membrane)
Membrane Specifications	
Membrane size	8inch
Length	40 inch.
Active surface area	34 m ²
Permeate flow rate	9500gpd.
Maximum feed flow rate	3.2m ³ /hr
Pressure drop	1 bar.
Number of RO elements used	2

2.1. Experimental method:

In reverse osmosis, desalination of well water sample was carried out. The feed water was pretreated in filter media that contains sand filter, graded pebbles and fine sand and then passed to the next filter media which contains carbon filter, activated carbon IV 900 grade to remove suspended solids, colour present in the feed water source, then it moves to micro filter cartridges, dosing tank and finally using high pressure pump sent to the hydrophilic thin composite cellulose polyamide membrane where distilled water received at the permeate tank. In the process of reverse osmosis, the feed well water is fed continuously in a single pass mode and after feed water pretreatment it passes to the hydrophilic membrane and the product water was collected for different time intervals at different operating pressure and feed flow rate for the calculation of flux performance. The effects of different input parameters like feed pressure, feed flow rate, feed TDS on an output variables like recovery ratio, permeate TDS and salt rejection were studied. Water efficiency for different operating pressure were studied for reverse osmosis and the experimental data obtained was shown in the Table.2.

**Figure 1. Photographs of Reverse osmosis plant****Table: 2 Feed pressure, Feed flow rate on water recovery ratio**

S. NO.	Feed pressure (Kg/cm ²)	Feed flow rate(lpm)	Product flow rate (lpm)	Permeate TDS(ppm)	Water recovery ratio(%)	Salt rejection
1	4	121	16	9.18	13	0.972
2	5	110	20	8.93	18	0.9732
3	5.5	103	23	8.37	22	0.975
4	6.5	98	26	7.9	27	0.9762
5	7	94	30	7.81	32	0.9765
6	7.5	92	33	7.7	36	0.9768
7	8.5	86	35	8.1	41	0.9756
8	9	82	38	7.84	46	0.9764
9	10	76	41	8.7	54	0.9738
10	10.5	70	44	9.26	63	0.97219
11	11	69	46	9.7	67	0.9708

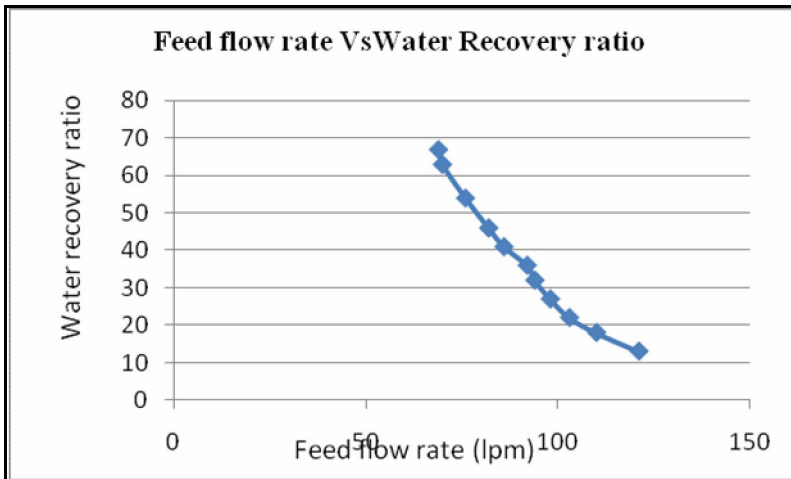


Fig.2 Feed flow rate Vs Water recovery ratio

Table : 3 Effect of operating time with permeate flux at feed pressure 9 Kg/cm2

S.NO	Time(min)	Product volume(l)	Flux(l/m ² hr)
1	1	23	21.13
2	2	52	25.49
3	3	79	23.23
4	4	112	27.45
5	5	132	24.26
6	6	158	23.23
7	7	186	23.60
8	8	214	24.20
9	9	242	23.72
10	10	270	24.81

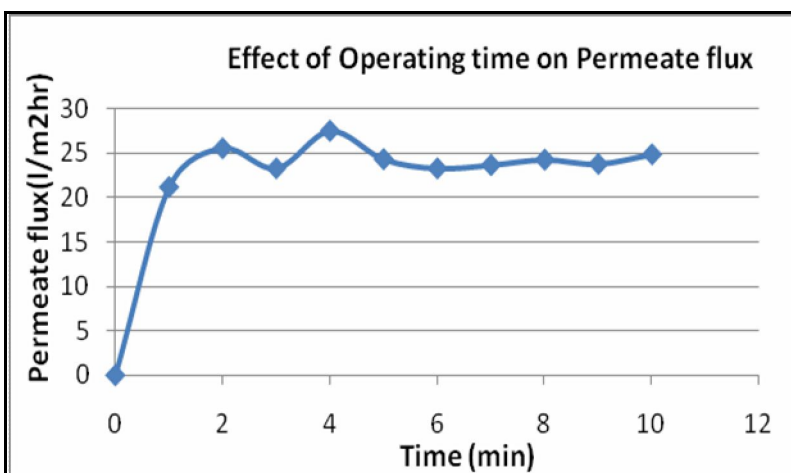


Fig . 3 Effect of operating time on Permeate flux at 9 Kg/cm2

3. Result and discussion

Experimental data for different feed pressure, feed flow rate and water recovery ratio was given in table.2. It was understood from figure. 2, as feed flow rate increases, sharp decrease in water recovery ratio. Experimental results for variation of flux at different intervals of time at optimum design feed pressure 9 kg/cm² was given in table.3 and increase of permeate flux on increase of time was shown in figure. 3.The

experimental results are in concordance with the literature³. Experimental calculation for permeate flux, water recovery ratio, salt passage and salt rejection was given in the following expressions, 1, 2, 3 and 4

Reverse osmosis performance was evaluated by measuring the permeate flux ⁹

$$\text{Permeate flux } J = \Delta V / AX \Delta t \quad \dots\dots\dots (1)$$

J is the permeate flux, 'V' is the volume of permeate water collected, 'A' is the area of membrane, 't' sampling time.

$$\text{Water recovery ratio} = \text{Permeate flow rate} / \text{Feed flow rate} \times 100 \quad \dots\dots\dots (2)$$

Salt rejection (R_s %) was measured by,

$$\text{Salt passage} = \frac{\text{TDS of permeate}}{\text{TDS of feed}} \quad \dots\dots\dots (3)$$

$$\text{Salt rejection } (R_s \%) = (1 - \text{salt passage}) \quad \dots\dots\dots (4)$$

3.1. Multiple linear regression

In Linear regression model, there is a single dependent variable or response 'y' that depends on k independent or regressor variables, x_1, x_2, \dots, x_k . The relationship between these variables is characterized by a mathematical model called a regression model.⁸ The regression model is fit to a set of sample data. If more than one regressor variable is present in the regression model, it is called as multiple regression model. It is given as, $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, where ' β_0 ' is a constant, ' β_1 ' is a regression coefficient and ' x_1 ' is an independent variable and ' β_2 ' is a regression coefficient and ' x_2 ' is an independent variable and ' ϵ ' is error.¹⁰ Here empirical model relates water recovery ratio to the input parameters like feed flow rate, feed pressure.

3.1.1. ANOVA

Linear multiple regression developed for water recovery ratio prediction was validated by analysis of variance. To study its significance the statistical estimators help in evaluating the goodness of the developed model, such as P-value, F-value, R^2 then the developed model was found to be highly significant. The importance of each factor to the response is indicated by its coefficient. The variable with the highest coefficient has greatest impact. Minitab output summary from an ANOVA study was given in table-4, using multiple regression analysis, first order regression model equation was found to be

$$Y = -60 + 9.56 \text{ Feed pressure (Kg/cm}^2\text{)} + 0.269 \text{ Feed flow rate (lpm)}$$

$R^2 = 0.98$, adj. $R^2 = 0.97$, validated by ANOVA test. Regression coefficients in the model equation shows the influence of each variable on the response, recovery ratio 'Y'. The regression variable which shows higher influence on the recovery ratio was found to be feed pressure ($X_1 = 9.56$) and lower impact on response was feed flow rate ($X_2 = 0.269$). $R^2 = 0.98$ shows the goodness of fit indicating the quality of regression and best model equation.

3.1.2. Model adequacy checking

Predicted regression model has to be checked by residual analysis for its adequacy^{11,13}.

3.1.3. Normal probability plot

In normal probability plot, residuals are normally distributed and form a straight line. If there is no departure of data from normality. Then the plot indicate no model inadequacy. Figure 4, shows the normal probability of the residuals.

3.1.4. Residual plots Versus fits

Residual plot should show a random pattern of residuals on both sides of 0. Error of data indicate if the residuals are distributed completely positive or negative side of the fitted line at the centre. Plot of residuals versus fits shown in figure. 4

3.1.5. Residual plots Versus Order

This plot is used to check any drift in the process for the observed experimental data. Plot of residuals versus order was shown in figure.4

3.1.6. Histogram

This plot shows the spread of data in the form of bar chart. If one bar is far from others then that point is considered to be outlier. Histogram bar chart shown in figure.4 respectively

Table : 4 Minitab Output for the water recovery ratio for regression model

Regression Equation is Water Recovery ratio = $-59.9 + 9.56 \text{ Feed pressure (Kg/cm}^2) + 0.269 \text{ Feed flow rate (lpm)}$

Analysis of Variance Summary

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	3181.33	1590.67	228.97	0.000
Feed pressure(Kg/cm ²)	1	70.20	70.20	10.11	0.013
Feed flow rate (lpm)	1	2.80	2.80	0.40	0.543
Error	8	55.58	6.95		
Total	10	3236.91			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.63571	98.28%	97.85%	94.68%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-59.9	61.6	-0.97	0.360	
Feed pressure(Kg/cm ²)	9.56	3.01	3.18	0.013	70.46
Feed flow rate (lpm)	0.269	0.424	0.64	0.543	70.46

3.1.7. Interaction plot of main effects:

Figure .5 shows the interaction plot of main effects of independent variables. Response increases as feed pressure increases. Decline of feed flow rate on higher response was shown by main effect plot of feed flow rate.

3.1.8. Contour plot:

The contour plot graphical representation of regression analysis was shown in figure.6, in order to visualise the response with experimental input variables. Contour plot exhibits 3- D relationship in 2 dimensions where x and y predictors plotted on x and y scales and response values represented by contour lines. As colour gets darker, response increases. Darker regions identify higher response values. At feed pressure 11 kg/cm², feed flow rate about 69-70 lpm, the response region shown in the range of 60-75 which is dark green in colour indicates higher response.

3.1.9. Response surface plot

It is used to explore the relationship between 3 variables (i-e) 2 predictor variables are on x and y-axis, response is represented by a smooth 3 D surface plot or 3 D wireframe plot.

It is used to examine the interaction of independent variables and to find the optimum value for each variable for high response value. Response plot for feed pressure, feed flow rate on water recovery ratio was shown in figure.7. Response plot gives clear concept than contour plot. Response surface characterises as plane since it is first order multiple regression model, response depends on feed pressure than feed flow rate.

3.2. Optimization using desirability function

Overlaid contour plot was showed in figure.8. This plot emphasise on a constrained region where all the responses are feasible within that region. Last step of the statistical regression analysis is to obtain best condition for the entire process. For this desirability function was employed. Response optimization plot was shown in figure.9. Optimization plot predicts optimum response as 77.9 when the desirability factor is 1.

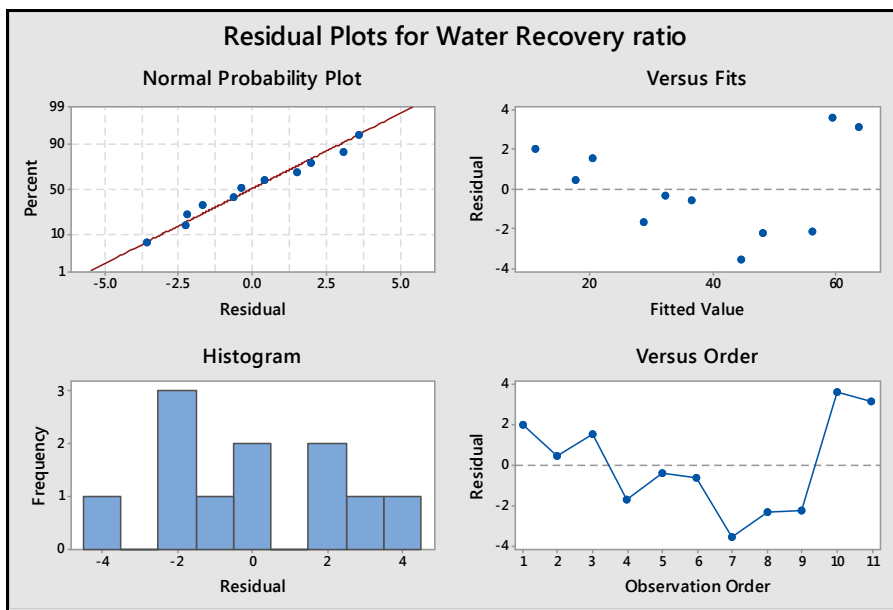


Figure .4 Residual plots (Ref. 12)

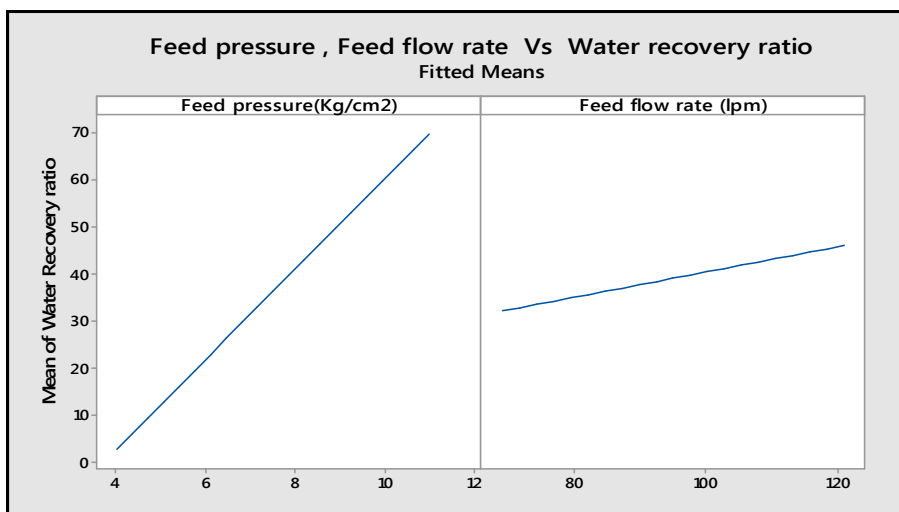


Figure . 5 Interaction plot of main effects (Ref: 12)

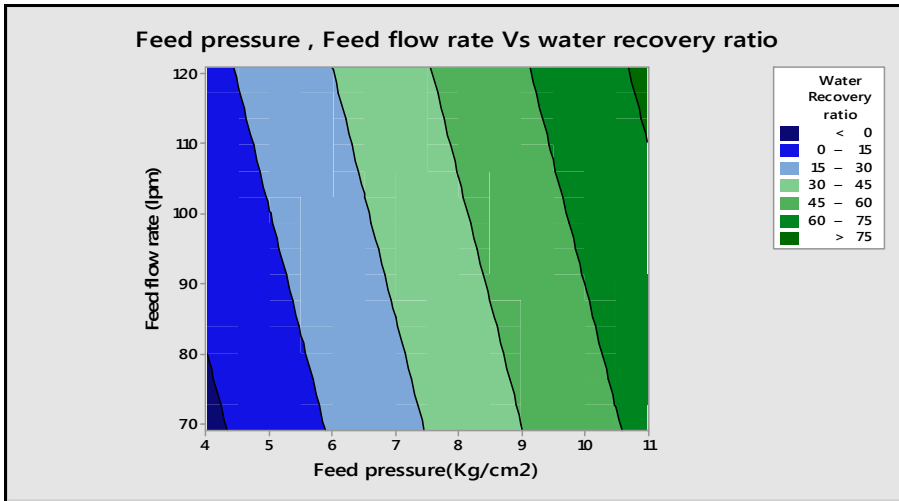


Figure. 6 Contour plot (Ref: 12)

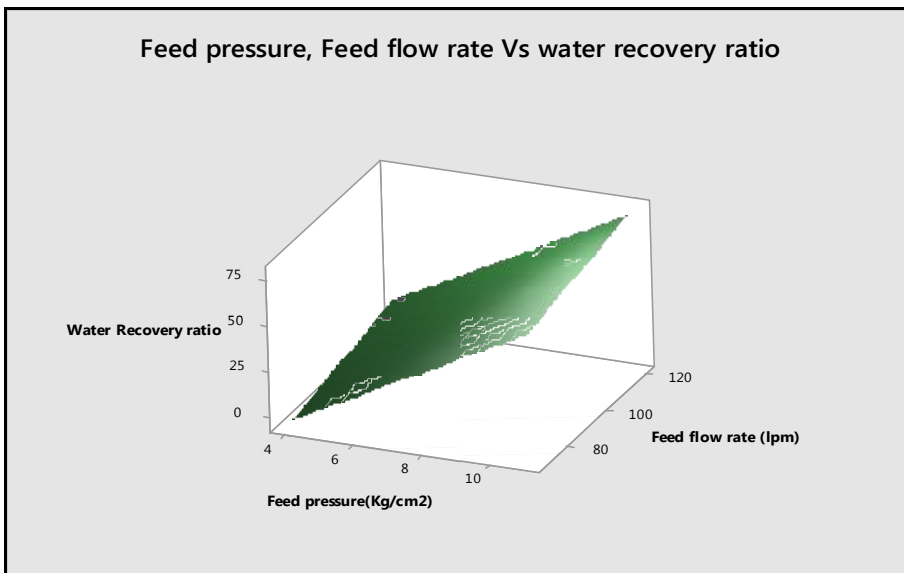


Figure .7 Response surface plot (Ref: 12)

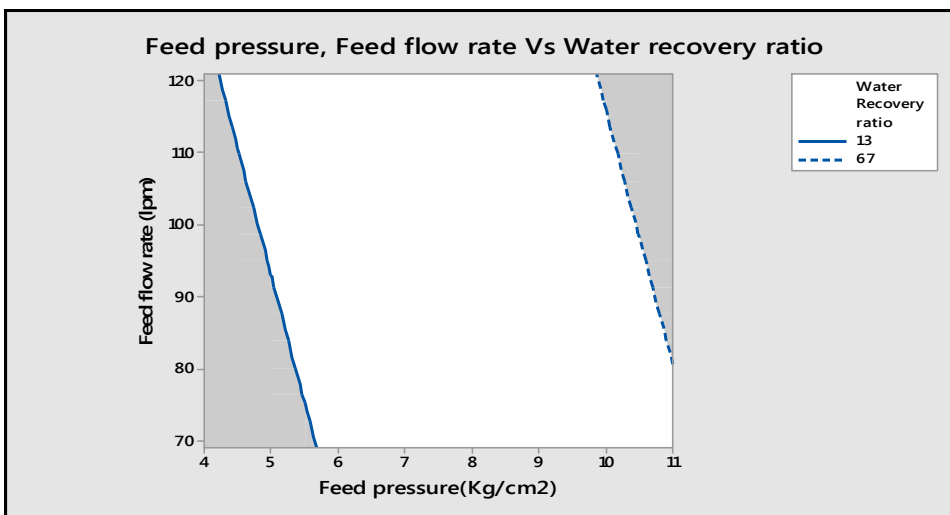


Figure . 8 Overlaid contour plot (Ref:12)

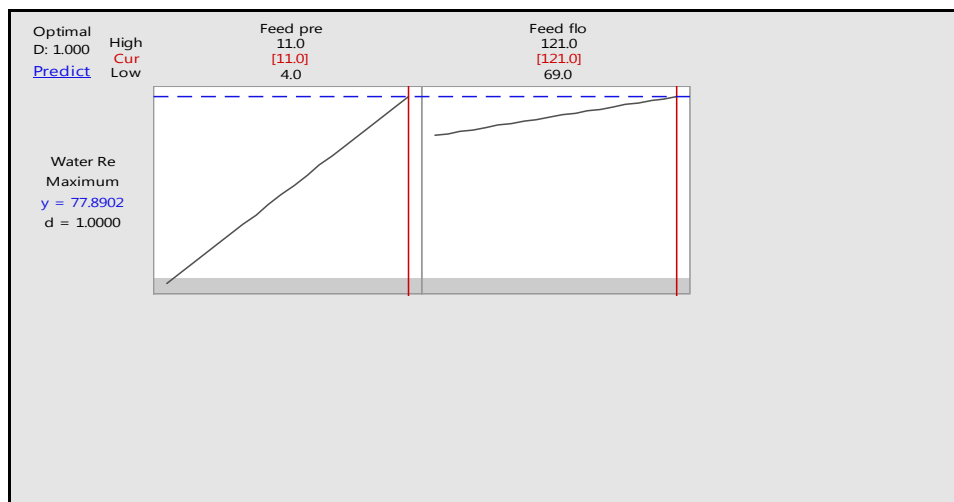


Figure .9 Response optimization plot(Ref: 12)

4. Conclusion

Multiple regression model fitting was done for the objective functions like feed pressure, feed flow rate on water recovery ratio for reverse osmosis using statistical software Minitab 17. By model adequacy checking, developed regression model was checked for accuracy using residual plots and normal probability plot. ANOVA estimates the model to be highly significant confirming by $R^2 = 0.98$, for regression constant coefficient P-Value -0.000, F-Value-228.9, which was found to be very high. Experimental response were compared with the predicted values of response of developed regression model equation and the results represents 1% deviation between predicted and experimental response. Multiple regression for water recovery ratio, feed pressure and product flow rate was also done. Regression model equation was found to be

$Y = -20.7 + 4.414 \text{ feed pressure (Kg/cm}^2) + 0.0125 \text{ product flow rate (lpm)}$. ANOVA estimates the model to be highly significant confirming $R^2 = 0.98$, for regression constant coefficient P-value - 0.000, F- 229.97 which was found to be very high.

Similarly other operating parameters can be performed to know the impact on the response and optimization study on design of experiments will always save cost and valuable time for any chemical process. Regression model can also be used in design of experiments in most of the chemical Engineering process.

Fouling of membrane in reverse osmosis leads to poor performance of product quality and decrease of flux. Maintenance of sand filter on regular basis is foremost important. By the application of antiscalants specifically for silica can be recommended. EDTA and soda solution can be applied to prevent salts and organic materials, citric acid can be used to remove metallic oxides. Thus statistical study on regression analysis is a common method to correlate the input variables on response and the influence of different independent variable on dependent variable for good performance of any process.

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References

1. Selvi.SR, Baskaran.R. Desalination of Well water by Solar power membrane distillation and reverse osmosis and its efficiency analysis. International Journal of ChemTech Research.2014;6(5):2628-2636.

2. Lilian Malaeb, George M. Ayoub. Reverse osmosis technology for water treatment: State of the art review. *Desalination* 2011; 267 : 1-8.
3. Al Ahamed.M.Prediction of performance of sea water reverse osmosis units. *Desalination*,2010; 261:131-137
4. David A. Ladner ,Arun Subramanian ,Manish Kumar, Samer S. Adham, Mark M. Clark. Bench-scale evaluation of seawater desalination by reverse osmosis. *Desalination*,2010; 250: 490-499
5. Pedhazur, ED. Multiple Regression in Behavioural Research: Explanation and Prediction, The Dryden Press, 2nd edition. ,1982, 720-721.
6. Sami Maalouf ,Diego Rosso ,William W.-G. Yeh. Optimal planning and design of seawater RO brine outfalls under environmental uncertainty, *Desalination*,2014;333: 134-145
7. Kaushik Nath , Membration separation processes, PHI publications.,2008, 58.
8. Selvi S.R, Baskaran.R. Application of regression modelling techniques in desalination of sea water by Membrane distillation. *International journal of Engineering science & technology*, 2015;7(8): 267- 278.
9. Abdel-hameed Mostafa A. El-Aassar. Polyamide Thin Film Composite Membranes Using Interfacial Polymerization: Synthesis, Characterization and Reverse Osmosis Performance for Water Desalination. *Australian Journal of Basic and Applied Sciences*, 2012;6(6):382-391
10. Douglas G. Montgomery, George .C.Runger, *Applied statistics and probability for Engineers*, John Wiley & sons,3rd edition, 2004, 411
11. Venkatesan.G, Kulasekharan.N, MuthukumarV ,IniyanS. Regression analysis of a curved vane demister with Taguchi based optimization. *Desalination* 2015; 370: 33-43
12. Minitab release 17, Minitab Inc ., Pennsylvania, USA.
13. Douglas Montgomery, *Design and analysis of experiments*, John Wiley & Sons , 8th edition, 2013,450
